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13. ABSTRACT (Maximum 200 words)

This ONR grant was to facilitate the meeting of an international group of researchers to examine the future of structural dynamics. A group of about 40 researchers (see Appendix 1) was brought together from industry, universities and government labs, incorporating several age perspectives. The hope of the group was to help point the way forward for structural dynamicists. Perhaps the greatest wisdom in this regard came from our most experienced member, Dr. Stephen Crandall. He pointed out that during World War II, he came in contact with the newly-formed discipline of automatic control and promptly dismissed it as an intellectual pursuit not useful in structural dynamics. In reflecting back over his career, he now views this as a mistake. We find ourselves exactly in this same situation. New technologies are emerging at a rapid pace (computing, communications, micro-mechanical devices, smart materials, mechatronics, etc.) and determining the appropriate interaction between these new advances and structural dynamics was one of the major concerns addressed by the forum. In addition, significant effort was spent assessing the current state of the art, examining developing technologies, deciding ways to improve the image of structural dynamics and formulating "grand challenge" problems for structural dynamics.

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## **FINAL REPORT ON SD2000 MEETING**

This ONR grant was to facilitate the meeting of an international group of researchers to examine the future of structural dynamics. A group of about 40 researchers (see Appendix 1) was brought together from industry, universities and government labs, incorporating several age perspectives. The hope of the group was to help point the way forward for structural dynamicists. Perhaps the greatest wisdom in this regard came from our most experienced member, Dr. Stephen Crandall. He pointed out that during World War II, he came in contact with the newly-formed discipline of automatic control and promptly dismissed it as an intellectual pursuit not useful in structural dynamics. In reflecting back over his career, he now views this as a mistake. We find ourselves exactly in this same situation. New technologies are emerging at a rapid pace (computing, communications, micro-mechanical devices, smart materials, mechatronics, etc.) and determining the appropriate interaction between these new advances and structural dynamics was one of the major concerns addressed by the forum. In addition, significant effort was spent assessing the current state of the art, examining developing technologies, deciding ways to improve the image of structural dynamics and formulating "grand challenge" problems for structural dynamics.

### **Summary of Objectives and Goals**

It is useful at this point to recall our objectives and goals. The basic objectives of the original program were to:

- Determine future directions of Structural Dynamics;
- Define/develop a strategic plan to advance the discipline;
- Overview the state-of-the-art in structural dynamics.

These broad and simply-stated objectives were expanded and more fully developed by the group as the week unfolded. The very nature of the forum dictated a fluid and open manner of determining the best path forward for the structural dynamics community. To further lend direction to the forum, the following goals were outlined:

- Determine what Structural Dynamics is expected to do for the engineering community;

- Determine how effective Structural Dynamics is in meeting these expectations;
- Determine what Structural Dynamics should do differently.

In addressing these objectives and goals, deliberation naturally became divided into three areas:

Developing technologies,  
Emerging technologies,  
The image and culture of Structural Dynamics.

Here, developing technologies refers to taking the current state-of-the-art forward by examining what can be done in the next five years in the context of the stated goals. As an example, consider the field of model updating, which proposes to develop systematic means to fine tune analytical models to bring them in line with measured data in one form or another. This area has been developing over the last 15 years, yet needs several accomplishments before becoming a mature and trusted discipline. One such identified need was to provide and develop measures of model integrity by defining “what makes a good model”. The topic of emerging technologies refers to advances in technology that have a potentially large impact on the nature and practice of structural dynamics. Some examples of emerging technologies currently are smart materials, MEMS and, in some sense, digital computing. The image and culture of structural dynamics refers to our perception in all communities including the public sector, students, other engineers, politicians, etc. Although the concept of addressing the culture and image was not an original agenda item, it rapidly surfaced as an important consideration during the meeting.

### **Expectations**

In assessing where to go next in structural dynamics, it is important to consider what is expected of structural dynamics. In many circumstances, the structural dynamics community is expected to provide failure-free host designs in lighter, smaller and more durable formats. As efficiencies increase, and mass and size limits become reduced,

more is expected from the structural design. This single expectation is a strong driver for many of the problems identified in the forum. In addition, structural dynamics is expected to provide models suitable for design that are computationally friendly, and predictive in a variety of circumstances. This expectation is often at odds with itself in the sense that the high-fidelity, predictive models are often too large and complicated to be of any use in design. Furthermore, the structural dynamics community is expected to provide understandable models and to educate students and practicing engineers in simplified ways that allow these models to be communicated to and understood by people with limited technical backgrounds.

### **Effectiveness**

In assessing how effective the community has been in meeting these expectations, it is clear that structural dynamics is very effective for low-frequency, linear, deterministic, low-order structures in nice environments. It is equally clear that the current state of practice in structural dynamics is not very effective at nonlinear, stochastic, mid-frequency, mixed-field problems. Our way forward was greatly influenced by the group's perception of what we have failed to be effective at. The following is a brief summary that emerged from the group of important future directions in structural dynamics, organized according to the needs in each of the three categories: developing technologies, emerging technologies, and image.

### **Developing Technologies**

The needs going forward for developing technologies center strongly on modeling issues. In particular, guidelines for procedures in testing and in analytical modeling are needed. An acceptance criterion for assessment of model validity in the presence of uncertainties is needed in order to formulate these guidelines.

Particularly difficult modeling tasks are those that involve joints and interfaces between components. Joints and interfaces are a significant source of both non-linearity and uncertainty. Better physics-based models of joints and interfaces are required that are presented in ways compatible with practice. In addition, new modeling techniques and procedural guidelines should be defined, verified and fine-tuned by a series of round-



round-robin modeling error exercises. Once these techniques are in place, the community needs to address methods for learning/teaching best practice procedures. Round-robin modeling exercises not only serve to focus attention on problems, but have the potential to encourage industries, universities and government labs to work together towards a common goal, increase communication and lead naturally to best practice procedures.

Other modeling issues focused on damping mechanisms other than those dominated by joints, mid-frequency structures, and general nonlinearity. It was clear throughout the discussions that structural dynamics needs to focus on developing non-linear capabilities from modeling through analysis. In particular, the non-linear capabilities we now have are not well-integrated into the educational system and are not perceived as simple enough for wide industrial acceptance. A second area of concern is in modeling and using uncertainty. Uncertainty modeling has found its way into practice in a number of areas (testing, fatigue), but in general is not firmly in place in the educational arena nor is it widely used in structural dynamics practice.

### **Emerging Technologies**

The needs going forward for emerging technologies focused on education. Emerging technologies will, of course, change over time. Current examples of emerging technologies are MEMS, CFD, Smart Materials, Mechatronics, Computing etc. Integrating into these new fields requires education at a variety of levels. Focus should be on identifying the physical models needed, and how to interface these models into structural dynamics. In addition, it is critical to integrate these new technologies into industrial practice. The combination of structural dynamics with new technologies has all the elements of defining a new engineering discipline within the expectations placed on structural dynamics solutions. It is clear that the integration of new technologies into structural dynamics will require changes in education, continuing education and greater cooperation between structural dynamicists and engineers and scientists from other fields.

There is an emerging trend to refer to the integration of new technologies into structural dynamics as "Multi-functional Structures". This phrase and point of view has emerged from aerospace considerations in trying to produce faster, cheaper better systems by integrating functions, such as antennas, into load-bearing components.

A major hindrance to integrating new and emerging technologies into structural dynamics is the lack of interdisciplinary studies into the mainstream education of dynamicists. Emerging technologies require knowledge of concepts ranging from solid-state physics to aerodynamics. This task is intended to encourage and promote the inclusion of courses in electrical components, solid-state physics, micro processors, active materials, micro techniques, computing (distributed, real time, etc). In addition, the more traditional related courses in aerodynamics, acoustics, thermal dynamics and design are encouraged. Alternately, the creation of specialized courses in MEMS, Smart Structures, and/or Mechatronics is encouraged. Hence, there is a strong need to use graduate education to effect the integration of emerging technologies and structural dynamics. Colleagues are encouraged to train their students at the graduate levels in these disciplines and to promote a broader view of structural dynamics, being keen to watch for new technologies as they become available.

### **Image**

Of great future importance is the need to improve the image and culture of the profession of structural dynamics. Peers, practicing engineers and students need an improved image of the culture and abilities of the structural dynamics discipline to attract the best students. Our image should be expanded by interfacing with other markets and disciplines. The general public needs to be made more aware of our successes. We need to define how to interact with the emerging technologies and inform these groups about what Structural Dynamics has to offer and why they should care about interfacing with us.

### **Summary of Findings**

In summary, the deliberations produced nine key issues to be examined in the future.

These are:

- Improved physical modeling;
- Impact of improved computing;
- Determining model limitations (e.g., linear vs. nonlinear, stochastic characterizations);

- Improvement in non-linear techniques and, in particular, folding non-linear analysis into practice;
- Mid- to high-frequency modeling;
- Structural Dynamics keyed to industrial design and maintenance (a current failure);
- Finding new frontiers for Structural Dynamics;
- Enhancing the image of Structural Dynamics.

Perhaps these problems should now be addressed on a global rather than a national level. It is clear that this set of issues crosses all boundaries: international and institutional. The final chapter of the way forward is not yet written, and will unfold only by continued self-examination and reflection as provided by forums such as this.

A copy of the proceedings of the forum is attached. A monograph will be published in the summer of 2000 based on the presentations made at the workshop.

### **SUPPORT**

The workshop was partially supported by ONR Grant No. N00014-99-1-0703, which was intended to focus on the summary activities. The travel support, subsistence and conference fees for the entire group was paid for (about \$75,000) by the Engineering Science Division of Los Alamos National Laboratories, who also hosted the event at their facility in New Mexico.

APPENDIX 1: Attendees List (p. 1)

Structural Dynamics 2000 Forum  
April 11-16, 1999  
Los Alamos, NM

Name	Institution	EMAIL	FAX	Mail Address	Phone
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Nayfeh, Dr Ali	Virginia Tech	anayfeh@vt.edu	540-231-2290	Virginia Tech, Engineering Science & Mechanics, Blacksburg, VA 24061	540-231-5453

**Structural Dynamics 2000 Forum**  
**April 11-16, 1999**  
**Los Alamos, NM**

**APPENDIX 1: Attendees List (p. 2)**

Name	Institution	EMAIL	FAX	Mail Address	Phone
Ohayon, Professor Roger	CNAM Struc Mech and Coupled Sys Lab	ohayon@cnam.fr	33 (1) 40 27 27 16	Prof, Ch of Mech, Structuram Mech & Coupled Systems Lab, Conservatoire National des Arts et Metiers, 2 rue Conte, 75003 Paris France	33 (1) 40 27 24 47
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Zimmerman, Professor David	University of Houston	dzimmerman@uh.edu	713-743-4503	University of Houston, Dept. Mech Engineering, Houston, TX 77204-4792	713-743-4520

***STRUCTURAL DYNAMICS 2000***

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April 11-16, 1999  
Los Alamos National Laboratory  
Los Alamos, NM 87545

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- **Exciting Vibrations: A Philosophical Approach to Resolving Structural Dynamics Problems**  
Prof. David Ewins, Imperial College, UK
- **Damping Description in Structural Dynamics**  
Prof. Lothar Gaul, University of Stuttgart, Germany
- **Non-Structural Dynamics**  
Prof. J.K. Hammond, University of Southampton, UK
- **Multifunctional Structures of the Next Millennium**  
Prof. Daniel J. Inman, Virginia Polytechnic Institute and State University, USA
- **Warping the time axes**  
Dr. Havard Vold, Vold Solutions, USA

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Dr. Charles R. Farrar, Los Alamos National Laboratory, USA
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Prof. Michael Link, University of Kassel, Germany
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Dr. Thomas L. Paez, Sandia National Laboratories, USA
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Prof. Bob Randall, University of New South Wales, Australia
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Prof. G R Tomlinson, University of Sheffield, UK

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- Structural Damage Identification
- Structural Minimisation of Noise Emission
- Planet Earth Seismic Array
- Micro Electro Mechanical Systems

### ***Questions (Groups C and D)***

### ***Brief Biographies of Participants***



## Schedule of Events

### Sunday 11 April

1730- : Reception followed by Dinner

2000- : Speakers on the history of Los Alamos: Dr. W. Bruce Masse, LANL Archaeologist

### Monday 12 April

0730-0830: Breakfast

0830-0900: Opening Remarks

0900-1230: **Session 1: Presentations**

1300-1600: Lunch and free time

1600-1900: **Session 2: Debate Planning**

1930 - After Dinner Speaker: Dr. Steve Younger (LANL)

### Tuesday 13 April

0730-0830: Breakfast

0830-1230: **Session 3: Presentations**

1300-1600: lunch and free time

1600-1900: **Session 4: Debate**

1930 .... After Dinner Speaker: Admiral Steidle, United States Navy

### Wednesday 14 April

0730-0830: Breakfast

0830-1130: **Session 5: Presentations**

1130-1230: Plenary discussion – review of first half: plans for second half

1300 - Lunch, visit to Santa Fe, Dinner -- no speaker!

## Thursday 15 April

0730-0830: Breakfast

0830-1230: **Session 6: Debate**

1300-1500: Lunch and Free time

1500-1600: Talk on large scale computing: Dr. Andy White (LANL)

1600-1900: **Session 7: Grand Challenge / Strategy Document**

1930- After Dinner Speaker: Dr. George Zweig, LANL

## Friday 16 April

0730-0830: Breakfast

0830-1230: **Session 8: Debate**

1300-1600: Lunch and free time

1600-1900: **Session 9: Closing Remarks and Discussion**

1930- After Dinner Speaker: Prof Stephen Crandall (MIT)

# Technical Program

## **Presentations**

*Monday 0900-1230*

**Session 1**

**Hammond  
Richardson  
Nayfeh**

**Vold  
Tomlinson  
Farrar**

*Tuesday 0830-1230*

**Session 3**

**Sestieri  
Bergman  
Brown**

**Gaul  
Hunter**

**Kirmejian  
Inman**

*Wednesday 0830-1130*

**Session 5**

**Schweitzer  
Paez  
Link**

**Ohayon  
Randall  
Ewins**

## **Debate Planning - Schedule for Session 2**

- 15m introduction, explanation of rules (PLENARY)
- 30m (6 x 5m) presentations by each C member of their proposal for a Theme (based on own or others questions) (3 GROUPS)
- 90m Debate to pick 3 Themes for Debate on Days 2-5 (3 GROUPS)
- 30m (3 x 10m) reviews from each Team of suggestions for 3 themes (PLENARY)
- 15m Decision of Themes for Debates (PLENARY)

## **Debate – Schedule for each of 3 debates Sessions 4, 6 and 8**

- 15 m review plan for debate. Definition of theme. Objectives of debate (PLENARY)
- 90m Team debate of key issues of Theme, leading to specification of key issues with a view to putting on list of things that must be done in the coming years (3 GROUPS)
- 45m (3 x 15) Summaries from teams of Major features of Theme (PLENARY)
- 15m Conclusion (PLENARY)

## **Session 7 What to Produce as a record of the Forum?**

- 45m Presentation of Grand Challenges (PLENARY)
- 90m Drafting Strategy Document Structure (Chapter headings) (3 GROUPS)
- 45m Review of Strategy Document headings (PLENARY)

## **Session 9 What to do in the future to benefit from the Forum?**

- 30m Decision on Grand Challenge
- Final discussion Strategy Document
- Comments from Participants about the Forum
- Follow up Actions (How to benefit from the Forum?)

## TEAMS for Group Activities

### Team 1

Cudney  
Avitavle  
Cogan  
Friswell  
Martinez  
Yang

Vold  
Schweitzer  
Paez  
Link  
Gaul  
Crandall

### Team 2

Hemez  
Balmes  
Cooper  
Imregun  
Pickrel  
Zimmerman

Tomlinson  
Richardson  
Ohayon  
Kermidjian  
Brown  
Mote

### Team 3

Doebling  
Bucher  
Davies  
Lieven  
Shaw  
Caeser

Sestieri  
Randall  
Nayfeh  
Hammond  
Bergman  
Hunter

## Participant List

NAME	Affiliation	Country	Group	Specialization (Group B Only)
<u>Avitabile, Dr Peter</u>	Univ. Lowell, Mass	USA	C	
<u>Balmes, Dr. Etienne</u>	Ecole Centrale de Paris	France	C	
<u>Bergman, Prof Larry</u>	Dept. Aeronautics, Univ, Illinois	USA	A	
<u>Brown, Prof Dave</u>	University of Cincinnati	USA	B	<u>Experimental Modal Analysis</u>
<u>Bucher, Dr Izak</u>	Technion, Insitiute of Techn. Israel	Israel	C	
<u>Caeser, Dr Bernd</u>	Daimler Bnz Aerospace	Germany	D	
<u>Coqan, Dr Scott</u>	University of Besancon	France	C	
<u>Cooper, Dr Jonathon</u>	University of Manchester	UK	C	
<u>Crandall, Prof. Stephen</u>	Massachusetts Institute of Technology	USA	A	
<u>Cudney, Prof Harley</u>	Virginia Polytechnic Institute	USA	C	
<u>Davies, Dr Patricia</u>	Purdue University	USA	C	
<u>Doebling, Dr Scott</u>	Los Alamos National Lab	USA	C	
<u>Ewins, Prof David</u>	Imperial College	UK	A	
<u>Farrar, Dr Chuck</u>	Los Alamos National Lab	USA	B	<u>Damage Detection</u>
<u>Friswell, Dr Michael</u>	Univ. of Wales, Swansea	UK	C	
<u>Gaul, Prof Lothar</u>	Inst. Mechanics, Univ. Stuttgart	Germany	A	
<u>Hammond, Prof Joe</u>	ISVR, Univ. Southampton	UK	A	
<u>Hemez, Dr. Francois</u>	Los Alamos National Lab	USA	C	
<u>Hunter, Mr Norm</u>	Los Alamos National Lab	USA	B	<u>Environmental Testing</u>
<u>Imregun, Dr Mehmet</u>	Imperial College	UK	C	
<u>Inman, Prof Dan</u>	Virginia Polytechnic	USA	A	
<u>Kiremidjian, Prof Anne</u>	Stanford University	USA	B	<u>Earthquake Engineering</u>

<u>Lieven, Dr Nick</u>	University of Bristol	UK	C	
<u>Link, Prof Michael</u>	University of Kassel	Germany	B	<u>Model Updating</u>
<u>Martinez, Dr. David</u>	Sandia	USA	D	
<u>Mote, Prof C Dan</u>	University of Maryland	USA	A	
<u>Nayfeh, Dr Ali</u>	Dept. ESM, Virginia Tech.	USA	B	<u>Nonlinear S. Dynamics</u>
<u>Ohayon, Prof Roger</u>	CNAM	France	A	
<u>Paez, Dr Tom</u>	Experimental Mech Dept. Sandia Lab	USA	B	<u>Random Vibration</u>
<u>Pickrel, Mr Charlie</u>	Boeing	USA	D	
<u>Randall, Dr Bob</u>	University of New South Wales	Australia	B	<u>Signal Processing</u>
<u>Richardson, Dr Mark</u>	Vibrant Technology	USA	B	<u>Structural Dynamic Measurements</u>
<u>Schweitzer, Prof Gerhard</u>	ETH, Zurich	Switzerland	A	<u>Mechatronics</u>
<u>Sestieri, Prof Aldo</u>	University Rome	Italy	B	<u>Vibroacoustics</u>
<u>Shaw, Prof Stephen</u>	Mech Eng. Dept, Michigan State Univ.	USA	C	
<u>Tomlinson, Prof Geof</u>	Dept. Mechanics, Univ. Sheffield	UK	B	<u>Damping</u>
<u>Vold, Dr Havard</u>	Vold Solutions	USA	A	
<u>Yang, Dr B</u>	Univ. of Southern California	USA	C	
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# Uncertainty Modeling in Dynamical Systems: A Perspective

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I suspect that all of us here have, at one time or another, been vexed by performance that quantitatively failed to match our analytical predictions. Despite our best modeling efforts and careful manufacture and assembly, we find ourselves cast into the position of reconciling our analyses in an attempt to bring the predicted behavior of our systems into line with observations. This apparently occurs with sufficient regularity to warrant giving the activity a unique identity in the literature: model updating, or some variation thereof.

Having found myself in precisely that position more times than I really care to admit to over the course of a career that spans more than thirty years, I've had plenty of time to think about these problems and have decided to use this opportunity to offer some insights into how we, as a community, might improve our capability to recognize and accommodate uncertainty in vibrating systems.

The modeling of uncertainty and its effects upon the response of dynamical systems of engineering interest has been of concern for many years. Professor Stephen Crandall coined the term "random vibration" more than 40 years ago, though the analysis of noise-driven systems was certainly a underway decades earlier (see, for example, the work of S. O. Rice, 1944-45), particularly in the field of what was then called "radio engineering".

In structural dynamics, we hold the view that uncertainty enters the typical system in at least one of three ways: through the input to the system, the coefficients of the differential equations governing the evolution of the system, and the initial and, possibly, boundary conditions. However uncertainties arise, their effect on system response and reliability can be profound.

One can envision a number ways in which uncertainty can be included in the analysis and design of a dynamical system, the "worst-case" and probabilistic approaches being two that have gained acceptance within the community. In the former, one examines some response of the system to a particular class of inputs or over a range of parameters and identifies the extreme case;

while in the latter, some response of the system to a class of inputs represented by a stochastic process of known probability distribution, is itself a stochastic process defined in terms of a probability distribution, from which statistics can be determined. As the notion of uncertainty is most familiar in the context of control, perhaps it will be instructive to couch the following discussion in that language, in particular as applied to structural control.

Model uncertainty, if ignored, can seriously degrade the performance of an otherwise well-designed control system. If the level of uncertainty is extreme, the system may even be driven to instability. In the context of structural control, performance degradation and instability imply excessive vibration and structural failure. Robust control has typically been applied to the issue of model uncertainty through worst-case analyses. These traditional methods include the use of the structured singular value, as applied to the small gain condition, to provide estimates of controller robustness (Doyle *et al.*, 1991). However, this emphasis on the worst-case scenario is without a probabilistic basis, from which a much more intuitive insight into controller robustness can be gained. A number of methods have been devised through which the robustness problem can be viewed in the context of probability, including Monte Carlo simulation, first and second order reliability methods adapted directly from system reliability resulting in the determination of a "stability index", and others based upon the use of  $\mu$ -synthesis procedures. A brief description of the problem follows.

The distribution of poles in uncertain dynamical systems and its relationship to the robustness issue have been topics of some interest in recent years. Stengel and Ray (1991) were among the first to use large-scale Monte Carlo simulation to estimate the robustness of uncertain controlled structural systems. Using this approach, one constructs a distribution of root loci simulating the stochastic behavior of the closed-loop pole locations. Because this is a graphical method, one gains an intuitive understanding of system robustness. The results reported were quite promising, but the large number of realizations required to attain a high degree of accuracy in the distribution of the tails of the closed-loop poles makes this approach computationally unattractive.

Spencer *et al.* (1992,1994) introduced a systematic approach for determining the probability that instability will result from the uncertainties inherently present in a controlled structure. This probability measure is a direct indication of the robustness of the closed-loop system. As

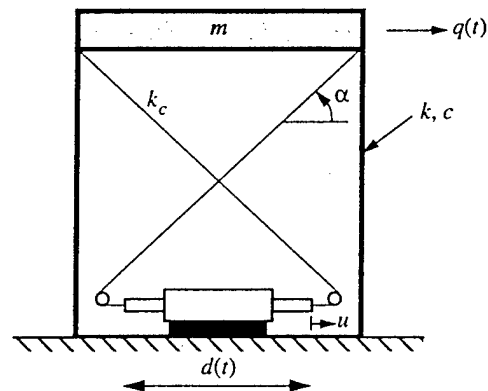


described by Spencer, this investigation into the probability of failure of controlled structures led to a method for characterizing the stability of the system based upon an eigenvalue criterion, namely the probability that the real part of every eigenvalue will be contained strictly in the left-half plane. First and second-order reliability methods (FORM/SORM), known to be accurate for series-type system reliability problems, were used for estimating the probability of system instability, and a series of numerical examples were constructed in which the stability of a controlled single degree-of-freedom system with four uncertain parameters was analyzed.

Traditional methods used to assess controller robustness, involving the use of the singular value of some mapping as applied to a small gain condition, are often conservative in nature. Therefore, most recent efforts involved comparing robustness estimates using the FORM methodology with those estimates obtained using the more traditional techniques. Field *et al.* (1996a,b) introduced an approach to adapt the robustness measure  $\mu$  gained from the structured singular value analysis into a probabilistic framework.

We thus have three methods that can be compared: Monte Carlo simulation, FORM, and probabilistic  $\mu$ ; and we can evaluate their comparative performance by application to the active control of a structure subjected to, for example, a seismic ground motion. In order to provide more realism, we can also consider the effects of controller time delay, which can be a significant contributor to system instability. Utilizing the three methods of assessment, we can also characterize the robustness qualities of several control law designs and conclude which will be most effective for this particular class of problems.

Figure 1 illustrates the SDOF model, as first reported by Chung *et al.* (1988). The control is



**Fig. 1 Single degree-of-freedom structure with active tendon control.**

applied to the structure using prestressed active tendons connected to a servocontrolled hydraulic actuator. The equation of motion of this system is given by

$$m\ddot{q}(t) + c\dot{q}(t) + kq(t) = -(4k_c \cos \alpha)u(t - \tau) + md(t), \quad (1)$$

where  $m$ ,  $c$ , and  $k$  are the mass, damping, and stiffness values, respectively, of the SDOF structure and  $\ddot{q}$ ,  $\dot{q}$ , and  $q$  are the acceleration, velocity, and position, respectively, associated with the rigid floor of the structure. Additionally,  $k_c$  and  $\alpha$  are deterministic parameters associated with the structure of the controller,  $u$  is the position of the controlling actuator, and  $d(t)$  represents the acceleration of the ground. In addition, when utilizing output feedback, the measurement signal is corrupted by noise

$$y(t) = \dot{q}(t) + n(t). \quad (2)$$

The parameter means are presented in Table 1.

Parameter	Mean, $m$
$m$ (lb-s <sup>2</sup> /in)	16.69
$c$ (lb-s/in)	9.02
$k$ (lb/in)	7,934
$\tau$ (ms)	20
$k_c$ (lb/in)	2,124
$\alpha$ (degrees)	36

**Table 1: Statistical Means of the Random Parameters**

The optimal controller for this problem is the Linear Quadratic Regulator; *i.e.*, a state feedback gain matrix,  $\mathbf{K}$ , that is obtained from the solution of a standard algebraic Riccati equation. Note that this controller is the same one that is obtained for the more usual (deterministic) system, where  $d(t) = \mathbf{0}$  and the objective is to minimize

$$J = \int_0^\infty [\mathbf{x}^T(\tau)\mathbf{Q}\mathbf{x}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\mathbf{u}(\tau)]d\tau, \quad (3)$$

for some initial defined state, *i.e.*,  $\mathbf{x}(0) = \mathbf{x}_0$ . We'll call this design the *nominal LQR* control. In

the presence of time delay, a simple modification of the optimal LQR gains can be introduced to account for any phase additions introduced by the delay. The corresponding solution is simply a revised gain set attempting to correct for phase additions due to controller delay and is termed the *phase-corrected LQR* control (LQRPC).

Usually in structural systems, state information is unavailable for direct measurement. In addition, measurement noise signals are present and tend to degrade the performance of the closed-loop. Modeling these added terms provides a much more realistic control problem. Here we'll consider two optimal output feedback design methods for the nominal system. The notion of optimality should be related to the particular assumptions on the system inputs as well as the cost objectives.

The first is the  $H_2$  (or LQG) optimal design, where it is assumed that both  $d(t)$  and  $n(t)$  are unit intensity, uncorrelated white noise processes. The objective is to minimize the steady-state variance of the output

$$\lim_{T_f \rightarrow \infty} E \left[ \frac{1}{T_f} \int_0^{T_f} z^T(\tau) z(\tau) d\tau \right]^{\frac{1}{2}} = \|w \rightarrow z\|_2. \quad (4)$$

The second is the  $H_\infty$  optimal design, where it is assumed that both  $d(t)$  and  $n(t)$  are energy bounded signals. The design goal is to minimize the “worst case” amplification of the energy of the output

$$\sup_{w \neq 0} \frac{\left[ \int_0^\infty z^T(\tau) z(\tau) d\tau \right]^{\frac{1}{2}}}{\left[ \int_0^\infty w^T(\tau) w(\tau) d\tau \right]^{\frac{1}{2}}} = \|w \rightarrow z\|_\infty. \quad (5)$$

Here,  $d(t)$  is an exogenous disturbance (e.g., a seismic excitation),  $n(t)$  models sensor noise,  $y(t)$  is the measured output vector, and  $u(t - \tau)$  is the input, subject to controller time delay. Let the input vector,  $w(t)$ , and the regulated output vector,  $z(t)$ , be given by

$$w(t) = \begin{Bmatrix} d(t) \\ n(t) \end{Bmatrix} \text{ and } z(t) = \begin{Bmatrix} Q^{\frac{1}{2}}x(t) \\ R^{\frac{1}{2}}u(t) \end{Bmatrix}, \quad (6)$$

where  $Q$  and  $R$  are positive semidefinite and positive definite matrices that weight the state of the structure and control input, respectively. There are two important things to note at this time. First, in order to apply the usual finite-dimensional  $H_2$  and  $H_\infty$  design methods, the time delay must be approximated by a rational transfer function. Second, the input/output pair given by (6) is identical for all of the control designs considered. This allows the best comparison of the robustness characteristics of each individual design since, in each case, an identical closed-loop transfer function is used.

Let's first consider the robustness of the system subject to a deterministic time delay,  $\tau = 20$  ms. MCS, FORM and  $\mu$ -analysis robustness estimates of the four controllers are presented in Table 2, where  $m$ ,  $c$ , and  $k$  are modeled as independent uniform random variables with a specified coefficient of variation (COV) of 25%.

Controller	$p_f^\mu$	$p_f^{FORM}$	$p_f^{MCS}$
LQR	0.3450	0.0	0.0
LQRPC	0.0	0.0	0.0
$H_2$	0.8586	0.2686	0.2683
$H_\infty$	0.9987	0.8792	0.8788

Table 2: Robustness estimates ( $\tau = 20$  ms) using FORM and  $\mu$ ,  $COV_{m, c, k} = 25\%$ .

The conservative nature of the  $\mu$ -analysis method is readily apparent from the data since, in all cases,  $p_f^{FORM} \leq p_f^\mu$ . However, while the two assessment methods provide different estimates of robustness, they predict a similar pattern. The data clearly illustrate the high level of accuracy encountered when using the FORM method.

With the addition of the random time delay, the robustness estimates using the  $\mu$ -analysis method become *even more conservative* as indicated in Table 3. Again, although both methods predict similar trends, the robustness estimates using the FORM method are clearly superior to

those utilizing the  $\mu$ -analysis method. This leads to the general observation that, when considering systems of this type and in this framework, the well-established reliability methods are more useful.

With regard to the robustness of the various control designs, it is clear from the data that the LQR and LQRPC control are the most robust in the presence of uncertainty. This can be attributed to the inherently good robustness properties of LQ regulators (*e.g.*, guaranteed phase and gain margins, *etc.*). None of the nominal output feedback designs can match the robustness of the LQ designs. Hence, the need for output feedback techniques that can directly incorporate uncertainty into the design process is apparent.

Controller	$p_f^\mu$	$p_f^{FORM}$	$p_f^{MCS}$
LQR	0.8575	4.69e-3	0.0
LQRPC	0.6395	0.0	0.0
$H_2$	0.9987	0.8342	0.8339
$H_\infty$	1.0	0.9971	0.9968

Table 3: Robustness estimates using FORM and  $\mu$ , random  $\tau$ ,  $COV_{m,c,k,\tau} = 25\%$ .

Traditional methods used to assess controller robustness may have precluded a probabilistic understanding of robust control. In this presentation, some ideas and results related to assessing controller robustness as a probability measure have been presented. With the use of these methods, one can classify the robustness characteristics of various control designs for a specific problem.

The first-order reliability method (FORM), used here to estimate robustness, is an approximation that relies upon two assumptions. First, the failure surface defining the onset of instability must be fairly linear. If it is not, the first-order curve fit to the failure surface may become inadequate. Second, the most probable failure condition must be sufficiently governed by a single mode. Highly uncorrelated failure modes may lead to inaccuracy of the FORM approximation, and more sophisticated FORM/SORM methods may be needed. However, for the problem considered herein, robustness estimates using the FORM method are adequate. This may be attributed to the high level of correlation between failure modes, typical of a structural system.

The use of the structured singular value,  $\mu$ , to assess controller robustness proved to be conservative when applied in a probabilistic framework. The main premise of this method operates only upon the maximum structured singular value and, hence, evaluates the "worst-case" scenario only. In addition, the fundamental robust stability criterion used is valid for uncertainty that may contain much more than the class of uncertainty considered here. The examples considered assumed only real parametric uncertainty, a small subclass. As a result, the analysis gave conservative, although qualitatively accurate, results. That is,  $\mu$ -analysis can correctly predict which controller is more robust probabilistically, yet the predicted probability of instability can be quite conservative. Therefore, it is possible to conclude that the FORM method provides superior robustness estimates when applied to structural control problems.

Implicit in all this is the assumption that the mean and variance of the parameters  $m$ ,  $c$ ,  $k$ ,  $\tau$  are well known and available and, in the case of a real structure, this is rarely the case. Effective utilization of the probabilistic framework will require a significant effort to characterize the material and physical properties of structural components as well as of the inputs to the system in a stochastic sense, far more than has been accomplished to date. Whether the benefits to be derived would justify the needed effort is an open question.

#### Issues Proposed for Discussion.

- Is the structural dynamicist equipped to view the his/her world stochastically?
- Are current analytical methods sufficiently capable?
- What would an adequate base of probabilistic data consist of?

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# Exciting Vibrations

## A philosophical approach to resolving structural dynamics problems

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### Summary and subject of the essay

This essay discusses three underlying principles of resolving structural dynamics problems:

- on the need for a systematic approach to identifying and understanding the problem to be solved ('**exciting**' here refers to the satisfaction of tracing the source of the problem);
- on recognising that all that really matters is how large the vibration response will be (or is) and so a knowledge of the real source of **excitation** becomes a primary requirement; and
- on the inevitability of having to undertake tests to supplement predictions since, no matter how good our computational skills become, our demands of them will always outstrip their performance: as a result we have to **excite** our structures deliberately in order to understand exactly how they behave in order to be able to design them to possess acceptable characteristics.

## **1. First point - What is the problem?**

For our first point, we shall consider the critical phase of tackling any problem, which is the part where we actually define the problem. This is a very important part of the overall problem-solving process and one which is often sabotaged by a tendency to initiate an over-hasty application of an available or familiar solution procedure before the correct approach has been properly identified. Such precipitate action can lead to much frustration and delay in obtaining a satisfactory solution and in order to avoid the associated disappointment which this can bring, we advocate here a more measured and systematic approach.

### ***1.1 Defining the questions***

The first issue concerns the need to define the correct questions. It is often the case that the hardest part of solving a difficult problem lies in the correct identification of the specific questions to be addressed. The solution, once the questions are properly formulated, is frequently the relatively straightforward application of quite well developed tools. We need to know, for example, whether to 'solve' a vibration problem by adding damping, by removing the excitation or by changing the natural frequencies. The correct solution depends on the correct identification of the root problem.

How can we achieve this? By being methodical and precise in describing the various features and phenomena that we observe. So, for example, we might assemble a list of questions which seek to get to the bottom of the problem:

- Is the problem ... that the component breaks?
- Is the problem ... the fact that the machine stops (because the component breaks)?
- Is the problem ... the fact that the vibration level was too high?
- Is the problem ... the fact that the excitation frequency was too close to a critical natural frequency?
- Is the problem ... that the excitation forces were too high? ...or that the structure was too responsive? ...or that there was insufficient damping?

Clearly, we must understand what the problem really is so that we can set about fixing that and not trying to deal with a consequence of the actual problem. There is no point in simply replacing the broken component with new one if the cause of its failure is not removed: that will only lead to a second failure. More often than not, the problem comes down to the fact that the excitation is too large (and we shall discuss that in more detail in the second part) but, of course, such a simple statement hides important issues about the definition of the complex phenomenon of 'excitation' and also of how we must determine what constitutes 'large'. So, we have established here the need to define the problem.

### ***1.2 Adopt a structured approach***

When formulating these questions, they should follow a logical sequence:

- What is the apparent problem?
- How to define what is causing it?
- If we need a model of the structure - how shall we obtain it? How to check that it is valid? How to use it to identify the problem?
- How to see whether we have correctly identified and corrected the problem?



We can often make use of 'road maps' to guide the analysis (dissection of a problem) of a problem and to formulate such a sequence of questions. This we can do with the aid of road maps and the like, perhaps themselves structured, or graded:

- route maps of major processes
- street maps of detailed procedures
- floor plans of algorithms

It is interesting to note that it is often appropriate to categorise the various issues we are dealing with here and that these often fall naturally into groups of three. This is found in many situations and, indeed, there is reference to 'the power of three' in some works of a philosophical nature. [It is perhaps no accident, therefore, that this essay has chosen to focus on three points or issues, or that the name of the original project at the site of the Forum was 'Trinity'.] While reflecting on this point, it is perhaps an appropriate time to present the corresponding classification for the types of analysis that we might undertake as part of our studies of structural dynamics. Of course, there are three types, or levels:

- Level 1 - Preliminary analysis, where order-of-magnitude answers are required to ascertain whether or not there might be a problem that demands a more thorough analysis. This is followed, in those cases where the results of the preliminary analysis indicate that there is a case to answer, by a more extensive, standard analysis:
- Level 2 - Standard analysis, where a conventional assessment is conducted to predict or to measure the extent of the problem; to determine the level of vibration or noise, for example. This may then be followed up in special cases by
- Level 3 - Advanced analysis, based on state-of-the-art techniques, which are often difficult and/or expensive to apply but which are reserved for those applications which demand the closest inspection because they represent critical cases where structural or competitive failure is at stake.

### *1.3 Use precise and consistent terminology and notation*

As an aid to the development of systematic and effective analysis of the problem, we should be sure to use precise definitions, and we should try to foster a set of terminology and notation which is more common across the various branches of the field. By way of examples, we can mention the frequent interchangeability of two or more words used for a specific item. Such a practice causes a loss of precision in the language and, eventually, demands longer expressions to define something precisely. Most of us have a number of 'pet hates' in this respect but there is a serious side to the issue. For example, we should be careful not to confuse **data** with **information**. The acquisition of a large quantity of data does not necessarily equate to the gaining of a corresponding amount of information and so the two words should be used distinctly. Indeed, it is part of the skill of the structural dynamicist (and other engineers) to maximise the information that can be extracted from the minimum of data.

Another distinction that should be clearly defined is the difference between **validation** and **verification**. The two words are widely used interchangeably and yet there are two distinct processes to which we need to refer and these two processes (and hence the words used to describe them) are not (and should not be) the same. One process is that of establishing that (a) the data supplied really has come from the transducer with which it is identified (and not from a different one by some error of labelling), or that (b) a computer routine correctly executes the algorithm which it

is supposed to perform. To my mind, these are examples of **verification** and they indicate that things are working or recorded correctly. However, there is no associated suggestion that the data provided by the transducer, or that the results of the computer routine are useful, or valid. That depends on the appropriateness of the measurement process or the input data to the computer routine and is a different matter altogether. The second process, **validation**, refers to the question of whether the results which are obtained (such as from a measurement or a computation) are good enough for the purpose for which they were sought. An eigenvalue routine can be working perfectly correctly (i.e. has been verified as correct) but the usefulness of the natural frequencies which are produced using it depend on the correctness of the model and the parameters used as well as on the correctness of the computations. These natural frequencies will only be valid if the model is good enough, and that has nothing to do with the eigenvalue routine. So, verification and validation are clearly different issues. Similar comments can be made for **comparison** and **correlation** and, no doubt, there are many other examples.

One area where different words (which actually mean different things) are used almost without discrimination is the experimental world where we use **test**, **measure** and **experiment** with equal ease. It is interesting just to see how the dictionary differentiates these and there we find:

**measure** // n. & v.

..

v.

1 tr. ascertain the extent or quantity of (a thing) by comparison with a fixed unit or with an object of known size.

..

[Middle English via Old French mesure from Latin mensura, from metiri mens- 'measure']

**experiment** // n. & v.

n.

1 a procedure undertaken to make a discovery, test a hypothesis, or demonstrate a known fact.

..

v.intr. // (often foll. by on, with) make an experiment.

..

[Middle English from Old French experiment or Latin experimentum (as experience)]

**test** // n. & v.

n.

1 a critical examination or trial of a person's or thing's qualities.

2 the means of so examining; a standard for comparison or trial; circumstances suitable for this ..

v.tr.

1 put to the test; make trial of (a person or thing or quality).

..

[Middle English via Old French from Latin testu(m) 'earthen pot', collateral form of testa test2]

While this discussion may seem like pedantry, I submit that the reliability and effectiveness of our communications can depend heavily on the precision of the language we use, and that we have in our language the capacity for greater precision and efficiency than we generally exploit.

Turning finally to more specific and technical terms, another worrying trend is found in an almost-universal tendency not to differentiate between **natural frequency** and

**resonance frequency** (or should it be **resonant frequency**? - no!). Clearly, these are different quantities - one referring to a system property and the other to a response characteristic - and in many cases, this difference is important, even if the numerical differences between resonance frequencies and natural frequencies is often insignificant. We, of all people, should be more precise in our use of the language we use in our subject.

So, to summarise this first part of the essay, it is my submission that in order to tackle our various problems with greater effect, we need:

- a structured approach to identifying the real questions;
- a more uniform use of notation, terminology and definitions
- a precise use of language

All of these are available: it is a matter of availing ourselves of them.

## **2. Second point - the 'Bottom Line'**

The second point to be discussed in this essay concerns the feature which is the focus of our interest in most structural dynamics problems. It is suggested here that we need to have a clear picture of what is the feature of ultimate interest and that this is generally the level of **response** suffered by the structure which is the subject of our studies - be that a building or vehicle whose integrity is at stake, a machine whose reliability is of concern, or a human being who is the end user (sufferer) of many dynamics-related phenomena.

### **2.1 Response as the ultimate measure**

These, and many other examples, indicate that the 'bottom line' on which we generally need to base all our decisions is a knowledge of the response levels and not simply of the system's modal properties of natural frequencies and mode shapes. Such modal properties are all very useful but as a means to an end rather than an end in themselves. It could be argued that FRFs are much more useful than modal properties since they demonstrate the real behaviour of the structure in a much more explicit and practically-useful form. Indeed, the widespread and extensive development of modal analysis has served to bridge the gap between modal and response properties very effectively. Thus, it is suggested that the maximum emphasis possible should be placed on the response function descriptions which are used in modal analysis, and that these data should not be discarded once the modal analysis has revealed the underlying modal properties: it must always be remembered that the modal properties are a subset of the information contained within the response functions.

If we accept the pre-eminent significance of response characteristics, this means that the excitation becomes as important a parameter as the mass and stiffness descriptions with which we are so preoccupied in our analysis of system properties. Accordingly, we should make sure that a balanced effort is made in the attempts to model the dynamic behaviour of real structures. It is possible that new developments on smart structures mean that we do not need to anticipate these excitation forces in quite the same way - perhaps, they can be simply be identified by the structure and counteracted automatically at source as a means of reducing response levels to an acceptable degree. Be that as it may, it still remains the case that the response is the ultimate measure of acceptability, or not, and must feature prominently in our studies.

By the same argument, if we are primarily concerned with the response levels, then it follows that the damping properties of the structure are also very important. Many vibration problems are related in some way to resonance and it is clear that the response suffered in this area is directly affected by damping. Hence, we need to be as thorough in our estimation of damping levels as we are in our analysis of the excitation forces. Both of these requirements place a considerable additional burden on the structural dynamicist: in one case (damping) because the necessary modelling techniques are very much more demanding than are those required for the inertia and flexibility behaviour, and in the other case (excitation) because this usually requires an excursion into a different discipline, such as fluid dynamics or electromagnetics.

## ***2.2 Questions concerning response levels***

If we follow the recommendations provided in the first part of the essay, we shall set about addressing the problem of response prediction by posing a series of questions. Perhaps the relevant sequence of such questions, assuming that a model of the structure already exists, is as follows:

- What will be source of the excitation?
- How do I determine what the excitation is/was by measurement?
- Why will the excitation be at that level?
- What controls the excitation level?
- What level of excitation is acceptable?
- What do I do to change the excitation level?

It can be seen that these questions add up to a requirement for a comprehensive understanding of the nature of the excitation forces, and of their analysis, leading to the possibility of modifying them as a means of reducing the response they inflict on the structure.

## ***2.3 Determining the excitation as a key to response***

Predicting the excitation may require access to a technology that we do not readily have. For example, fluid dynamics is a major source of excitation in many machines and structures, including offshore platforms (waves) buildings (wind) aerospace structures and many machines, including turbomachinery, which have major and complex sources of excitation from the working fluid. In all of these examples, relatively advanced methods of analysis may be required to establish the excitation forces imposed on the structures of interest to us. Indeed, in one example - that of the excitation of blade vibration in turbomachines - the aerodynamics of relevance (unsteady) is more complex than that used for the performance analysis (steady flow aerodynamics). Thus, the implications of the emphasis which, it is proposed here, should be put on the dynamic response of structures may be quite significant in terms of the efforts that must be made to achieve the necessary balance.

In some cases, attempts may be made to establish the excitation in complex situations by measuring it, either directly or indirectly. Direct measurement may be as difficult as direct analysis in many cases, and even more expensive as it is usually extremely difficult to place transducers close to the source of the excitation. Instead, there is a widespread interest in using the [excitation/system properties/response] relationship as a means to deduce excitation forces by an identification process. The basic concept

is simply stated: if we measure the response of a structure or machine under operating conditions, and if we have previously obtained a mathematical model of the structure (by measurement or by analysis), then it should be possible to determine the excitation forces which must be causing the observed response. As with many inverse problems, this process is inherently ill-conditioned and suffers from the weakness that it cannot yield meaningful results if the set of assumed (but initially unquantified) excitation forces is incomplete. If one force which exists in reality is omitted from the set of forces to be determined by this inverse identification approach, then there will be the strong risk of physically-meaningless results being obtained, even though they may well satisfy the narrow requirement that they are capable of reproducing the observed response. It must always be remembered that we are obliged to use incomplete data in these processes and the fact that we do not have a measure of the response at point X does not mean that this quantity is zero. It **does** mean that we cannot tell whether our identified forces reproduce the correct response there (at point X) as well as at the few points we have measured. No, the determination of realistic excitation forces in most typical structures and machines is a frequently-underestimated task, but a critical one nevertheless.

There are, however, subtleties to the excitation which we need to appreciate so that we can exploit them whenever possible. There is a tendency to regard the excitation forces as having simply a characteristic magnitude and frequency. The magnitude of the response is directly related to the magnitude of the excitation, and is strongly related to the relationship between the frequency (content) of the excitation and the characteristic frequencies of the structure. This is clear. However, it must also be noted that the excitation in most practical situations is not confined to a single or even a few points, but is often distributed widely across the exposed surface(s) of the structure. Thus, we must take due note of the 'shape' of the excitation forcing as well as its magnitude and frequency. This can be an extremely critical feature because the magnitude of the response, in addition to its dependence on the force amplitude and frequency, can be dramatically influenced by the spatial distribution of the forces. In the same way that response magnitude is very sensitive to the relationship between the excitation force frequency and the structure's natural frequencies, so also is that response strongly influenced by the relationship between the structure's mode shapes and the excitation's spatial distribution (which is, in effect, a 'mode shape' of forces rather than displacements).

This aspect can be illustrated by reference to certain the vibration properties of rotating machines. A typical turbomachine blade installed in its working habitat of a compressor or turbine finds itself a participant in a very large number of modes of vibration (those of the bladed assembly of which it is one component). These modes, when charted on the typical Campbell or interference diagrams used in this application, find themselves vulnerable to an alarming number of potential resonances as a result of excitation by non-uniformities in the gas flow - known as 'engine-order' forced response. If all those coincidences of a natural frequency (from a mode of the bladed assembly) and an excitation frequency (from an integer multiple of rotation speed) converted into a significant resonance, then there would be no suitable operating region within the machine's speed range. However, because each of the potential resonances involves a particular mode shape together with a particular spatial feature of the excitation (an excitation 'shape'), it is discovered that the vast majority of the potential resonances result in near-zero response levels because these

two shapes are incompatible with each other (i.e. they are orthogonal). This phenomenon constitutes a major feature in the vibration response characteristics of turbine blades and is responsible for the vibration environment of these critical components being tolerable. Nor are such phenomena unique: there are many other examples - the fact that a only certain modes of a rotating shaft can be excited by out-of-balance forces (forward whirl modes can be excited by out-of-balance; backward whirl modes cannot) is another example of orthogonality between a mode shape and an excitation shape overriding the potential danger of a resonance resulting from the coincidence between an excitation force frequency and a structure's natural frequency.

So, to summarise the second part of this essay, it is my submission that we need:

- To focus more on response levels than modal properties;
- To pay more attention to the analysis of excitation forces and damping; and
- To understand and thus to control and exploit the spatial aspects of excitation as well as the magnitude and frequency properties

All of these are possible: it is a matter of affording them the necessary priority.

### **3. Third point - 'Hair of the dog'**

The third point to be made here is, simply, the case for continued development of experimental as well as theoretical methods for dealing with structural dynamics problems. The thesis is that no matter how refined our theoretical prediction techniques become, they will not be able to satisfy the associated demands placed on them. As a result, it is inevitable that as long as we are concerned with the design of structures that are to be built and used, and therefore to exist, it will be necessary to conduct practical tests on them or on their components. While the nature and detail of these tests may well evolve to accommodate the ever-improving theoretical predictions, their future demand is considered to be inevitable. What must happen is that the concept and detail of experimental methods must be refined so as to maximise the benefit that can be derived from them, and that is the challenge for the experimentalist in the coming decade. So, in order to be able to design safe, reliable, noise-free structures we have to deliberately provoke them to vibrate (a version of the old Scottish belief that you can avoid evil consequences resulting from a dog bite by applying a few hairs from the offending dog to the wound).

#### ***3.1 The failure of predictions to provide all***

The trend towards better computer techniques offers the promise of designs which rely only on theory-based predictions, and do not need validation by any kind of experimental measurement. This ideal state is unlikely to be reached in the foreseeable future for the simple reason that no matter how well we are able to predict these characteristics, it will not be good enough. Expectations will always lead capabilities and so we shall always be chasing that next improvement. If the requirement now is to predict vibration response levels to within, say, 10%, then as soon as that objective is realised it will be replaced by a requirement to be within 3%, and so it will continue. However, the only way in which the performance of these predictions can be assessed is by quantitative observation of the actual behaviour and that can only be done objectively by suitable test techniques.

There are a number of ways in which the theoretical predictions (as we shall call this type of result) fall short of their mission. The first is that their numerical values are inaccurate, and can differ from reality by a margin which is unacceptable from the engineering viewpoint. The second is that they fail to predict all the events or phenomena which occur in practice. It is not satisfactory if those events which are predicted have an acceptable accuracy if others, which may be just as important, are overlooked. Third, they can only be as accurate as the input data used and, in many cases, it is necessary to supply numerical values for parameters which cannot be predicted (material density, Poisson's ratio, etc). In these and other respects, theoretical predictions remain restricted in the extent of their capabilities.

### ***3.2 Practical limitations mean that tests will 'always' be required***

In reality, there will be limitations imposed on our ability to predict a structure's dynamics simply because of our inability to manufacture such structures to the precision required to ensure repeatable behaviour. One area in particular is in the characterisation of joints. We are already at the stage where, although we can predict the modal properties of most complex-shaped components quite well, we cannot predict their response anywhere nearly as accurately, and we cannot predict their modal behaviour when connected together to form the engineering structure in which we are ultimately interested without some additional information on the joints – information which can usually only be supplied by tests. It is believed that this situation currently poses a serious limitation to our ability to predict the dynamic response of many real engineering structures and in order to develop a mathematical model upon which to base the design of a critical component or structure, then it will continue to be necessary to build a hybrid model which relies on theoretical modelling for those areas which are amenable to this approach but which makes recourse to empirical data for other regions (such as joints) which are notoriously difficult to analyse by theory. Of course, one alternative approach to resolving this difficulty (because it is true that empirical data are expensive to obtain and they are generally not available until very late in the design cycle) is to change the design of these joints and other areas that are difficult to model so that they **can** be analysed using the same techniques as are used with success on other structural elements. That is sometimes opposed on the grounds that it is the 'tail wagging the dog' (it is not clear if it is the same dog whose hair is mentioned in the title) but it is an inevitable suggestion for a way to avoid this particular - and very real - problem.

### ***3.3 Testing remains an essential technology***

From the above comments, we see that tests remain an essential feature in the vibration engineer's armoury of tools to tackle the various problems he faces in designing, developing or curing the dynamic properties of a wide range of structures and machines. In coming to terms with this situation, we can see that there are (at least) three distinct types of test, which can be classified as follows:

- Tests to measure parameters which are, in effect, unpredictable: empirical data, material properties, etc. (these are measurements, pure and simple);
- Tests to validate predictions of dynamic behaviour, but made under controlled excitation conditions and designed to check specific features only, usually of the structural model, rather than of the excitation forces; and
- Tests to record what actually happens in service, generally under uncontrolled and unknown excitation conditions (these may include those tests carried out to measure excitation indirectly).

It is important to be clear which type of test is being carried out in each case and what is the specific objective sought from it. For example, full validation of the accuracy of the theoretical model of the structure's dynamics cannot be expected without knowledge of the **complete** excitation and response quantities.

It is another important feature of all measurements that we should ensure that all the available information is extracted from the measured data (see earlier comments regarding the difference between data and information). There is always the danger that we shall extract (or 'see') what we are looking for in test results, especially in a validation test, and fail to extract (or 'see') other indications, especially if these risk contradicting the behaviour that we are seeking to validate. This failing is not necessarily the result of a wilful ignoring of contrary signs but it is a well-known characteristic that 'once we have seen what we came for, we stop looking any further'. That is a weakness in any investigation, and especially so in experimental studies of structural dynamics where there is always a real risk that our theoretical models may fail to predict all the phenomena which actually occur.

In the same vein as ignoring unexpected, unwanted or inexplicable results is the need to ensure that 'impossible' results are properly investigated and not simply dismissed as being due to 'noise' or 'non-linearity' - two common scapegoats for data which do not conform to our preconceptions. This comment is prompted by two recent experiences of the author relating to the apparently successful measurement of (a) frequency response functions of an unstable rotor system (why did the excitation used to measure the response functions not induce an unstable response in the structure?) and (b) frequency response functions of heavily non-linear structures in which the intermediate (unstable and unattainable?) section of the overhanging region of the curve. There are usually good explanations for such 'impossible' results and these should be sought out.

### *3.4 Need to make testing more efficient*

The final point to make regarding the ongoing and crucial role to be played by experimental techniques in structural dynamics is the urgent need to make testing just as efficient as theoretical analysis. To this end, we need to optimise the testing procedures in just the same way that we have optimised the numerical algorithms used for computational studies. Of course, there have been a number of dramatic technique developments in the recent past - the FFT is a good example from 30 years ago - but this comment is less about the measurement processes themselves and more about the design of the test and the selection of the most appropriate data to be measured. Although there are always exceptions, there is a widespread philosophy where tests are involved 'to measure as much data as possible and then to see what information can be extracted after the test is complete'. The author can recall a series of tests which were made on an annual basis in which the preceding year's data had only partly been analysed (because of its volume) by the time it was required to plan the next year's tests. Clearly, this is sub-optimal in terms of the benefit being derived from the experiments which, as is often the case, were very expensive to conduct.

The philosophy of optimising tests can best be illustrated in the case of tests which are undertaken to validate theoretical models that have been constructed for design calculations of a particular structure. These are typically modal tests in which the modal properties of the test structure are derived from measurements of its response



to a controlled excitation and are then compared with the corresponding data predicted by the theoretical model. The resulting comparisons are intended to provide some systematic feedback to the modeller so that the model can be refined (or updated) until the agreement between the test data and the predictions are within an acceptable tolerance. An obvious way to proceed in this application, in order to maximise the benefit which can be gained from the measured data and to minimise the amount of data to be acquired, is to use the theoretical model which is to be validated to 'rehearse' the test in a computer simulation of the test itself. In this way, the effectiveness of having certain items of data when trying to perform the validation and updating procedure can be explored much more cheaply than is possible with actual test data. In principle, unnecessary or ineffective data can be identified prior to the proposed tests being carried out so that time is not wasted measuring data which serve relatively little useful purpose.

It is becoming clear that the timing of these validation tests (in the overall design cycle) is an increasingly critical factor - test data frequently arrive too late to permit full benefit to be gained from them. Thus, we need to re-think the strategy of validation tests and this may tend to press towards earlier and earlier testing which means, in turn, that effort should be put on validating the individual components as they are manufactured, rather than waiting for the fully assembled structure which can only be done when the last item is available. Thus, validation testing needs to work through a series of philosophical issues in an attempt to improve the efficiency of the process by an order of magnitude. Not until this has been done, is it more likely that testing will be accepted as an integral part of the design process.

So to summarise the third part of this essay, it is my submission that we need:

- To accept that experimental techniques will continue to play a central role in the development of structures which are acceptable from the dynamics viewpoint;
- To be more objective and analytical in our design of tests;
- To optimise tests for validation by using the model to be validated in a rehearsal of the proposed tests; and
- To incorporate dynamic testing as an integral part of structural design, and not an afterthought.

All of these are possible: but may require holding back the tide.

# DAMPING DESCRIPTION IN STRUCTURAL DYNAMICS

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## 1. Introduction

Wave propagations and vibrations are associated with the removal of energy by dissipation or radiation. In mechanical systems damping forces causing dissipation are often small compared to restoring and inertia forces. However their influence can be great and is discussed in the present survey paper together with the transmission of energy away from the system by radiation. Viscoelastic constitutive equations with integer and fractional time derivatives for the description of stress relaxation and creep of strain as well as for the description of stress-strain damping hysteresis under cyclic oscillations are compared. Semi-analytical solutions of wave propagation and transient vibration problems are obtained by integral transformation and elastic-viscoelastic correspondence principle. The numerical solution of boundary value problems requires discretization methods. Generalized damping descriptions are incorporated in frequency and time domain formulations for the boundary element method and the finite element method.

## 2. Classification of Damping, Solutions of Viscoelastic Boundary Value Problems

Vibrating mechanical systems are primarily governed by cyclic transformation between potential and kinetic energy. Associated elastic and inertia properties of lumped or continuous models allow to explain natural frequencies and modes of vibrating systems or group and phase velocities of wave propagating systems. Additional mechanisms remove energy and cause the so called damping, which is responsible for the amplitude decay of free vibrations or propagating waves, for finite amplitudes and phase shifts of

systems excited near resonance, for the need of external work to maintain forced vibration amplitudes and the changes in stability of non-conservative systems (Crandall 1970, Bert 1973).

Damping in metals can be caused by combinations of physical mechanisms such as thermal currents on both the micro and macro scale, grain boundary viscosity, point-defect relaxations, eddy-current effects, stress induced ordering and electronic effects (Lazan 1968).

For an important class of non-metallic materials, namely polymers and elastomers, considerable phenomenological data have been obtained. Due to the long range molecular order associated with their giant molecules, polymers exhibit rheological behavior as a combination of a reversible elastic deforming solid and a dissipative viscous liquid (Ferry 1980). The marked frequency and temperature dependence as well as relaxation and creep phenomena of such viscoelastic materials are of particular importance. Industrial applications of polymers and elastomers are widespread such as for damping layers, coatings, absorbers, resilient mounts. The purpose of developing mathematical models for rheological behavior of solids is to permit realistic response predictions from the analysis of complicated structures undergoing various conditions of loading.

TABLE 1. Definition and classification of damping

<b>Definition of Damping</b>	
Energy dissipating property of materials and members undergoing time dependant deformations and/or displacements. Damping is primarily associated with the irreversible transition of mechanical energy into thermal energy. The energy radiation into a surrounding domain is called radiation - or geometric - damping.	
<b>Classification of Damping</b>	
<b>Material damping</b>	Energy dissipation by deformation in a medium (Irreversible intercrystal heat flux, grain boundary viscosity, etc.)
<b>Structural Damping</b>	Damping in assembled structures including: Material damping of members; Frictional losses (Microslip, macroslip) at contacting surfaces (Bolted, riveted, damped, welded connections); Dissipation in medium between surfaces in relative motion (Gas pumping, squeeze film damping, lubricated bearing)
<b>Radiation damping</b>	Energy radiation into surrounding medium
<b>Active/passive damping</b>	Damping with/without external energy and control
<b>Internal/external damping</b>	Damping inside/outside defined system boundary

According to the definition in Tab. 1 damping is the removal of energy from a vibratory system. The energy loss is either dissipated within the system or transmitted away by radiation.

The purpose of the present paper is threefold. Firstly to compare linear material damping descriptions by conventional with generalized viscoelastic constitutive equations. Replacing integer time derivatives in the conventional equations by fractional time derivatives in the generalized equations leads to improved curve-fitting properties of measured data with less parameters and assures causality (Torvik and Bagley 1987, Gaul, Klein and Kempfle 1991) which is violated by the so called constant hysteresis damping model (Crandall 1962, Gaul, Bohlen and Kempfle 1985). Unique selection of complex roots in frequency domain of Fourier transformation is gained by mathematical and physical judgment (Gaul et al. 1991). It has been shown (Torvik and Bagley 1987) that fractional derivatives cannot be viewed merely as effective means of providing curve-fits to data, but their presence has been predicted by accepted theories for the behavior of polymer solids without cross linking for example. A comprehensive review about the application of fractional calculus to dynamic problems of linear and nonlinear hereditary mechanics of solids has been published by Rossikhin and Shitikova (Rossikhin and Shitikova 1997).

The second purpose of the paper is to derive semi-analytical solutions of one dimensional wave propagations by integral transformation and adopting the elastic-viscoelastic correspondence principle (Gaul et al. 1991, Crandall 1962, Nakagawa and Kawai 1980, Beyer and Kempfle 1995, Kempfle and Gaul 1996).

The third purpose is to incorporate generalized damping descriptions in frequency and time domain formulations of discretization methods for solving 2-d and 3-d boundary value problems. This is shown for the boundary element method (BEM) in frequency domain (Gaul and Chen 1993, Gaul 1991) and in time domain (Gaul and Schanz 1994, Gaul and Schanz 1997) and the finite element method (FEM) as well (Krings 1976, Carpenter 1972, Pilkey and Pilkey 1995). Radiation damping for problems with semi-infinite or infinite exterior domains is described by surface discretization with the BEM (Gaul and Schanz 1998).

### 3. Viscoelastic Constitutive Equations

The purpose of developing viscoelastic constitutive equations is to describe the rheological stress-strain hysteresis, relaxation and creep behavior as well as the temperature influence for a homogeneously loaded volume element of material. Implementation into field equations of motion and integra-

tion over inhomogeneously loaded domains leads to response predictions of members or structures under different loadings such as sinusoidal, transient or random. Other than the microscopic approach which includes the internal structure of matter in terms of atomic and molecular interactions, the present formulation is a macroscopic approach based on phenomenological aspects of physics and the laws of thermodynamics of irreversibility. If small vibratory deflections are superimposed on static predeformations a linear description for the low-stress regime is sufficient.

Hooke's law for an elastic isotropic material relates the deviatoric state of stress and strain with shear modulus  $G$

$$s_{ij}(t) = 2Ge_{ij}(t) \quad (1)$$

and the hydrostatic states of stress and strain with bulk modulus  $K$

$$\sigma_{kk}(t) = 3K\varepsilon_{kk}(t) \quad (2)$$

where

$$\sigma_{ij} = s_{ij} + \frac{1}{3}\sigma_{kk}\delta_{ij}, \quad \varepsilon_{ij} = e_{ij} + \frac{1}{3}\varepsilon_{kk}\delta_{ij} \quad (3)$$

The viscoelastic equation corresponding to (1) of differential operator type with time derivatives  $D^k = d^k/dt^k, k \in N$  is

$$P_D s_{ij}(t) = Q_D e_{ij}(t) \quad (4)$$

with differential operators

$$P_D = \sum_{k=0}^N p_k D^k, \quad Q_D = \sum_{k=0}^M q_k D^k \quad (5)$$

or of hereditary integral types

$$\begin{aligned} s_{ij}(t) &= 2 \int_{-\infty}^t G(t-\tau) \frac{de_{ij}}{d\tau}(\tau) d\tau \\ e_{ij}(t) &= 2 \int_{-\infty}^t J(t-\tau) \frac{ds_{ij}}{d\tau}(\tau) d\tau \end{aligned} \quad (6)$$

with the relaxation modulus  $G(t)$  and the creep compliance  $J(t)$  kernels describing the fading memory of the material with respect to the load history. Relaxation and creep standard tests are described by Ferry (Ferry 1980).

Creep compliance and relaxation modulus are connected by the relation (Gaul 1980)

$$\frac{d}{dt} \int_0^t 2G(\tau)J(t-\tau)d\tau = 1 \quad (7)$$

The equations for the hydrostatic state are of the same structure as (5, 6). The differential operator type of (2) is

$$P_H \sigma_{kk}(t) = Q_H \varepsilon_{kk} \quad (8)$$

Anisotropic constitutive equations are given by Christensen (Christensen 1971). Thermorheologically simple materials allow the introduction of the non-uniform temperature in (6) by replacing the independent variable time  $t$  by a reduced time

$$\zeta(t) = \int_0^t \Phi[T(x, \eta)]d\eta \quad (9)$$

which is based on the shift function  $\Phi$  determined from experimental data (Christensen 1971, Ferry 1980).

A fruitful generalization of viscoelastic laws replaces the integer order time derivatives  $D^k$  by those of fractional order  $D^{\alpha_k}$ , where  $\alpha_0 = \beta_0 = 0$ ,  $0 < \alpha_k$ ,  $\beta_k < 1$  and  $D^\alpha$  is defined either by the nonlocal convolution operator

$$D^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(t-\tau)}{\tau^\alpha} d\tau \quad (10)$$

with the gamma function

$$\Gamma(1-\alpha) = \int_0^\infty e^{-x} x^{-\alpha} dx \quad (11)$$

as the inverse operation of fractional integration attributed to Riemann and Liouville (Oldham and Spanier 1974), or alternatively by a definition based on generalized finite differences according to Grünwald (Grünwald 1967)

$$D^\alpha x(t) = \lim_{N \rightarrow \infty} \left( \frac{t}{N} \right)^{-\alpha} \cdot \sum_{j=0}^{N-1} \frac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)} x \left[ t \left( 1 - \frac{j}{N} \right) \right] \quad (12)$$

This discrete definition is more convenient in constitutive equations treated by time stepping algorithms (Padovan 1987) and can be shown to be equivalent to the definition in (10). Fractional derivatives replace the differential operators in (4) by

$$P_D = \sum_{k=0}^N p_k D^{\alpha_k}, \quad Q_D = \sum_{k=0}^M q_k D^{\beta_k} \quad (13)$$

and change the relaxation function  $G(t)$  in (6) such that sums of exponential functions for integer time derivatives are replaced by gamma functions for fractional time derivatives. A correspondence between elastic and viscoelastic equations can be found by Laplace transformation

$$\mathcal{L}(D^\alpha x(t)) = s^\alpha \mathcal{L}(x(t)) - \sum_{k=0}^{n-1} s^k D^{\alpha-1-k} x(0), n-1 \leq \alpha < n \quad (14)$$

or Fourier transformation

$$\mathcal{F}(D^\alpha x(t)) = (i\omega)^\alpha \mathcal{F}(x(t)). \quad (15)$$

This is why steady state time dependency such as  $e_{ij}(t) = \text{Re} [e_{ij}^* \exp(i\omega t)]$  or the frequency domain of (15) convert (6) and (13) to

$$s_{ij}^*(\omega) = 2G^*(\omega)e_{ij}^*(\omega) \quad (16)$$

with

$$G^*(\omega) = i\omega \mathcal{F}(G(t)) = \sum_{k=0}^M \frac{q_k(i\omega)^{\beta_k}}{p_k(i\omega)^{\alpha_k}}. \quad (17)$$

The real part of the complex shear modulus (17) is the storage modulus  $G'(\omega)$ , the imaginary part is the loss modulus  $G''(\omega)$ , their ratio is the loss factor  $\eta(\omega)$

$$G^*(\omega) = G'(\omega) + iG''(\omega) = G'(\omega) [1 + i\eta(\omega)]. \quad (18)$$

If we decompose the stress relaxation modulus  $G(t) = G_\infty + \hat{G}(t)$  in an equilibrium modulus  $G_\infty$  and  $\hat{G}(t)$  where  $\hat{G}(t \rightarrow \infty) \rightarrow 0$ , the complex modulus is related to the relaxation modulus according to (17) by

$$G'(\omega) = G_\infty + \omega \int_0^\infty \hat{G}(\tau) \sin(\omega\tau) d\tau, G''(\omega) = \omega \int_0^\infty \hat{G}(\tau) \cos(\omega\tau) d\tau. \quad (19)$$

An interpretation of the loss factor as energy quotient is obtained by dividing the dissipated work per cycle  $T$ , e.g. in the plane  $x_1, x_2$ ,

$$D(\omega) = \int_0^T s_{12}(t) \dot{e}_{12}(t) dt = \pi G''(\omega) \hat{e}_{12}^2, \hat{e}_{12} = |e_{12}^*| \quad (20)$$

by the  $2\pi$ -fold of stored energy

$$U(\omega) = \frac{1}{2} G'(\omega) \hat{e}_{12}^2 \quad (21)$$

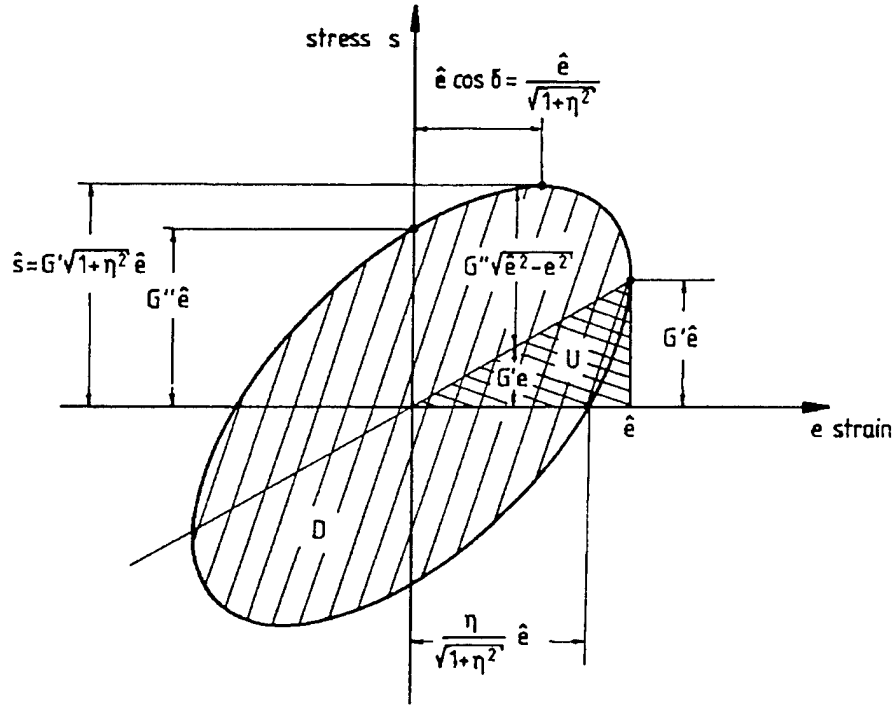


Figure 1. Elliptical hysteresis loop characteristic of linear damping

$$\eta(\omega) = \frac{D(\omega)}{2\pi U(\omega)} = \frac{G''(\omega)}{G'(\omega)} = \frac{1}{Q(\omega)} = \tan\delta(\omega) \quad (22)$$

The energy ratio of (22) can as well be generalized to nonlinear stress-strain constitutive equations. Other damping measures are the quality-factor  $Q(\omega)$  and the loss tangent  $\tan\delta(\omega)$  defined in (22). A graphical representation of equations (20), (21), (22) is given by the elliptical hysteretic loop in Fig. 1.

One advantage of the fractional derivative model of viscoelasticity are improved curve-fitting properties for relaxation and creep functions and complex moduli. This is demonstrated by an example with data taken for a high damping polymer from (Cupial 1998) in Fig. 2. All 5 parameters of the fractional derivative model (17) with  $M = N = 1$

$$G^*(\omega) = G \frac{1 + b(i\omega)^\beta}{1 + a(i\omega)^\alpha} \quad (23)$$



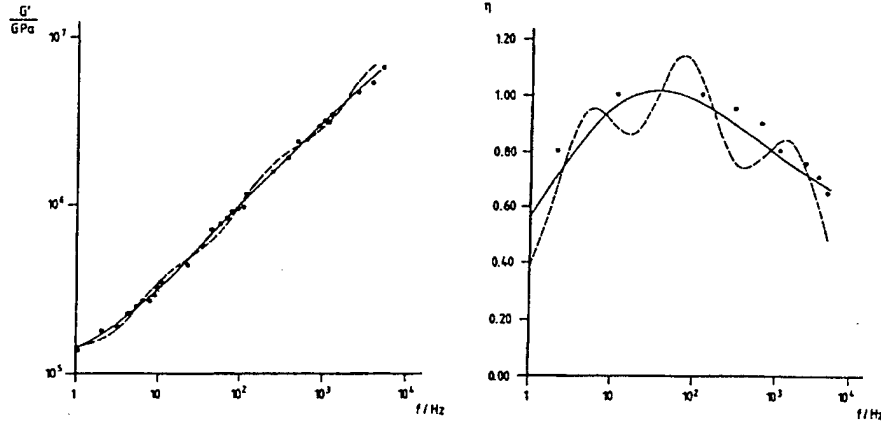


Figure 2. Storage modulus  $G'(\omega)$  and loss factor  $\eta(\omega)$ ; •••• Experimental data; — 5 parameter fractional derivative model ( $G = 0.87 \cdot 10^5 \text{ GPa}$ ,  $a = 0.039$ ,  $\alpha = 0.39$ ,  $b = 0.38$ ,  $\beta = 0.64$ ); - - - Conventional 9 parameter model

with selected principal values of multi-valued function  $(i\omega)^\beta$  have been found by least squares fit of the real part of (23) in the interval  $1 < f/\text{Hz} < 10^4$ :

$$\sum_i \left( \frac{G'(\omega_i) - G'_i}{G'_i} \right)^2 \rightarrow \min. \quad (24)$$

Proper selection of complex roots for the range of positive and negative frequencies of Fourier transform is addressed below (Gaul et al. 1991).

For comparison Fig. 2 shows results of the fitted conventional 9 parameter model  $N = M = 4$  with all exponents being integers

$$G^*(\omega) = G \frac{1 + \sum_{k=0}^4 b_k (i\omega)^k}{1 + \sum_{k=0}^4 a_k (i\omega)^k}. \quad (25)$$

Only the real part was fitted by (24) with little oscillation, whereas the loss factor was generated poorly. Thus the imaginary part had to be fitted as well and still gives rise to oscillations.

The restrictions of non-negative internal work and non-negative rate of energy dissipation (Torvik and Bagley 1987, Schanz 1994) in accordance with the second law of thermodynamics and the requirement of finite viscoelastic wave speed (Schanz 1994, Gaul and Schanz 1997) reduce (23) to

a 4 parameter model

$$aD^\alpha s_{ij}(t) + s_{ij}(t) = G \left[ e_{ij}(t) + bD^\beta e_{ij}(t) \right] \quad (26)$$

if the parameters fulfill the constraints

$$0 < \alpha = \beta < 2 \quad G \geq 0 \quad b > a > 0. \quad (27)$$

A powerful tool for calculating viscoelastic behavior from a known elastic structural response is the elastic-viscoelastic correspondence principle. According to this principle (Flügge 1975) the viscoelastic solution is calculated from the analytical solution by replacing the elastic moduli in the Laplace transformed domain by the transformed impact response functions of the viscoelastic material model. The viscoelastic solution is then obtained by inverse transformation.

For the above mentioned generalized 4 parameter model the elastic-viscoelastic correspondence under the assumption that the same dissipation mechanisms act in the hydrostatic and the deviatoric states is given by

$$3K \rightarrow 3K \frac{1 + qs^\alpha}{1 + ps^\alpha}, \quad 2G \rightarrow 2G \frac{1 + qs^\alpha}{1 + ps^\alpha}, \quad (28)$$

where  $K$  is the elastic bulk modulus and  $G$  the elastic shear modulus. In Equation (28) the transformation (14) is used for vanishing initial conditions.

An alternative formulation of the correspondence principle adopts Fourier transformation and associated complex moduli.

#### 4. Transient Waves in Viscoelastic Solids

Transient uniaxial wave propagation in a viscoelastic rod is calculated for generalized constitutive equations by the correspondence principle before the 3-d generalization is treated numerically. The rod (Fig. 3) is fixed at  $x = 0$  and loaded by the force  $F(t)$  at  $x = \ell$ . The equation of motion

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad c = \sqrt{\frac{E}{\rho}}, \quad (29)$$

boundary conditions

$$u(0, t) = 0, \quad EA \frac{\partial u(\ell, t)}{\partial x} = -F(t) \quad (30)$$

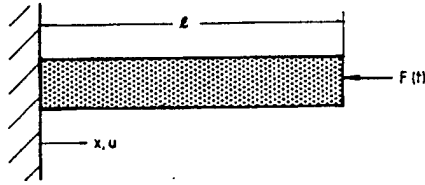


Figure 3. Fixed-free thin rod with coordinate  $x$ , displacement  $u$

and initial conditions

$$u(x, 0) = \frac{\partial u(x, 0)}{\partial t} = 0 \quad (31)$$

lead to the elastic displacement field in terms of right and left propagating waves excited by the Heaviside step function  $F(t) = H(t)$

$$u(x, t) = -\frac{1}{\rho c} \sum_{n=0}^{\infty} (-1)^n \left[ \left( t - \frac{(2n+1)\ell - x}{c} \right) H \left( t - \frac{(2n+1)\ell - x}{c} \right) - \left( t - \frac{(2n+1)\ell + x}{c} \right) H \left( t - \frac{(2n+1)\ell + x}{c} \right) \right] \quad (32)$$

The viscoelastic solution is obtained from the Laplace transform of (32)

$$u^*(x, s) = -\frac{1}{\rho c} \frac{1}{s} \sum_{n=0}^{\infty} (-1)^n \frac{1}{s} \left[ \exp \left\{ -\frac{(2n+1)\ell - x}{c} s \right\} - \exp \left\{ -\frac{(2n+1)\ell + x}{c} s \right\} \right] \quad (33)$$

Replacing Young's modulus in the elastic wave speed  $c$  by the corresponding viscoelastic impulse response of the 4 parameter fractional derivative model (28)

$$E \rightarrow E \frac{1 + qs^\alpha}{1 + ps^\alpha}, \quad c = \sqrt{\frac{E}{\rho}} \rightarrow c \sqrt{\frac{1 + qs^\alpha}{1 + ps^\alpha}} \quad (34)$$

leads to the displacement field

$$u^*(x, s) = -\frac{1}{\rho c} \frac{1}{s} \sqrt{\frac{1+ps^\alpha}{1+qs^\alpha}} \sum_{n=0}^{\infty} (-1)^n \frac{1}{s} \left[ \exp \left\{ -\frac{(2n+1)\ell - x}{c} \sqrt{\frac{1+ps^\alpha}{1+qs^\alpha}} s \right\} \right. \\ \left. - \exp \left\{ -\frac{(2n+1)\ell + x}{c} \sqrt{\frac{1+ps^\alpha}{1+qs^\alpha}} s \right\} \right]. \quad (35)$$

The normal stresses follow with the constitutive equation

$$\sigma^*(x, s) = \frac{1+qs^\alpha}{1+ps^\alpha} E \frac{\partial u^*(x, s)}{\partial x}. \quad (36)$$

The inverse Fourier transformation is carried out numerically by an adapted version of the method of Talbot (Gaul and Schanz 1997), (Schanz 1994). Jump relations (Flügge 1975) show that the viscoelastic wave front travels with a speed calculated with the initial value  $E(t=0)$  of the relaxation modulus  $c_v = \sqrt{E(t=0)/\rho}$ . The initial value theorem of Laplace transform governs the initial relaxation modulus derived from the 5 parameter model (26) by

$$E(t=0) = \lim_{t \rightarrow 0} E(t) = \lim_{s \rightarrow \infty} sE(s) = \lim_{s \rightarrow \infty} s \left( E \frac{1+qs^\beta}{1+ps^\alpha} \frac{1}{s} \right). \quad (37)$$

Only for  $\alpha = \beta$  the initial relaxation modulus  $E(t=0) = E_p^q$  exists and leads to the wave speed

$$c_v = \sqrt{\frac{E}{\rho} \frac{q}{p}} = c \sqrt{\frac{q}{p}}. \quad (38)$$

This is a different justification for the reduction of the 5 parameter model (26) to a 4 parameter model.

The time dependent tip deflection of the rod in Fig. 4 shows the superposition of right and left traveling waves. After traveling the distance  $2\ell$  the tip displacement vanishes. Compared with the elastic solution two changes indicate the influence of material damping. Due to viscoelastic stiffening and dissipation, the wave speed increases according to (38) because the constraint (27) requires  $q > p$  and the tip displacement is reduced.

The effect of viscoelasticity on pulse propagation is discussed next in terms of spectral analysis (Doyle 1998). The equation of motion (29) appears in spectral form

$$\frac{E^*(\omega)}{\rho} \frac{d^2 u^*}{dx^2} + \omega^2 u^*(x, \omega) = 0 \quad (39)$$

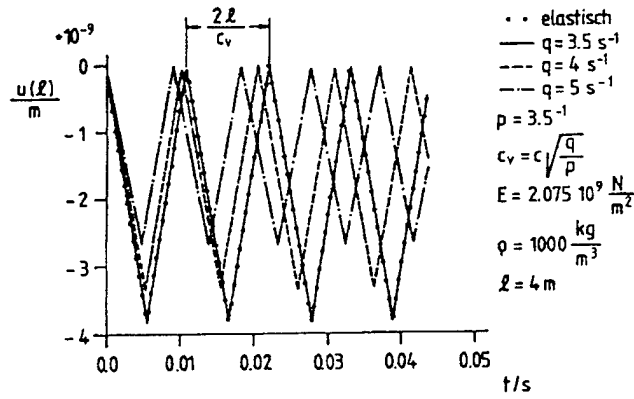


Figure 4. Tip deflection due to wave propagation in elastic and viscoelastic solid

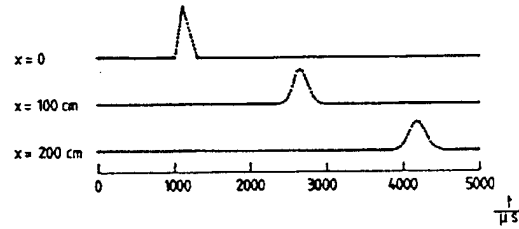


Figure 5. Effect of viscoelasticity on pulse propagation

with dispersive spectrum relation from

$$u = A e^{i(k^* x \pm \omega t)}, \quad k^* = \pm \omega \sqrt{\frac{\rho}{E^*(\omega)}}. \quad (40)$$

The nonzero imaginary part of the complex wave number  $k^*$  means there is attenuation and the effect of this is seen in a time reconstruction of a propagating pulse in Fig. 5.

If we consider the viscoelastic modulus (34)

$$E^*(\omega) = E \frac{1 + q(i\omega)^\alpha}{1 + p(i\omega)^\alpha} \quad (41)$$

it has very slow and very fast purely elastic limits

$$E^*(0) = E, \quad E^*(\infty) = E \frac{q}{p}. \quad (42)$$

Consequently the viscoelastic dissipation occurs only in the middle range. The phase and group velocities  $c$  and  $c_g$  respectively are in general given as

$$c = \frac{\omega}{k^*} = \sqrt{\frac{E^*}{\rho}}, \quad c_g = \frac{d\omega}{dk} = \frac{\sqrt{\frac{E^*}{\rho}}}{\left(1 - \frac{\omega}{2E} \frac{dE^*}{d\omega}\right)} \quad (43)$$

and have the same limiting values. The effect of viscosity on the propagation of a pulse is to decrease the amplitude and spread the pulse out because of the spectrum of speeds. Knauss (Knauss 1968) discusses uniaxial wave propagation in a viscoelastic material using measured material properties.

## 5. Viscoelastic Solids treated by the Boundary Element Method

The simulation of 3-d dynamic response of members with relevant material damping requires discretization methods to cope for complex geometry, boundary conditions and loading. The boundary element method (BEM) provides one powerful tool (Gaul and Fiedler 1997) for the calculation of elastodynamic response in frequency and time domain. Field equations of motion and boundary conditions along with initial conditions in time domain are cast into boundary integral equations (BIE), which are solved numerically by discretization of the boundary only, thus reducing the problem dimensions by one. The boundary data are often of primary interest because they govern the transfer dynamics of members. This substructure behavior can as well be implemented in finite element models or multi-body systems (Gaul and Chen 1993). The application of BEM for modal analysis has been treated by Lanzerath (Lanzerath 1996). Another advantage of the BEM is the treatment of energy radiation into a surrounding medium of infinite or semi-infinite domains, the so called radiation or geometric damping which does not incorporate dissipation. Simple surface discretization around the radiating source provides a non-reflecting boundary because the fundamental solutions of the BIE fulfill the Sommerfeld radiation condition excluding reflections. For the material damping described by viscoelastic constitutive

equations, the implementation in BEM formulations is demonstrated by the elastic-viscoelastic correspondence principle. Among numerous practical applications are the descriptions of elastomer resilient mounts or soil half-space presented herein.

### 5.1. ELASTIC BE-FORMULATION IN TIME DOMAIN

The Lamé field equations of a homogeneous isotropic elastic domain  $\Omega$  with boundary  $\Gamma$  are given by

$$(c_1^2 - c_2^2) u_{i,ij} + c_2^2 u_{j,ii} + b_j = \ddot{u}_j(x, t) \quad x \in \Omega \quad (44)$$

with displacement coordinates  $u_j(x, t)$  and wave speeds

$$c_1^2 = \frac{K + \frac{4}{3}G}{\rho}, \quad c_2^2 = \frac{G}{\rho}. \quad (45)$$

The corresponding traction and displacement boundary conditions are

$$\begin{aligned} t_i(x, t) = \sigma_{ik} n_k &= p_i(x, t) & x \in \Gamma_t, \\ u_i(x, t) &= q_i(x, t) & x \in \Gamma_u \end{aligned} \quad (46)$$

and the initial conditions are

$$\begin{aligned} u_i(x, 0) &= u_{0i}(x), \\ \dot{u}_i(x, 0) &= v_{0i}(x) & x \in \Omega. \end{aligned} \quad (47)$$

The 3-d Stokes fundamental displacement tensor of the Lamé equation (44) in an unbounded space, excited by the volume force  $b_j(x, \xi, t, \tau) = \delta(t - \tau) \delta(x - \xi) e_j$  is given by (e.g. (Schanz 1994))

$$\begin{aligned} \bar{u}_{ij}(x, \xi, t, \tau) = & \frac{1}{4\pi\rho} \left\{ \frac{t - \tau}{r^2} \left( \frac{3r_i r_j}{r^3} - \frac{\delta_{ij}}{r} \right) \left[ H\left(t - \tau - \frac{r}{c_1}\right) - H\left(t - \tau - \frac{r}{c_2}\right) \right] \right. \\ & \left. + \frac{r_i r_j}{r^3} \left[ \frac{1}{c_1^2} \delta\left(t - \tau - \frac{r}{c_1}\right) - \frac{1}{c_2^2} \delta\left(t - \tau - \frac{r}{c_2}\right) \right] + \frac{\delta_{ij}}{rc_2^2} \delta\left(t - \tau - \frac{r}{c_2}\right) \right\}, \end{aligned} \quad (48)$$

where  $r = \sqrt{r_i r_i}$ ,  $r_i = x_i - \xi_i$  is the Euclidean distance between the field point  $x$  and the load point  $\xi$ . The corresponding fundamental stress vector components are obtained by substituting (48) into the constitutive equation and adopting Cauchy's stress formula.

The dynamic extension of Betti's reciprocal work theorem combines two states of displacements and tractions,  $(\tilde{u}_{ij}, \tilde{t}_{ij})$  and  $(u_i, t_i)$  respectively, and leads to the boundary integral equation

$$c_{ij}u_j = \int_{\Gamma} [\tilde{u}_{ij} * t_j - \tilde{t}_{ij} * u_j] d\Gamma + \int_{\Omega} \rho [\tilde{u}_{ij} * b_j + \tilde{u}_{ij}v_{0j} + \dot{\tilde{u}}_{ij}u_{0j}] d\Omega \quad (49)$$

where  $c_{ij} = \delta_{ij}/2$  if  $\xi$  is located on a smooth boundary. The  $*$  denotes the convolution with respect to time. The integral equation (49) contains boundary integrals only if the volume forces  $b_j$  and the initial conditions vanish. Discretization of the boundary integral equation in space and time leads to the boundary element formulation. Only the time discretization by  $n$  equal steps  $\Delta t$  is discussed here. The simplest nontrivial choice ensuring that no terms drop out in the boundary integral equation are linear shape functions for the displacements  $u_i$  and constant shape functions for the tractions  $t_i$  in time domain

$$u_i(x, \tau) = \left( U_{il}^{m-1} \frac{t_m - \tau}{\Delta t} + U_{il}^m \frac{\tau - t_{m-1}}{\Delta t} \right) \eta_l(x) \quad (50)$$

$$t_i(x, \tau) = T_{il}^m \cdot \mu_l(x). \quad (51)$$

The actual time step is  $m$ . The nodal values are  $U_{il}^m, T_{il}^m$  for the corresponding boundary element  $\Gamma_l$  at time  $t_m = m\Delta t$ . After substituting equations (50) and (51) the boundary integral (49) reads

$$\begin{aligned} \int_0^t \int_{\Gamma} [t_i(x, \tau) \cdot \tilde{u}_{ij}(x, \xi, t - \tau) - \tilde{t}_{ij}(x, \xi, t - \tau) \cdot u_i(x, \tau)] d\Gamma d\tau = \\ \sum_l \sum_{m=1}^n \int_{\Gamma_l} \int_{t_{m-1}}^{t_m} \left[ \tilde{u}_{ij}(x, \xi, t - \tau) \mu_l(x) \cdot T_{il}^m \right. \\ \left. - \tilde{t}_{ij}(x, \xi, t - \tau) \cdot \eta_l(x) \cdot \left( U_{il}^{m-1} \frac{t_m - \tau}{\Delta t} + U_{il}^m \frac{\tau - t_{m-1}}{\Delta t} \right) \right] d\tau d\Gamma. \end{aligned} \quad (52)$$

Equation (52) can be integrated analytically (Schanz 1994). After substituting  $t_m = m\Delta t$ ,  $t_{m-1} = (m-1)\Delta t$  and  $t = n\Delta t$  it can be seen that the functions depend on the difference  $(n-m)$ , between the observation and the excitation time only. After time and space discretization a system of algebraic equations (Schanz 1994) is obtained

$$\left[ \frac{1}{2} I + \begin{pmatrix} 1 \\ T \end{pmatrix} \right] \begin{pmatrix} u \\ \end{pmatrix} + \sum_{m=1}^{n-1} \begin{pmatrix} n-m+1 \\ T \end{pmatrix} \begin{pmatrix} u \\ \end{pmatrix} = \sum_{m=1}^n \begin{pmatrix} n-m+1 \\ U \end{pmatrix} \begin{pmatrix} t \\ \end{pmatrix}, \quad (53)$$



where  $I$  is the identity matrix,  $T^{(m)}$  and  $U^{(m)}$  are the influence matrices of stresses and displacements at the time step  $m$ . The vectors  $u^{(m)}$  and  $t^{(m)}$  contain all nodal displacements and tractions of the time step  $m$ . Unknown boundary data are calculated in terms of known boundary data after re-ordering (53).

## 5.2. VISCOELASTIC BE-FORMULATION IN TIME DOMAIN

In order to obtain a viscoelastic boundary integral formulation from the elastic formulation (53) the elastic-viscoelastic correspondence principle (28) is applied. This requires the Laplace transformation of (53) first. The kernels of the matrices consist of the fundamental solutions of the displacements and tractions after time integration. For the sake of brevity the procedure of deducing a viscoelastic formulation is explained for the first term on the right side of (52) only.

The one sided Laplace transformation of the first term in (52) leads to

$$\begin{aligned} \int_0^\infty \sum_{m=1}^n \int_{t_{m-1}}^{t_m} \bar{u}_{ij}(x, \xi, t - \tau) d\tau e^{-st} dt = \\ \frac{1}{4\pi\rho} \sum_{m=1}^n \left\{ \int_{t_{m-1} + \frac{r}{c_1}}^{t_m + \frac{r}{c_1}} \left[ f_0(r) \frac{1}{2} \left( (t - t_{m-1})^2 - \left( \frac{r}{c_1} \right)^2 \right) + f_1(r) \right] e^{-st} dt + \right. \\ \int_{t_{m-1} + \frac{r}{c_2}}^{t_m + \frac{r}{c_2}} f_0(r) \left( t t_m - t t_{m-1} - \frac{t_m^2}{2} + \frac{t_{m-1}^2}{2} \right) e^{-st} dt + \\ \left. \int_{t_{m-1} + \frac{r}{c_2}}^{t_m + \frac{r}{c_2}} \left[ f_0(r) \left( t t_m - \frac{t_m^2}{2} - \frac{t^2}{2} + \frac{1}{2} \left( \frac{r}{c_2} \right)^2 \right) + f_2(r) \right] e^{-st} dt \right\}, \quad (54) \end{aligned}$$

with  $f_0$ ,  $f_1$  and  $f_2$  depending on spatial coordinates only. After integrating one time interval of (54), the elastic constants are replaced by the viscoelastic impact response functions. The elastic bulk and shear moduli appear only in the compression wave speed  $c_1$  and the shear wave speed  $c_2$ . Thus

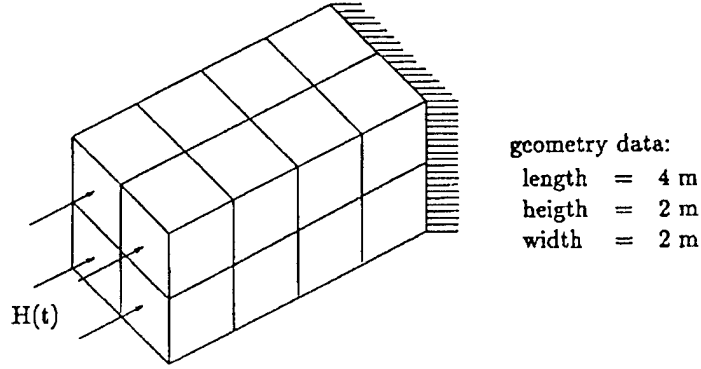


Figure 6. Step function excitation of a free-fixed bar

the corresponding viscoelastic expressions

$$\begin{aligned} c_{1v}^2 &= \frac{1}{\rho} \left( K \frac{1+qs^\alpha}{1+ps^\alpha} + \frac{4}{3} G \frac{1+qs^\alpha}{1+ps^\alpha} \right) \\ c_{2v}^2 &= \frac{1}{\rho} G \frac{1+qs^\alpha}{1+ps^\alpha} \end{aligned} \quad (55)$$

are inserted into equation (54) after integration. The method of Talbot has been selected for numerical inversion (Schanz 1994).

#### 5.2.1. Numerical example: Waves in a 3-d viscoelastic continuum associated to the 1-d formulation in section 4

The propagation of waves in a 3-d continuum has been calculated by the present boundary element formulation in time domain. The problem geometry and the associated boundary discretization are shown in Fig. 6. Linear shape functions in space have been used. The free end is excited by a pressure jump according to a unit step function  $H(t)$ . The opposite end is fixed at the nodes. The time step size  $\Delta t$  has been chosen close to the time it takes for the viscoelastic compression wave to travel across one element.

The viscoelastic material data of a corning glass at  $550^\circ C$  (Tab. 2) are used in this example.

Fig. 7 shows the longitudinal displacement in the center of the free end cross section versus time for several values of the damping parameter  $q$  in

TABLE 2. Material data for corning glass

$E = 2.075 \cdot 10^9 \frac{N}{m^2}$	$p_K = p_G = 3.5 s^{0.635}$
$\nu = 0.25$	$\alpha_K = \alpha_G = 0.635$
$\varrho = 1000 \frac{kg}{m^3}$	$q_K = q_G = q s^{0.635}$

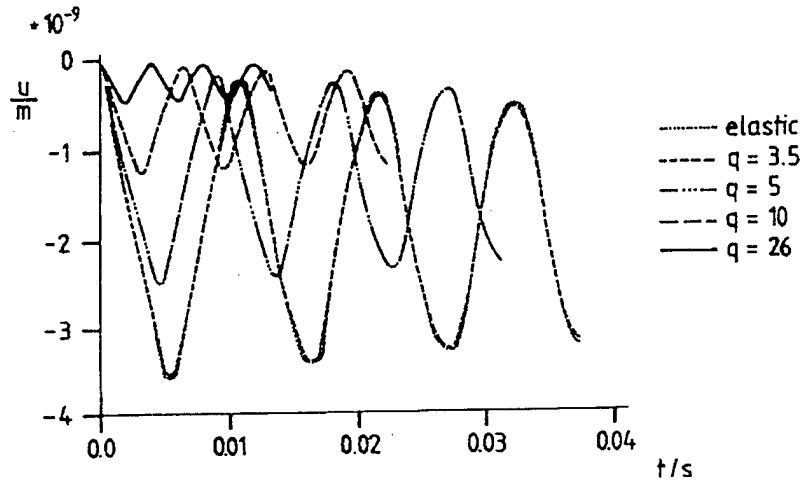


Figure 7. Step function response of a free-fixed bar

the constitutive equation (45). Wave reflections at the fixed and the free end show up. The viscoelastic wave speed of the compressional wave front is given by

$$c_{1v}^2 = \frac{1}{\varrho} \left( K \frac{q_K}{p_K} + \frac{4}{3} G \frac{q_G}{p_G} \right) \quad (56)$$

Increasing damping parameters  $q$  stiffen the solid and increase the wave speed. Similar to the 1-d solution in Fig. 4 shorter travel times show up in Fig. 7 and the stiffening leads to smaller deflections.

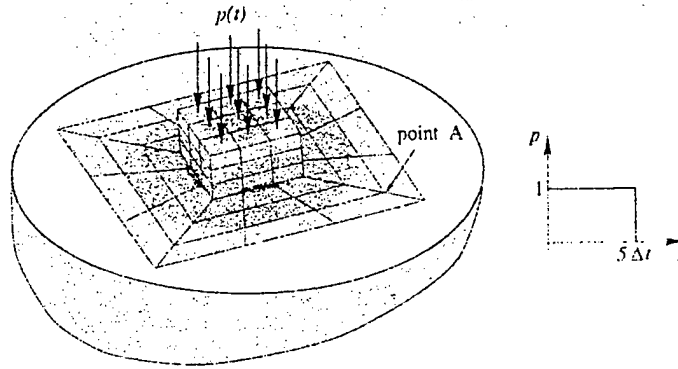


Figure 8. Elastic concrete slab on viscoelastic halfspace: boundary element discretization, surface displacement wave fronts, loading function

#### 5.2.2. Numerical example with radiation damping: Waves in semi-infinite soil

The propagation of waves in an elastic concrete foundation slab ( $E = 3 \cdot 10^8 \text{ N/m}^2$ ,  $\rho = 2000 \text{ kg/m}^3$ ,  $\nu = 0.2$ ) bonded on a viscoelastic soil half-space ( $\nu = 0.35$ ,  $E = 1.38 \cdot 10^8 \text{ N/m}^2$ ,  $\rho = 1966 \text{ kg/m}^3$ ,  $\alpha = 1.3$ ,  $p = 1 \text{ s}^{-1.3}$ ) has been calculated by the presented BEM in time domain. Both domains are coupled by a substructure technique based on displacement- and traction-continuity at the interface. The assumption of welded contact does not allow the nonlinear effect of partial uplifting.

The problem geometry and the associated boundary discretization are shown in Fig. 8. The soil discretization is truncated after a distance of the foundation length. The surface of the foundation slab is excited by a positive and negative pressure jump according to Fig. 8. Linear spatial shape functions have been used. Similar to the Courant criteria, the time step size  $\Delta t$  has been chosen close to the time needed by the viscoelastic compression wave to travel across the largest element.

In Fig. 9 vertical surface displacement at point A is plotted versus time for different values of the constitutive parameter  $q$ . Obviously the wave speeds increase for higher values of  $q$ , because of a stiffening of the material with growing influence of viscoelasticity. This is associated with a significant displacement reduction. Propagation of the wavefronts across the boundary of the discretized area shows the capability of the BEM to describe radiation damping without reflection (Schanz 1994).

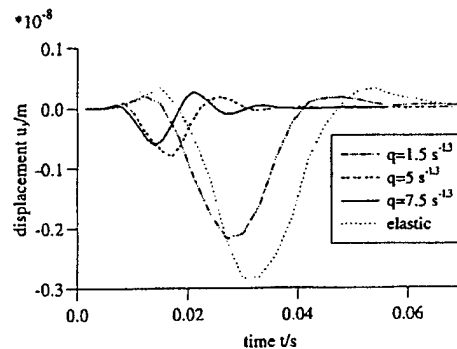


Figure 9. Displacement  $u_3$  perpendicular to surface on point A for different values  $q$

## 6. Damping Description for the Finite Element Method

The finite element method (FEM) is a well established domain discretization approach. It has found increasing applications in damping design of complex structures. Several approaches exist for damping modeling in the context of mdof structures discretized by FEM (Pilkey and Pilkey 1995, Garibaldi and Onah 1996) such as

- viscoelastic material behavior
- hysteretic damping with assumed frequency dependence
- local damping mechanisms .

A special case of viscoelastic behavior is the viscous damping assumption. This model extends the mass and stiffness matrix in the FE equation of motion by a viscous damping matrix. Various forms of constructing this matrix exist. Several of these such as the so called proportional damping (Rayleigh damping), the Caughey series (general Rayleigh damping), the direct procedure which constructs the damping matrix with real modes and modal damping ratios and the hysteretic damping allow to decouple the equations of motion by modal transformation (Garibaldi and Onah 1996). The application of fractional derivatives to modal analysis is discussed in (Bagley and Calico 1989, Maia, Silva, Ribeiro and Leitão 1996). A variety of solution procedures is based on real modes in time or frequency domain. If the damping mechanisms of the structures do not validate the decoupling assumption, complex modes exist. Decoupling can be obtained by rewriting the equations of motion in first order state space notation and transformation with right and left eigenvectors (Krämer 1984). Radiation

damping can be simulated in finite element models by means of infinite elements (Bettes 1992). The near field between the radiating surface and an artificial boundary has to be discretized. On the artificial boundary one can prescribe boundary conditions which incorporate (exactly or approximately) the far field behavior into the FEM model (Givoli 1992). Infinite elements use for example in frequency domain analytical solutions in radial direction and shape functions in circumferential direction.

### 6.1. FINITE ELEMENT FORMULATION OF VISCOELASTIC SOLIDS

The implementation of viscoelastic constitutive equations of conventional differential operator type (60) or generalized type (67) as well as associated hereditary integral formulations (61) into the FE formulation for an elastic structure is discussed next. Spatial discretization reduces the elastodynamics of a continuous structure to the FE equation of motion

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\} \quad (57)$$

with  $n$  dofs in the displacement vector  $\{u\} = [u_1 \ u_2 \ \dots \ u_n]^T$  with mass and stiffness matrix  $[M]$  and  $[K]$  respectively and external force vector  $\{F\}$ . The stiffness matrix is assembled of element stiffness matrices  $[K]_e$  which relate nodal dofs and nodal forces

$$[K]_e\{u\}_e = \{F\}_e. \quad (58)$$

The principles of mechanics lead to the stiffness matrix

$$[K]_e = \int_V [B]^T [E] [B] dV \quad (59)$$

by volume integration over the element domain. The elasticity matrix  $[E]$  relates the stress state with the strain state

$$\{\sigma\} = [E]\{\epsilon\}. \quad (60)$$

In displacement theory, the element displacements  $\{v\}$  are expressed in terms of the nodal displacements as

$$\{v\} = [N]\{u\}_e \quad (61)$$

by the matrix of shape functions  $[N]$ . The strain displacement relation with the differential operator matrix  $[D]$

$$\{\epsilon\} = [D]\{v\} = [D][N]\{u\}_e = [B]\{u\}_e \quad (62)$$

defines the element strain displacement matrix  $[B]$ . The elastic-viscoelastic correspondence principle is adopted to implement viscoelastic constitutive equations. A matrix notation of Hooke's law (57), (58), (59) and (60) is given by

$$\{\sigma\} = (2G[E]_G + 3K[E]_K) \{\varepsilon\} \quad (63)$$

with the vectors

$$\begin{aligned} \{\sigma\} &= [\sigma_{xy} \sigma_{yz} \sigma_{zx} \sigma_{xx} \sigma_{yy} \sigma_{zz}] \\ \{\varepsilon\} &= [\varepsilon_{xy} \varepsilon_{yz} \varepsilon_{zx} \varepsilon_{xx} \varepsilon_{yy} \varepsilon_{zz}] \end{aligned} \quad (64)$$

and the matrices

$$[E]_G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \text{sym.} & & & & \frac{2}{3} & -\frac{1}{3} \\ & & & & & \frac{2}{3} \end{bmatrix}, [E]_K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \text{sym.} & & & & \frac{1}{3} & \frac{1}{3} \\ & & & & & \frac{1}{3} \end{bmatrix} \quad (65)$$

## 6.2. DIFFERENTIAL OPERATOR FORMULATION

The corresponding viscoelastic constitutive equation in differential operator notation is obtained by replacing the bulk modulus  $K$  by  $\frac{1}{3}(Q_H/P_H)$  and the shear modulus  $G$  by  $\frac{1}{2}(Q_D/P_D)$ . As division with differential operators is not defined, (63) is rewritten as

$$P_D P_H \{\sigma\} = ([E]_G Q_D P_H + [E]_K Q_H P_D) \{\varepsilon\}. \quad (66)$$

The elastic constitutive equation (60) is now replaced by the viscoelastic one (66). The obtained equation of motion of one finite element with the mass matrix  $[M]_e$

$$[M]_e P_D P_H \{\ddot{u}\}_e + ([K]_G Q_D P_H + [K]_K Q_H P_D) \{u\}_e = P_D P_H \{F\}_e \quad (67)$$

includes the volume integrals

$$[K]_G = \int_V [B]^T [E]_G [B] dV, \quad [K]_K = \int_V [B]^T [E]_K [B] dV. \quad (68)$$

The equation of motion of the structure associated to (57) is obtained by assembling the finite element equations of motion (67).

### 6.3. HEREDITARY INTEGRAL FORMULATION

The equivalent hereditary integral formulation of the constitutive equation is the integrated differential operator formulation (66)

$$\{\sigma\} = (2G_0[E]_G + 3K_0[E]_K) \{\varepsilon\} + \int_0^\infty \left( 2\tilde{G}(\tau)[E]_G + 3\tilde{K}(\tau)[E]_K \right) \{\dot{\varepsilon}(t - \tau)\} d\tau \quad (69)$$

with relaxation modulus for shear  $G(t) = G_0 + \tilde{G}(t)$  and compression  $K(t) = K_0 + \tilde{K}(t)$ . The associated finite element equation of motion is an integro-differential-equation

$$[M]_e \{\ddot{u}\}_e + (2G_0[K]_G + 3K_0[K]_K) \{u\}_e + \int_0^\infty \left( 2\tilde{G}(\tau)[K]_G + 3\tilde{K}(\tau)[K]_K \right) \{\dot{u}(t - \tau)\}_e d\tau = \{F\}_e. \quad (70)$$

Both formulations, the differential operator type and the hereditary integral type, have in common that the element matrices  $[M]_e$ ,  $[K]_G$  and  $[K]_K$  in (67), (68) and (70) can be generated by commercial FEM software with elastic constitutive equations.

### 6.4. SOLUTION APPROACHES FOR FE EQUATIONS

Available approaches for solving the structure equation of motion (Carpenter 1972, Krings 1976, Padovan 1987) often prefer the differential operator formulation 13 for numerical reasons. Such approaches for calculating the displacement response  $\{u\}$  are:

- Numerical integration (Padovan (Padovan 1987) uses the Grünwald definition of fractional derivatives in (12) for time-stepping algorithms)
- Finite time elements
- Integral transforms (Laplace, Fourier)
- Modal analysis (A summary of approaches is presented in (Pilkey and Pilkey 1995, Garibaldi and Onah 1996, Maia et al. 1996, Bagley and Calico 1989))
- Matrix functions

After the displacement response is known, differential equations for the stresses have to be integrated (Carpenter 1972, Krings 1976).

#### 6.4.1. Example: Viscoelastic finite rod element

The equation of motion of a finite rod element in Fig. 10 shall be adapted to a viscoelastic conventional 3 parameter model with uniaxial constitutive



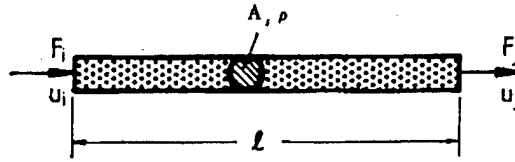


Figure 10. Rod element

equation

$$p_1 \dot{\sigma}_{xx} + \sigma_{xx} = q_0 \epsilon_{xx} + q_1 \dot{\epsilon}_{xx} . \quad (71)$$

One corresponding rheological model consists of a spring  $E_0$  in parallel with spring  $E_1$  and a viscous dashpot  $R_1$  in series such that the parameters are related to (71) by

$$p_1 = \frac{R_1}{E_1} \quad q_0 = E_0 \quad q_1 = R_1 \left(1 + \frac{E_0}{E_1}\right) . \quad (72)$$

For the stress and strain state of the slender rod the correspondences

$$E \rightarrow \frac{Q}{P} = \frac{q_0 + q_1 \frac{d}{dt}}{1 + p_1 \frac{d}{dt}} \quad 2G \rightarrow \frac{Q_D}{P_D} = \frac{1}{1 + \nu} \frac{Q}{P} \quad 3K \rightarrow \frac{Q_D}{P_D} = \frac{1}{1 - 2\nu} \frac{Q}{P} \quad (73)$$

with real Poisson's ratio  $\nu$  hold.

Linear shape functions lead to the consistent mass matrix  $[M]_e$  and elastic stiffness matrix  $[K]_e$

$$[M]_e = \frac{\rho A \ell}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad [K]_e = \frac{E_0 A}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (74)$$

with modal displacement and force vectors

$$\{F\}_e = \begin{bmatrix} F_i & F_j \end{bmatrix}^T \quad \{u\}_e = \begin{bmatrix} u_i & u_j \end{bmatrix}^T \quad (75)$$

according to Fig. 10.

The differential operator type FE equation of motion (67) leads to

$$p_1 [M]_e \{\ddot{u}\}_e + [M]_e \{\ddot{u}\}_e + \frac{q_1}{E_0} [K]_e \{\dot{u}\}_e + [K]_e \{u\}_e = \{F\}_e + p_1 \{\dot{F}\}_e . \quad (76)$$

With the relaxation modulus

$$E(t) = E_0 + E_1 e^{-\frac{t}{\tau_1}} \quad (77)$$

and relaxation moduli

$$2G(t) = \frac{E(t)}{1 + \nu} \quad 3K(t) = \frac{E(t)}{1 - 2\nu} \quad (78)$$

the hereditary integral type FE equation of motion (69) leads to

$$[M]_e \{\ddot{u}\}_e + [K]_e \{u\}_e + \int_0^\infty \frac{E_1}{E_0} e^{-\frac{\tau}{\tau_1}} [K]_e \{\dot{u}(t - \tau)\}_e d\tau = \{F\}_e. \quad (79)$$

The Kelvin-Voigt model of spring  $E_0$  and dash-pot  $R_1$  in parallel is obtained from (76) and (79) by taking the limit  $E_1 \rightarrow \infty$

$$[M]_e \{\ddot{u}\}_e + \frac{R_1}{E_0} [K]_e \{\dot{u}\}_e + [K]_e \{u\}_e = \{F\}_e. \quad (80)$$

The differential operator type FE equation for the constitutive equation (71) but with fractional order time derivatives  $\alpha = 1/2$  has been treated by Bagley (Bagley and Torvik 1985).

## 7. Summary

The paper provides a unified approach for conventional and generalized linear models of viscoelastic constitutive behavior. Creep, relaxation and hysteresis effects of materials and structures are described consistently. Advantages of the fractional derivative concept are outlined. Mathematical consequences resulting from operator non-locality in time domain and uniqueness questions arising in frequency domain are addressed. The elastic-viscoelastic correspondence principle serves as a tool to obtain as well analytical and numerical BEM and FEM solutions of wave propagation and vibration problems by transform methods. Characteristics of viscoelastic waves and vibrations are discussed. The paper is focussed on material damping but includes aspects of radiation damping description by discretization methods as well.

Important aspects of damping description are beyond the scope of selected topics of this survey paper. This is why additional reading is recommended on the following subjects: Thermo-viscoelasticity and nonlinear viscoelasticity (Christensen 1971, Ferry 1980, Lazan 1968), determination of mechanical properties by experimental methods (Ferry 1980, Garibaldi and Onah

1996, Mahrenholtz and Gaul 1997, Bert 1973), damping devices and surface damping treatment (Nashif, Jones and Henderson 1985, Cremer, Heckl and Ungar 1973, Garibaldi and Onah 1996), material damping data (Lazan 1968, Nashif et al. 1985) and structural damping (Ruzicka 1960) including the nonlinear dissipation in mechanical joints such as bolted or riveted connections (Gaul and Lenz 1997, Gaul and Sachau 1997, Ottl 1985). A list which is by far not complete.

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## Essay on

# NON-STRUCTURAL DYNAMICS

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## 1. Introduction

Dynamics is solidly based in Newtonian physics, admitting modelling by equations representing force balancing. Whilst practitioners readily acknowledge that models are 'approximate' in some sense, great effort and faith is put into deterministic models, predominantly linear, unchanging and often finite dimensional - and even with these restrictions, great insight into fundamentals and practicalities has resulted. However, the complexity of even the simplest real systems soon reveals the limitations of these approximations. Accordingly, structural dynamics is evolving along lines that incorporate conceptual, mathematical and probabilistic ideas that allow inclusion of more realism. These are generally referred to in a way that implies exclusion from the accepted and well-developed formalism - and are generally prefixed with - NON - *non-stationary; nonlinear; non-Gaussian; non-deterministic*. This essay comments on aspects of these *non* developments and speculates on, and questions, their relevance and importance for the future of structural dynamics.

## 2. Non-stationarity

The wide ranging and topical subject of condition monitoring and fault diagnostics/classification is essentially aimed at quantifying changes in systems. Information on these changes is assumed to be contained in measured signals and particular changes may have revealing signatures. Fundamental to monitoring the condition of a system is the implication that *temporal* changes will manifest themselves in the modes of operation, i.e., if the system changes, then measurements based on system behaviour will be correspondingly non-stationary.

For example, progressive failures (e.g., developing cracks) in mechanical or structural systems can result in changing signatures. These are non-stationary and/or nonlinear features. For example, a cracked vibrating beam exhibits weakly nonlinear characteristics and the resulting dynamical behaviour may be related to crack characteristics [1]. Additionally, crack identification may be perceived through changes

in natural frequencies [2]. If changes occur over short time scales then continuous monitoring may be needed. Non-stationarity in signals and systems can be addressed in the time domain or in transform domains. In particular, time-frequency analysis is playing an increasing role in practical dynamics analysis.

## 2.1 Time-frequency Analysis

The essence of this form of analysis is to consider how the "frequency" of a signal or system "changes with time". This notion is readily accepted but introduces very fundamental considerations.

A starting point is to decide just what is meant by 'frequency' in a varying context. The concepts of instantaneous amplitude, phase and frequency of a signal lead to the analytic signal  $\sigma_x(t) = x(t) + j \hat{x}(t) = A(t)e^{j\phi(t)}$ , where  $\hat{x}(t)$  is the Hilbert transform of  $x$ ,  $A(t)$  is the instantaneous amplitude and  $\phi(t)$  is the instantaneous frequency. This is, for many 'simple' signals (i.e., monocomponent), an intuitively attractive alternative to Fourier methods. The term 'instantaneous frequency' provides the link between frequency and time. Another link between time and frequency is based on Fourier analysis through the concept of group delay, leading to an alternative time-frequency descriptions of a signal. The two descriptions are equivalent (roughly) if a signal is monocomponent and has a large bandwidth-time product. These apparent *alternatives* to linking time and frequency are a basis for considering these more deeply.

Fundamental considerations are given in a text edited by Boashash [3] and a survey paper is [4]. A recent special issue of Mechanical Systems and Signal Processing [5] has been devoted to time-frequency methods with several of the contributions directly related to condition monitoring. The choice of which time-frequency approach is to be used involves deciding the relative merits of linear decomposition (e.g., short time Fourier transform, Gabor forms, wavelet transform) *vs* quadratic forms (Wigner distributions, Cohen class) or nonlinear forms. Presentations in the special issue relate to:

- discussions of providing a rationale for automatic interpretation of time-frequency maps characterising damage which incorporate both physical modelling and pattern recognition.
- The analysis of gearbox faults involving the three stages of obtaining a time-frequency representation with good 'readability', interpretation of the main

components with respect to models of gearbox vibration and, finally, validation of the identified models with respect to the original data.

- Sometimes there is a need for clear identification of indicators of early machine degradation when faults may be of such brief duration that they occupy only a small fraction of the analysis interval. The Cohen-Posch distribution is used to locate such events.
- The analysis of speed-dependent vibration signals measured on rotating machinery is accomplished using several sensors giving methods for directional and spatial decompositions using time-frequency plots.
- Gear vibration manifesting itself in amplitude and phase modulation of tooth meshing vibration is analysed using wavelets.
- Higher Order Spectral methods (see Section 3) using the Wigner Higher Order Moment Spectra (WHOMS) are used to detect faults in rotating machinery.

Other time-frequency applications include: Dalianis *et al* [6], who use time-frequency methods to characterise abrupt structural changes in the response of a vibrating beam, and Bonato *et al* [7], who use the cross Wigner distribution to characterise vibrating systems under non-stationary excitation. The class of signals that are termed cyclostationary (i.e., those for which the correlation functions are periodic) are clearly of relevance to rotating phenomena with stochastic elements. There is an extensive bibliography on the subject and we note Hardin and Miamee [8], in which a general class of processes called correlation autoregressive (CAR) is described, which include cyclostationary processes. This is applied to data arising from helicopter noise. Other papers addressing these issues include [9,10] and higher order cyclostationarity has also been discussed [11].

## 2.2 Time Domain

Purely *time domain* approaches to monitoring changes are essentially aimed at (adaptive) methods for fitting of parametric models to such observations include Prony analysis (where sums of damped exponentials are fitted to ring down data or autocorrelation), maximum entropy spectral analysis, in which AR models are fitted to random time series (assuming white excitation of the system), maximum likelihood estimation using FIR filter forms and others. In general, systems are ARMA in form and the problem of estimating coefficients from noisy output measurements has been

given significant attention. The thesis by Kim [12] gives a good overview of the methods. For non-Gaussian inputs, interest has centred on cumulant-based (see Section 3) parametric estimation methods. Model-based fault detection methods are described in [13] and [14]. Basseville *et al* [15] consider a system identification approach for damage monitoring in vibrating structures.

### 2.3 The Future; *Speculation I*

Detection of non-stationarity is vital for condition monitoring, but is it fundamental to the development of structural dynamics? *Specifically*, is modelling of 'short-time' changes in dynamical behaviour important (or even feasible?) for structural dynamics, or should we rely on statistical/time series methods to detect signature changes? In short, is non-stationarity a red-herring for fundamental structural dynamics?

## 3. Nonlinearity and Non-Gaussianity

Nonlinear dynamics is enormous in scope and has a long history - ranging from classic nonlinear differential equation theory to topological methods and chaotic dynamics. The describing differential equations are nonlinear, superposition is not applicable, and multiple solutions can exist. Structures under harmonic excitation can exhibit sub- and superharmonic, quasi-periodic and chaotic responses. There have also been many attempts to generalise the well understood approaches that serve linear systems well. For example, the notions of 'modes' and 'mode shapes' have been re-appraised. The paper by Vakakis [16] is an overview of so-called nonlinear normal modes.

In this discussion, we focus on the generalisation of the linear system input-output form to the Volterra form leading to higher-dimensional impulse response functions and corresponding frequency response forms. This formalism is well-established but limited in its more general practical utility in dynamics. (Why?) It is also appropriate to note the inclusion of the concept of non-Gaussianity. The response of a nonlinear system driven by a Gaussian input is in general non-Gaussian. Accordingly, the two concepts are sometimes linked - that is, the detection of non-Gaussianity *may* be considered as evidence of nonlinearity.



The full probabilistic structure of a random process is often too difficult to compute but moments of the process can sometimes be useful and more easily obtained. Second order properties, namely, auto- and cross-correlation functions and spectra, are used for Gaussian processes. Higher order moments, cumulants and spectra are used for non-Gaussian processes. This area of study is referred to as higher order statistics (HOS), e.g., [17]. The linking of non-Gaussianity to nonlinearity was emphasised by Rao [18]. The use of either moments or cumulants is one of choice. The key characteristic of cumulants is that they are blind to Gaussianity whilst moments are not. Accordingly, if cumulant-based methods are applied to signals composed of a non-Gaussian process and additive Gaussian noise, then the cumulants are (asymptotically) unaffected by the noise.

Furthermore, cumulants and higher order spectra contain amplitude and phase information about the process in contrast to auto-correlation and power spectra. This additional information contained by HOS is vital to system identification (e.g., identification of non-minimum phase systems), signal reconstruction (of non-minimum phase signals) and inverse problems.

The first of the higher order spectra is the third order spectrum, the bispectrum. However, the bispectrum only yields information in cases where the random process has a skewed distribution. In a significant number of physical problems, systems are symmetrical and yield unskewed output signals; in these circumstances the bispectrum is an uninformative measure; see [19] which shows the concepts associated with the bispectrum carried over to the fourth order spectrum, referred to as the trispectrum, and discusses how the trispectrum can be used to analyse symmetric nonlinearities.

Higher Order Statistics are relevant in the exploitation of non-Gaussianity even in the context of linear systems in relation to system identification and in inverse problems. In surveillance, extraction of the 'cause' of a measured signal is often necessary. This area of signal processing is called inversion and in the linear case may be expressed as

$$y = Ax + n$$

with the aim of recovering  $x$  from measurement  $y$  in the presence of noise  $n$ . The operator  $A$  may or may not be known. The problem may be expressed in the time or frequency domain. This involves the inversion (in some sense) of the operator  $A$  such that the recovered signal (say  $\hat{x}$ ) is a good approximation to  $x$ . Generally, inverse

problems are ill-conditioned, in that small perturbations in  $y$  or  $A$  lead to large changes in the recovered estimate of  $x$ . The aim therefore is to find methods to create inverse operators that are not unduly sensitive to these perturbations. The procedure for achieving this is 'regularisation'.

This is a widely studied problem resulting in various approaches to the construction of the pseudo-inverse to matrix  $A$ , e.g., using Tikhonov regularisation, singular value decomposition, generalised cross validation, etc; see [20]. However, if the operator  $A$  is unknown, this is the *blind* inversion problem and now higher order statistics again come into play, in which the success of the procedure is dependent on the non-Gaussianity of the input to be recovered. The use of cumulant based objective functions in the design of linear inverse filters for impulsive type excitation is described in [21].

### 3.1 The Future; *Speculation II*

Nonlinearity can occur in unlimited forms. However, many real systems are an assemblage of essentially linear systems interconnected through *local* nonlinear connections.

Is there an effective and practical approach to this problem, and what role does HOS have to offer/play here?

## 4. Non-determinism

By a 'deterministic' structure we mean one that is described by a mathematical model that (subject to the fact it is an approximate description in some sense) we treat as exact. For *forward prediction* of the response, we solve the equations for the particular inputs and boundary conditions; for *determination of unknown systems* we use input and output data to estimate *parameters* of the model; for the *determination of the input* we use the measured output and invert the system in some way. This section discusses the practical issue of lack of knowledge of the system and the measured signals, i.e., non-determinism. We distinguish between two types of system non-determinism.

Non-determinism from *Complexity*

Distributed systems are infinite dimensional and are often approximated by a high order finite dimensional model. Detailed consideration of such systems becomes impractical and approximate methods have been introduced and are currently of great interest. These include Statistical Energy Analysis (SEA), fuzzy structure theory, and approaches combining modal analysis and SEA.

#### Non-determinism from *Variety*

When there are many possible configurations of a system, writing down a set of equations *a priori* may not be definitive. Indeed, a variety of models may be candidates. Under such circumstances, system identification and parameter estimation is resorted to, i.e., experimental 'verification' of the model. *But, which model?* The problem of fitting parametric models to data is well worn, including aspects of model order determination, but more formalised approaches to model selection in addition to order determination and parameter estimation are needed. Recently, Bayesian methods have been demonstrated to offer great insight into signal processing/time series problems and new computational methods for numerical integration and optimisation have made the procedures increasingly relevant.

The remainder of this paper is aimed at suggesting that these concepts arising from dynamics and from probabilistic/time series methods could come together to provide a powerful and effective approach to dealing with real dynamical systems.

#### 4.1 Complexity

It is often the case that determination of the individual modes is unrealistic owing to the 'high modal density' arising with increasing frequency. This has led to Statistical Energy Analysis, in which a system is modelled as a collection of coupled subsystems characterised by their vibrational energy; each subsystem must contain a number of resonant modes in the analysis bandwidth. The Proceedings of a recent Symposium on SEA gives a current overview of theory, experiment and applications [22]. Langley and Bremner [23] point out that this implies that the wavelength of subsystem deformation must be less than or of the same order of the dimensions of the subsystem. This is not necessarily always the case, e.g., bending motion may satisfy this, but in-plane motion may not. Accordingly, they propose an approach that combines conventional modal analysis for modelling long wavelengths and SEA for modelling short wavelengths. This approach complements the fuzzy-structure approach in which a master-structure has attached to it a set of uncertain or fuzzy

attachments. Use of this formulation allows representation of the dynamics in matrix partitioned form for global response and (coupled) local response. The global analysis essentially follows the 'usual' procedures and are linked in with the local responses (described using SEA methods) with due allowance for the coupling existing between the two types of response. An important general result of this is that if the local modes have a high degree of modal overlap then the main effect of the local mode dynamics is to add damping and an effective mass to the global co-ordinates, in line with fuzzy-structure theory.

Fuzzy structure theory was introduced by Soize in 1986 and described in a book [24]. The aim of this is to predict the medium-frequency local response of a 'master' structure coupled to a large number of subsystems or secondary structures. Owing to their complexity these subsystems are called fuzzy substructures. [Note: [24] that fuzzy structure theory does not correspond to a classical dynamics problem with random uncertainties nor does 'fuzzy' have anything to do with the mathematics of fuzzy sets and logic. Fuzzy structure theory introduces a random boundary impedance operator to model the effects of the fuzzy substructures on the master structure.]

#### 4.2 Variety

We now describe further non-determinism by including within the mathematical framework the possibility of different models, each with different parameters of unknown order, being candidates to describe a data set. This formalism is set within a time series and probabilistic context and recent theoretical developments, matched by major steps in algorithm development have led to some impressive practical applications - reported mainly in the signal processing literature. This is the Bayesian approach led by the Cambridge University Signal Processing Group over a number of years. The book by O'Ruaniidh and Fitzgerald [25] has arisen from this work.

#### Bayesian Analysis [25]

Two basic problems exist in data analysis. First, we need to choose (from a set of possible models) that which is supported by the data - called model selection. Second, we wish to determine the values of the model parameters - called parameter estimation. The essence of the method may be briefly summarised as follows.

We start with the likelihood function. For an observed data set,  $d$ ,  $p(d|\theta, M_k)$  denotes the probability of realising data given  $d$  given parameters  $\theta$  (of model and noise)

and model  $M_k$  ( $k = 1, \dots, M$  denotes candidate models). The maximum likelihood approach estimates parameters  $\theta$  by locating the supremum of  $p$ , i.e., by maximising the probability of observing the data by selecting  $\theta$ , but (i) this does not use prior information that may be available; (ii) the likelihood on its own does not limit the number of parameters that may 'fit' the data, i.e., it does not control complexity of the model. Ways of controlling complexity are well known, e.g., Akaike's Information Criterion (AIC) and the Minimum Description Length (MDL), amongst others [26].

The Bayes' approach builds in *prior* information expressed as

$$p(\theta | d, M_k) = \frac{p(d | \theta, M_k) p(\theta | M_k)}{p(d | M_k)}$$

or in words                      posterior =  $\frac{\text{likelihood} \propto \text{prior}}{\text{evidence}}$

i.e.,  $p(\theta | M_k)$  is the prior probability summarising knowledge of the parameters prior to observing the data. The denominator  $p(d | M_k)$  is called the *evidence* which is of interest in model selection. The quantity  $p(\theta | d, M_k)$  is the posterior density summarising the state of knowledge about the parameters after the data is observed, i.e., Bayes provides an approach by which prior information is updated on the basis of new data. The posterior density may be used for parameter estimation, i.e., Maximum a Posteriori (MAP) estimation.

The Bayesian approach offers much for model selection, parameter estimation and a rational basis for compromise between accuracy of fit to the data and parametric complexity (Ockham's razor), but it introduces difficulties too. What form should priors take? How can we compute the conditional probabilities and find optima? The book [25] and associated and more recent results from the Cambridge Group are interesting demonstrations of the power of this approach.

#### 4.3 The Future; *Speculation III*

Can the Bayesian approach and the existing methods of SEA, fuzzy structures, uncertain parameters, etc., be brought together to provide a unified approach to model determination and parameter estimation for identification of classes of structural systems?

## 5. Concluding Remark

This essay has noted three speculations and they should be ranked in importance. They have been presented in the sequence: non-stationarity; nonlinearity and non-Gaussianity; non-determinism. Whilst not diminishing the intrinsic importance of each, it is the author's *speculation* that their importance to the future development of structural dynamics is in the reverse order - what do you think?

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# Multifunctional Structures of the Next Millennium

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This paper is an essay on my view of one possible way forward in structural dynamics. Smart materials or material transducers are materials that contain sensors, actuators, and control systems that allow structures to respond or adaptively change as the result of external conditions. Such materials form transducers that are able to convert electrical or magnetic energy into mechanical motion or force and vice versa. Structures made of these materials present new opportunities for the structural dynamics community. Piezoelectric patch and stack actuators, shape memory alloys and electrorheological fluids, when incorporated into structures, can provide a myriad of benefits and solutions to vibration problems such as vibration suppression, aeroelastic control, shape control, and diagnostics. It is clear that new technologies and materials form a key to creating improved structures. It is also clear that our user community is extremely conservative and resists change. A discussion of smart structures is presented followed by comments on dealing with change, with the hope of provoking action and response from the structural dynamics community.

## Introduction

In the 90's many of those working in structural dynamics started to seek solutions to various vibration and design problems using unusual materials (Rogers, 1993, and Tzou, 1998). These material systems and structures, known as smart structures, consist of a variety of components that have the ability to change one or more of their physical properties as the result of an externally applied field (usually magnetic or electric). Conversely these systems can produce currents as the result of mechanical forces or strains. Such devices have been used for years as accelerometers and shakers, but when fully integrated through out a structure in some composite fashion, form many new possibilities for structural dynamics. For example, currently several air frame manufactures are looking into the possibility of removing flaps from an aircraft wing in favor of an "adaptive wing" which is capable of changing its shape. This "new" technology has forced the integration of the field of structural dynamics with the fields of control, materials and circuit theory.

In a similar way, MEMS (Micro Electro Mechanical Systems) have produced the ability to integrate sensing and actuation functions into structures, forming new possibilities. An example of a MEMS device is an accelerometer made at the molecular level. Both smart structures and MEMS technologies require knowledge of mechanical and electrical components, highlighting the importance of the field of Mechatronics, which has emerged

in the last decade. Mechatronics is the discipline of integrating mechanical and electrical components at an initial design stage rather than treating them separately. This trend to integrate the functions of sensing, actuation and electronics into the structure itself has raised interest in defining Multifunctional Structures with the hope of advancing the performance and utility of structures. Multifunctional Structures may be loosely defined as the integration of devices into structures and structural design. A simple example of a multifunctional structure is the integrating the antenna functions of an aircraft into the skin of the airframe. All of these technology areas have important consequences for structural dynamic engineers.

The hope of modern structures is application driven. The goal is to increase the performance of structures by reducing their weight, increasing strength and utility. This is not profound. An examination of the history of engineering shows that most great breakthroughs in design, modeling and analysis came as the result of a focused, application driven projects: flight, reaching the moon, spanning rivers, etc. The exception may be computer technology which has developed under its own steam, yet provided advances in structural dynamics analysis and measurement by making new tools available.

The concept of working in structural dynamics is an evolving one. Between the first and second World Wars, it simply met that you knew how to compute dynamic safety factors from static calculations. At one time, the forefront of structural dynamics research implied the ability to write large FORTRAN codes. Important sub sets of structural dynamics required and understanding of fluid dynamics which eventually led to the field of aeroelasticity. An engineer working in structural dynamics for the space program soon found a need to couple an understanding of structural dynamics with thermodynamics. As applications changed, performance demands increase and the abilities required of structural dynamics engineering have evolved into numerous related areas. The goal of this article is to suggest that the next generation of structural dynamics engineers will need expertise in smart materials and generally be adapt at integrating electric, thermal, fluid and magnetic properties into their skills.

Several examples of the use of smart materials integrated into structures to form smart material systems or smart structures are presented to provide the flavor of some possible uses of this technology in the structural dynamics community. Many other examples may be found in the literature. Hopefully these examples will motivate the integration of these materials into standard practice in the structural dynamics community.

### **Smart Damping**

This is the notion that many of these new materials can provide passive damping by changing mechanical motion into an electrical field, then using a circuit, or shunt, to dissipate electric energy as heat. The smart material that has received the most attention is the piezoceramic shunt that has come to the attention of vibration experts in the early 90's. Specifically the piezoceramic effect may be used along with a passive circuit to

produce a damping mechanism with unique capabilities comparable to the damping properties of viscoelastic based constrained layer damping treatments commonly in use.

Passive piezoceramic materials have illustrated the ability to modify the resonant response of a structure. Forward (1981) demonstrated the use of a resistive shunt placed across a piezoceramic to provide damping and verified this result experimentally. Since then, a number of researchers have modified and developed this simple idea into a variety of applications (Lesieutre, 1998).

Rather than applying a viscoelastic layer to a host structure a piezoelectric device (usually a piezoceramic) is layered into or on a host structure and shunted to a resistor or resistor and inductor. As the host vibrates, the piezoelectric effect changes the induced strain into a voltage that is then dissipated as heat through the resistive shunt circuit. The result is a system that produces a loss factor versus frequency curve much like that of a viscoelastic material. Only loss factors of about 0.45 for longitudinal vibration and just 0.08 in the transverse direction are obtained when practical values of resistance are used. However, the peak value of the loss factor can be easily change from one value of frequency to another providing increased design flexibility.

If an inductor is added to the shunt, it behaves like a vibration absorber and can produce larger loss factors at resonance than a comparably sized constrained layer damping treatment, but over a narrow band. In addition, the shunted piezoceramic system is not as temperature dependent as the viscoelastic counter part, and is much stiffer than a viscoelastic material (VEM). Piezoelectric materials also do not suffer from creep, common to viscoelastic materials. While shunted piezoceramics do not directly compete with VEM for adding damping, they do offer more design flexibility, increased stiffness and temperature stability. This approach may also provide some advantages in terms of temperature response, mass location and design parameters.

One of the technical difficulties with designing a vibration suppression system with a shunt is that the size of the inductance required increase with the level of damping added. This requires the use of synthetic inductors. The use of additional capacitors can reduce the required inductance. This sort of problem points directly to the importance of the interactions between structural dynamics and circuit theory.

A variety of other smart materials, such as shape memory alloys or magnetostrictives, may provide improved damping solutions in passive settings when combined with a shunt of some sort. This is largely unexplored, but several researchers are working in this direction. Already those who work on building dampers and vehicle shock absorbers have exploited use of more exotic materials and this has resulted in improved damping.

### **Adaptive Wing Programs**

One of the more exciting prospects for the use of smart materials is in structural dynamics of airfoils. Here the goal is to remove traditional control surfaces in favor of wings that

change shape and other properties to provide improved performance as well as to provide maneuver control. Some of the motivation for this is the desire to produce an all-electric aircraft. Adaptive wings use various active materials (shape memory alloys, piezoceramics, electrostrictives, etc.) to bend, twist and change the surface of a wing in flight in order to obtain more favorable lift and drag, to replace flaps, prevent flutter, etc. Most of the world's airframe manufactures have some sort of adaptive wing program under way and some have even moved into the flight test stages (Kudva, et. al 1997). However, the most probable use in practice will come from unmanned aircraft, the development of which is being supported by the US Air Force

Current wing and control surface design may eventually give way to wings that change shape to replace hydraulically activated flaps or to provide multiple wing profiles. Airfoil design is always a compromise and the concept of adaptive wings might allow less compromise. For example, through proper heat treatment and processing, a shape memory alloy 'memorizes' a configuration. After large plastic deformation, greater than 1% strain, the alloy will return to its memory shape upon heating above a characteristic transition temperature. This memory effect is a result of a phase transformation from a low modulus martensitic phase, at low temperatures, to a high modulus austenitic phase, at high temperatures. The change in modulus can be as much as a factor of 2.4 (e.g. from 35 GPa to 83 GPa). These materials when properly integrated into the wing design may allow for new flight regimes and drastic reductions in limitations on fighter and commercial aircraft.

One of the major factors in airfoil design is the wing chord line: the line between leading and trailing edge. Flaps were invented to provide a way to change the chord line of an airfoil during flight. In a sense, flaps are really a lumped parameter approximation to changing the chord line, based on the technology available in the 20's. With the new technology of smart materials and structures, it may be possible to perform this same task in a distributed fashion and greatly enhance the performance and capabilities of flight control systems by having an advanced and distributed mechanism for changing camber.

Tab Assisted Control (TAC) is a technique for avoiding lock up of control surfaces on submarines. Here again, the concept of using an active material may be ideal for performing the same task without adding another mechanical surface to the structure. The use of shape memory alloy (SMA) in particular may provide a variety of solutions to engineering problems which require actuators which can deliver high force, high stroke, high energy densities, and, most significant for this research, high force-to-volume (or weight) ratios. As noted above, recent studies have shown the suitability of SMA actuators to aerodynamic problems that require large torque from actuators that must be housed within an airfoil's cross-section. These studies used SMA torque tubes as actuators, and were able to achieve high torque, 500--2000 in lbf, due to the high recovery stress available from SMAs. Similarly, large stroke actuators are possible due to the high recoverable strains, as much as 4 to 8%. The extension of that research to tab-assisted control (TAC) surfaces is natural.

There are several possible designs that a distributed TAC may use. Two torsional SMA actuator designs: *torque tubes* and *torque rods* may be suitable for this application. If these actuators are twisted resulting in 'plastic' deformation, then upon heating the deformation is recovered due to the memory effect of the material. Torque tubes have been used in a variety of aerodynamic applications to effect the twist of lifting and control surfaces. Torque rods are a recent development and are attractive because they can be designed to exhibit two-way memory. Normally, shape memory actuators return to their original state at an uncontrolled relaxation time making it difficult to provide actuation in two directions. Torque tubes are constructed from thin walled alloy stock; this ensures plastic deformation of all of the material. These actuators exhibit one-way memory and are used in an agonist-antagonist configuration; in this configuration, as the heated actuator cools, the cold actuator supplies a necessary bias stiffness restoring the active actuator to its initial position. Torque rods, however, can be designed for two-way memory. If properly designed, the central section remains elastic and acts a bias stiffness for the actuator. The size of this elastic region, and hence the bias stiffness, is dependent upon the applied load. As a torque rod actuator begins recovery upon heating it cannot fully recover because of the internal bias. Because the center section has stored some potential energy, the stored energy will be released upon cooling and the actuator will rotate back to its initial position.

Two examples of applications using smart structures have been mentioned here: submarines and aircraft. However many other application areas are possible. The point is to illustrate that current technology in structural dynamics has traditionally focused only on structural properties. Going forward it appears that the integration of control and sensing functions into structures and hence into the discipline of structural dynamics is a viable possibility.

### Self-Diagnostic Structures

Smart materials are keenly suited for integrating into health monitoring and fault detection systems based on vibration signals. They are small and unobtrusive and come in a variety of sizes and abilities, allowing them to be placed almost anywhere. The most interesting aspect of smart materials however is that they can also serve as actuators to provide driving signals as well as sensing, for systems that do not contain natural excitation forces, or for diagnostic algorithms that require a known, well controlled excitation. Thus by combining smart structures technology with diagnostics, one can imagine structural systems in the future that have self contained and self-diagnostic components minimizing maintenance and inspection cycles. Three examples are summarized here, one which is based solely on signal processing and involves no model of the structure ending with one which requires a substantial model of the structure. A summary of recent health monitoring and damage detection methods can be found in Doebling, et. al. (1998).

In general the diagnostic or damage detection methods seek to answer the questions:

Is their damage?

Where is it?

How big is it?  
What is its orientation?  
What should be done about it?

The first of these questions can be answered without having much of a model of the structure or part of interest. The need to know more about the nature of the defect however requires an increasing knowledge of the model of the structure that is not always possible. Thus methods that strive to answer a number of these questions quickly become intertwined with modeling issues. Modeling in structural dynamics is often not compatible with actuation, nonlinearity, damping, interaction with acoustic or fluid phenomena. Our traditional modal model is not good beyond a certain frequency for most structures. What we need is something (test and analysis) in the mid-range such as Statistical Energy Analysis, wave propagation models, impedance modes, etc.

First consider the idea often used in damage detection that a defect will produce a small change in stiffness and/or mass, and hence frequency. Early vibration based damage detection methods often looked at frequency response function (FRF) data for a small change in frequency. However small changes in frequency are generally difficult to measure using FRF data. One method that avoids this difficulty is to continuously compare the healthy time signal of the structure under study to the current time signal of the structure. If a small difference in frequency exists between these two signals, then when they are combined they will produce the beat phenomena that serves to magnify small differences in frequencies. This effect has been used successfully to detect the presence of small amounts of damage (less than 0.1% change in mass) in plates and in helicopter blades by using internal piezoceramic materials to both excite and sense the various time histories.

This approach illustrates a time-domain procedure that is totally self-contained in a moving structure, capable of self diagnostics using embedded piezoceramic sensing and actuation at relatively low power costs. The procedure depends only upon subtracting two signals, does not require any modeling of the structure and hence is simple enough for onboard use (Cattarius and Inman, 1997).

A second example of a diagnostic procedure is impedance based and may also be used in a self diagnostic configuration by incorporating local, embedded and/or surface mounted piezoceramic sensors and actuators. This approach is a high frequency impedance based method that looks for a shift in electrical impedance measurements as an indicator of damage. In the impedance-based qualitative health monitoring technique, the real-time implementation relies on a simple scalar damage index that can be easily interpreted. Using this damage index in conjunction with a damage threshold value, the approach can warn the operator in a green/red light form, whether or not the threshold value has been reached. This approach has been used on numerous bolted joint structures and in large concrete structures. Again, this system uses the integration of sensors and actuators into the structure forming a smart structure capable of self-monitoring (Park, et. al., 1998).

Banks, et.al. (1996) presented a method which takes advantage of a detailed and well identified model of the structure. The results are mentioned here to provide experimental proof that a piezoceramic based diagnostic system can not only determine the existence of damage, as can the previous methods, but is also able to determine the size and location of the damage. The method is based on using a partial differential equation model of a structure that is partially layered with piezoceramic patches which again forms a "self-diagnostic" structure. The algorithm uses a spline-based approximation of the equations of vibration and successfully identifies the existence, size and location of holes in a beam. The method works by estimating functions of the longitudinal direction of the beam corresponding to the damping parameters (both Kelvin-Voigt and air damping), the modulus and the density. Each of these is allowed to be discontinuous in order to allow holes to be included in the solution set of functions.

It is important to note that while this method works very well, it has the disadvantage of requiring a very detailed model of the structure including a model of the damping mechanisms. The parameters are allowed to have some measure of uncertainty as they are estimated in the inverse procedure used to identify the damage. However, the form of the governing differential equation must be known in substantial detail. With this noted, the method very effectively identifies the damage, where it is and possibly its orientation. Furthermore, the results are consistent across several different types and sets of measurements.

Another important feature of the method is that it does not use modal data, but rather is based on time domain measurements. The experiments were repeated with traditional excitation means (an instrumented hammer) and response measurement (accelerometer, and position probe) as well as with the internal self-sensing actuation scheme offered by the integrated piezoceramic patch. Besides verifying a damage detection algorithm, the test shows conclusively that it is possible to use piezoceramic materials to form self-monitoring devices.

Almost any diagnostic or monitoring system can be integrated into a structure to make it a self-diagnostic system. Here, three examples have been mentioned which make the point that the integration of sensors and actuators directly into a structure provides interesting new problems for the structural dynamics community. In addition, the three examples illustrate that increased knowledge of the structure allows more information regarding the health of the structure to be determined.

### **Self-Repairing Structures**

The area of smart materials also holds some promise for carrying diagnostics one step farther by adding an ability to perform self-repair. Because smart materials are able to change their stiffness and damping properties according to an applied voltage, it is thought that they may be able to compensate for some types of limited damage, allowing the continued safe use of a damaged structure. One possibility under consideration is the use



of woven fabric like composites with shape memory alloy fibers. When damaged, the fibers can be activated to increase strength.

Another example of a self-repairing structure made possible by smart materials is the smart bolt. A smart connection, or bolt, consists of structural members joined together by bolt and nut combinations fitted with piezoceramic elements as washers. These combinations can be used to monitor bolt tension and connection damage. Suppose that torque is used as a measure of the health of the system. Then weak impedance signals can be sent out from the piezoceramic patches and sensed by the opposing washer to determine the level of torque on the bolt. In some configurations, temporary adjustments of the bolt tension can be achieved actively (and thus remotely). Experimental results to date clearly show that changes in the mechanical characteristics of bolted connections can be identified through localized impedance measurements of connection components. Initial experiments indicate that changes of the order of an inch pound can be determined in the Coulomb region. In particular it may be possible to use a washer system made from an active material such as piezoceramics. Then as the bolt becomes out of torque, changes in the impedance of the system will result. It may then be possible to tighten the bolt by applying a voltage to cause the piezoceramic to expand. This idea incorporates the aspects of self-diagnostics with active control to improve the behavior of a bolted structure under dynamic load. Details of such systems are being worked on by a small company in the US and by Prof. Gaul of the University of Stuttgart who has invented the concept of a semi-active bolt.

### **Biologically Influenced Structures**

One approach to formulating smart structures has been to mimic nature. In fact many of the first attempts at flight sought to mimic nature. Of course flapping wings were not to succeed, but birds did provide motivation and eventually lead to successful ideas. Perhaps more appropriately then, biological systems have been used to motivate new smart structures and material systems as well as uses of these materials. This has lead to a number of different mechanisms. Some have used shape memory alloys in an attempt to mimic muscle. Others have attempted to use piezoceramics to form miniature bugs that walk about (smart ants) with miniature video cameras in hostile environments.

The need for large solar panels and antennas in space coupled with the need to have very small packages to send into orbit has produced a focus on inflatable structures and a class of structures known as tensegrity structures. Tensegrity, is an example of structural advantage gained by biological motivation. Tensegrity refers to a repeated structure that stabilizes itself because of the way in which tension and compression forces (stresses) are distributed and balanced within the structure. It is possible for a system of wires and rods to then be configured in a plane such that they will deploy to a three dimensional configuration that is rigid (stable).

Inflatable structures are also under consideration for use in expanding components in modern satellites. The idea is to use a collapsed Mylar like material stowed for launch,

then expanded in orbit to provide the structural framework for satellites and antennas. Both types of configurations are motivated by nature's unfolding of planar systems into three-dimensional formations. Both types of structures will also benefit from the inclusion of smart materials to serve as actuation or muscle imitating functions and as sensors. Such structures offer a challenge to structural dynamics in terms of providing predictive models that may be used in design and analysis.

The examination of biological functions may motivate others in structural dynamics to invent new structural forms that solve existing problems or create new products. In thinking about combining structures with sensing and actuation it is important to match requirements with abilities. Actuators are usually designed in terms of force, stroke and time constant. Sensors are designed in terms of band width and sensitivity. Locations of sensors and actuators inside a structure are also extremely important.

### **Historical Perspective**

In looking to the new fields of structural dynamics it is important to review the past. Defining the elements of success and failure is best tried in a historical setting. The history of aerospace structures provides a nice setting for this. Much of the success of the aerospace business is summarized in an excellent collection of articles edited by Flomenhoft (1997). Many of the successes in structural dynamics in the early days were driven by failures in practice. In particular, flutter of aircraft drove the discipline of aeroelasticity and many innovations related to the structural dynamics areas. This greatly improved our understanding of structures. However, as applications became more complex, knowledge became more specialized and the discipline of structural dynamics became essential narrow-minded. To advance, the discipline has to return to an interdisciplinary state.

Another important lesson from history is the role of computation, simulation and experiments. Today we have intense computational abilities. Often one can simulate hundreds of designs before actually building anything. On the other hand it is very important to perform experiments to verify theories. It may not be enough to develop good models, techniques and designs that work. In addition they must also be user friendly and this often means simple. Not all problems have simple solutions however, but we can as a group develop better techniques for communicating with practicing engineering professionals.

History points out that ideas are often discarded prematurely. Think for example of steam powered automobiles. Critiques of the time used the problems of steam power to cast stones at the concept of an automobile. Yet a few years later the solution appeared in the form of gas powered engines. Thus some previous ideas should be considered again in view of current technological advances. This is particularly true with regard to computational methods in structural dynamics.

### **The Way Forward**

The optimal way forward is not clear, but it must begin with educational aspects, short courses, funding agencies and corporate tolerance of high-risk research. Our society has evolved into a stage of immediate gratification. Corporations now want to support research only if it will turn a profit by the next stockholders report. This is like a farmer with a four month growing season, plowing up his plantings after the first quarter because he hasn't sold any vegetables.

How can we change the current industrial "research" mode? We need to follow the model of the drug and computer industries where research directly drives their products and hence enjoys much greater support. A tough mind set exists in industries using structural dynamics when it comes to new materials and designs: if we haven't used it yet, we don't want to! If it's a new idea, competition with existing technology is often a problem of importance to consider meaning that new ideas are often suppressed because they do not offer an immediate advantage over existing systems.

What we need for structural dynamics to grow is to create challenge problems, benchmarks and to pursue crazy ideas. Those of us in university settings have to be creative and exotic in our approach because no other institutions (corporate research or government labs) are willing or able to take the risk of failure. The need for funding is obvious and critical, yet the university community needs to carry the ball even with out support from external sources.

As engineers we often are not the best at communicating or reading. We need to refocus our concept of information dissemination. We have new tools (i.e., the WEB) and we need to learn to take advantage of this. We tend to reinvent the wheel in a cyclic fashion. How do we avoid this? How do we make it more efficient? Realize that some things need to be revisited on purpose (eigensolvers and parallel computing). However we do need to guard against blindly ignoring previous results. Net searches should help this situation as do review articles. We have much more information at our disposal, but we also have efficient ways of searching for this information.

### **Summary**

The message or goal of this report is to convince the reader that the next generation of structural dynamicists will have to have skills that allow them to comfortably work with electronics, controls, and materials as well as traditional topics in structural dynamics. We need to replace the idea that stiffness and damping properties are all that is important with the notion that structures are integrated components of a total system.

While several successful examples of the use of structural systems integrated with smart materials have been presented, it is important to note that these devices all have limitations and one must examine the force, deflection and time constant needed to form a successful and useful system. The inclusion of smart structures into the discipline of structural

dynamics offers new opportunities and responsibilities. It as requires the integration of a more multidisciplinary role for the structural dynamics community.

Smart materials and structures should provide motivation for structural dynamics engineers to expand their expertise to include electromechanical devices. Many improvements are possible with these new materials as well as many new opportunities, and it is reasonable to view them as one possible advancement in structural dynamics for the future.

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Warping the time axes,  
or,  
Non-linear is what we call that which we do not  
understand

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This author has butted his head into many a ceiling in trying to fit elaborate parametric models to test. In his younger days he looked towards colleagues and mentors for help, and more often than not, the source of the trouble was labeled non-linearity. As the author grew older, wiser, but not as smart as before, he realized that this kind of non-linearity has a very pragmatic definition: Non-linear is what we call that which we do not understand. This definition is also subjective; your linearity may be my non-linearity.

The basic rule in statistics and empirical science is that the more that we invest in up front assumptions and parameters, the more resolution our estimation models will have. This is called statistical degrees of freedom, and shows itself, for example in the increased spectral resolution of a maximum entropy spectrum over a discrete Fourier transform spectrum. The problem is that as we add a priori assumptions, our models become less robust, and we often have severe problems explaining the lack of fit of models to experimentally acquired data. What our disciplines really need are good ways of visualizing test data without requiring restrictive assumptions, such as linearity.

One of the more challenging aspects of experimental dynamics is the description of transient noise and vibration phenomena. Transient here means that the system, loads or boundary conditions change with time. The usual approaches to analyzing such data are to perform successions of partially overlapped myopic analyses, using tools that assume stationary behavior. The usual effects are smearing and poor resolution. This may be acceptable if the non-stationarity is mild.

The author proposes a general approach for an important subclass of transient phenomena, namely those that can be reduced to a quasi-stationary case by suitable monotonic transformations of the time axis. Quasi stationary means here that the apparent variations of systems properties are more benign than the original formulation, and that hence, stationary analysis tools have a wider range of applicability.

The prime example of such a transformation is in the author's opinion the resampling of rotating machine data from the time domain into even angle increments around the primary shaft. In this angle domain, frequency analysis tools may be applied, because all rotation-induced phenomena seem to happen at fixed frequencies; all significant frequencies separated by a common step. Another application is to resample data from a moving microphone source according to an instantaneous Doppler shift correction factor. Frequencies on the ground become the same as frequencies at the moving source, and, to compound this, another resampling may be performed to transform to the crankshaft angle domain. Phase lock between moving data acquisition stations may be achieved by tracking out of band reference tones.

The point is that by choosing appropriate transformations of the time axes, transient and bewildering phenomena may become tractable with classical tools. Linearity has nothing to do with this, we have just taken some tricks from relativity theory and applied them to classical mechanics. Much insight can be gained by elementary means by investing into an accurate determination of the time axis transformations.

This author believes that a ruthless exploitation of time axis transformations, coupled with classical data analysis methods will enable a large class of phenomena to be visualized and quantified. The challenge is to define standard procedures for identifying such transformations and then applying them.



# SD2000 Review –Experimental Modal Analysis

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## Introduction

This paper is a review of the current state-of-the-art technologies in the area of modal analysis. Modal analysis refers to the process of estimating either analytically or experimentally a set of basic characteristic functions or solutions (modal parameters) which can be used to describe the static and dynamic response of a system. All possible responses of the system can be estimated by a superposition of these characteristic solutions. These characteristic functions are the eigenvalues, eigenvectors and scale factors of the equations of motion of the system. In this paper the precise mathematical description of the area of modal analysis will be sacrificed and a more conceptual presentation will be made for brevity.

This review will be presented with a historical perspective because the evolution of a technology often will fold over on itself. Methods which were discarded are often rediscovered in the presence of evolving technologies.

## Acronyms and Nomenclature

[A]	Residue Matrix
{f} or {F}	input vector (forces)
[h(t)]	unit impulse response matrix
j	square root of minus one
[H(ω)]	frequency response matrix
[L]	modal participation matrix (right hand eigenvectors–input characteristic functions)
p	index for response point
q	index for input point
t	time
{x} or {X}	response vector (displacements)
ω	frequency
[λ]	diagonal matrix whose elements are functions of time or frequency (eigenvalues–temporal characteristic functions)

$$\lambda_{rr}(t) = e^{\lambda_r t}$$

$$\lambda_{rr}(\omega) = \frac{1}{j\omega - \lambda_r} + \frac{1}{j\omega - \lambda_r^*}$$

r index for eigenvalues

[Ψ]	modal matrix (left hand eigenvectors–response characteristic functions)
ADC	Analog Digital Converter
CEA	Complex Exponential Algorithm



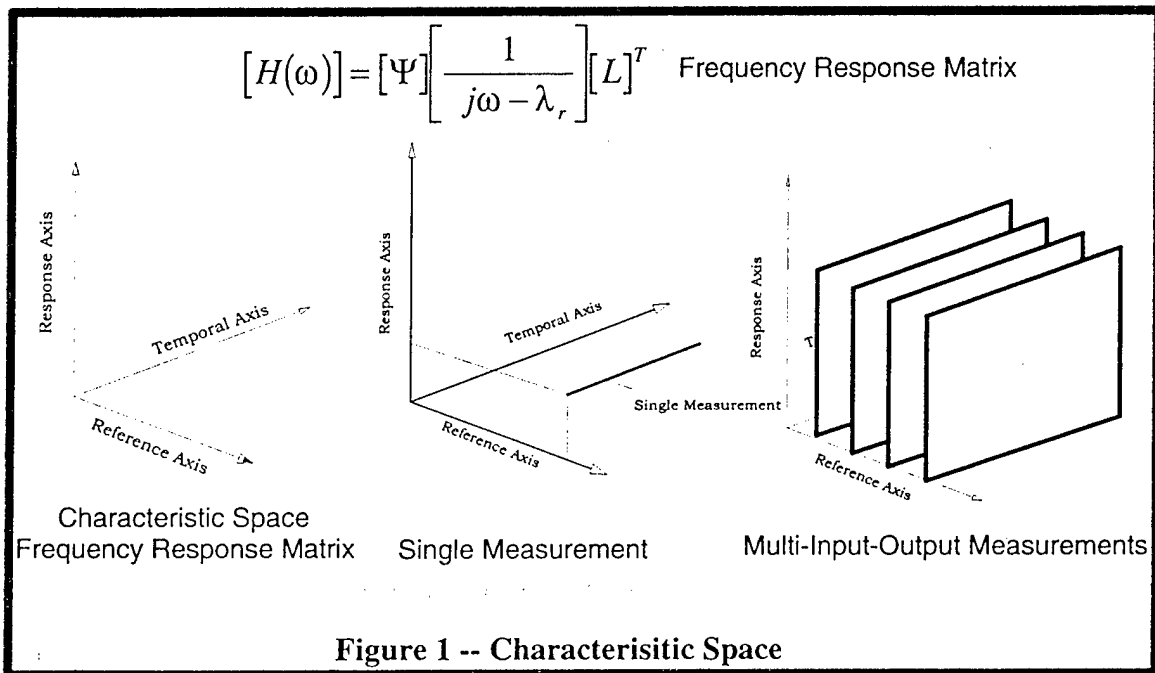
CMIF	Complex Mode Indicator Function
DAC	Digital Analog Converter
DFT	Discrete Fourier Transform
DOF	Degree-of-Freedom
eFRF	Enhanced Frequency Response Function
EMIF	Enhanced Mode Indicator Function
ERA	Eigenvalues Realization Algorithm
FFT	Fast Fourier Transform
LSCE	Least Squares Complex Exponential
MAC	Modal Assurance Criteria
MDOF	Multiple Degree-of Freedom
MIMO	Multiple Input Multiple Output
MRIT	Multiple Reference Impact Testing
IMAC	International Modal Analysis Conference
ITD	Ibrahim's Time Domain
PC	Personal Computer
PFD	Polyreference Frequency Domain
PTD	Polyreference Time Domain
SDOF	Single Degree-of-Freedom
SV	Singular Values
SVD	Singular Value Decomposition
UMPA	Unified Matrix Polynomial Approach

## Concepts

In this section the concepts of modal analysis will be addressed. Modal parameters are characteristic functions which are the building blocks for all possible solutions to the equations of motion of a system. For mechanical systems, all possible input-output (force-displacement) solutions are simple superpositions of these characteristic functions. Think of these characteristic functions as Legos (children's building toys). Complicated structures can be built from a small set of different Lego elements. These Lego elements are the building blocks, and superposition of the blocks can be used to construct complicated structures. Since the geometry of these elements is known, it is easy to design a structure. This leads to one of the important assumptions of modal analysis. The system must be linear. For the Lego case, if the geometry of the elements depends upon their position in the structure or a point in time, it would be difficult to design the structure. This would be the case if the system was nonlinear or non-stationary. The characteristic functions would not be constants of the system and superposition would not work.

The characteristic functions are functions of input-output locations and time/frequency. In other words, they describe the spatial and temporal information about the system. There is one set of characteristic functions which describes the input space, a second set which describes the output space, and a third set which describes the temporal space (time-frequency). The temporal characteristic function is represented by the system eigenvalues, and the input-output functions by sets of left hand and right hand eigenvectors. A three dimensional characteristic space can be defined with the input coordinate along one axis, the output coordinate along the second axis, and the temporal information (time/frequency) along the third axis. This characteristic space

corresponds to the three dimensional frequency response matrix or unit impulse matrix. Figure 1 is a graphical representation of the characteristic space. Any point in the frequency response matrix has the value of the frequency response function at a given frequency for a given input coordinate and given output coordinate. At first glance, this characteristic space would appear to contain nearly infinite information, but in reality it can be generated with only a small number of eigenvalues and eigenvectors.



Mathematically, the response of the system can be computed from the system characteristic functions by the following relationships:

Time Domain

$$\{x(t)\} = [h(t)] * \{f(t)\} \quad \text{Convolution} \quad (1)$$

Frequency Domain

$$\{X(\omega)\} = [H(\omega)] \{F(\omega)\} \quad (2)$$

where

$$[h(t)] = [\Psi][\lambda(t)][L]^T \quad \text{Unit Impulse Response Matrix} \quad (3)$$

and

$$[H(\omega)] = [\Psi][\lambda(\omega)][L]^T \quad \text{Frequency Response Matrix} \quad (4)$$

In equations (3) and (4), the columns of the  $[\Psi]$  matrix are the left hand eigenvectors which are the output space characteristic functions. The columns of the  $[L]$  matrix are the right hand eigenvectors which are the input space characteristic functions. These functions are spatial characteristics and are only functions of geometry. The  $[\lambda]$  matrix is a diagonal matrix whose diagonal elements are determined by the eigenvalues and are functions of time or frequency. In order to experimentally determine these characteristic functions, it is important to observe all three dimensions of the characteristic space.

This means that a set of measurements would have to be made by exciting at all potential input points and measuring the response at all output points. These measurements would also have to span the temporal space. In other words, the system has to be observable. Experimentally, this would be a formidable if not impossible task. Fortunately, the equations of motion for most mechanical systems are symmetric. This results from the fact that the effective stiffness and damping which act between point p and point q are the same as what acts between point q and point p. In other words, most mechanical structures obey reciprocity. Analytically, this means the left and right hand eigenvectors are the same or transposes of each other, depending upon how the eigenvectors are expressed mathematically. As a result, it is only necessary to completely span either the input space or the output space.

Theoretically, this means that when experimentally testing a structure, it is only necessary to excite at a single input point with the following exceptions:

- The excitation point is not at a node of one the eigenvectors.
- There are no repeated eigenvalues.

This reciprocity attribute makes modal testing a conceivable process.

Practically, many eigenvectors are non-observable in the presence of measurement noise or are so closely coupled that they appear to be repeated. Therefore, it is generally necessary to use multiple excitation points.

The above concepts are used to explain three of the important assumptions of experimental modal analysis:

1. The system has to be linear.
2. The system has to be time invariant.
3. Experimentally, the system has to be observable.

Another conceptional point of interest is that modal parameters that can easily be generated analytically are often very difficult to measure experimentally. The modal parameters are the result of an eigenvalue-eigenvector solution technique. This method performs a coordinate transformation, which uncouples the equations of motion. The modal parameters are solutions in this transformed coordinate system (modal coordinates). When testing a system, all forces are applied in the physical coordinate system. In a physical coordinate system, the forces are applied at discrete input points on the structure, while in modal coordinates the forces are distributed over the structure. Distributing forces proportional to the eigenvectors is difficult to do experimentally.

For modal coordinates, the transformation vectors are the eigenvectors. In this coordinate system, the forcing function is proportional to the given eigenvector of the system. In order to measure one of the characteristic solutions, the system has to be excited with a modal forcing function. As result, directly measuring the modal parameters is difficult.

When the structure is excited at a discrete point, a large subset of the modal characteristic functions are present in the data set. The modal parameters must be extracted from this data set by a parameter estimation process. In recent years, the largest research effort in experimental modal analysis has been the development of parameter estimation algorithms to extract modal parameters from the measured input-output responses.

The problem of matching analytical and experimental conditions is an important consideration that has historically created problems between the analyst and the experimentalist. It is important when developing a test that the limitations of the testing be understood. For example, a common testing requirement is to specify that a test object be tested in a fixed mounting condition. The test object, most likely, is not mounted in this configuration in practice but is analytically modeled in this configuration. In the laboratory it may not be possible to support this test article in a fixed condition, therefore, the test results will not reflect a fixed boundary condition. Analytically, the test object can easily be constrained in another manner which can be more easily tested. It is important for the analyst and the experimentalist to understand the limitations of both technologies. Of course, these considerations are important for other parties that are involved in planning a test. An example where this can be a problem is when an interested party insists that the object be tested in an operating configuration. If there are large nonlinear components in this configuration, then it can be difficult to perform a modal test of the complete test object. By removing these components and replacing them with linear simulators, the test can be performed. The linear characteristics of the complete test object can be verified. Auxiliary testing can be performed to identify the nonlinear characteristics of components which were removed.

## **Experimental Testing Procedures**

Modal testing has become a fairly mature technology and there are numerous testing methods which can be used to perform a test. There are many reasons for performing a modal test, and some test methods are better suited to certain tests. However, in most cases, even for a given test there are several choices that are available. In the next several sections, modal testing

procedures will be reviewed. Detailed information on each technique will not be included. Only a general description of the technique will be given.

## Generalized Procedures

In experimental modal testing, the response to external forcing functions is measured along with the external forcing functions when possible. In most cases the input forces can be measured, but there are cases where unmeasurable operating forces or free decays may be used. The modal parameters are estimated from the output response data and the input data. Historically, two basic techniques have been used. In the first technique, a modal force is applied and the response corresponds to one of the system eigenvectors. In the second technique, forces are applied at discrete input points so that the response is the summation of a subset of the modal parameters. For this case, the modal parameters are extracted by curve fitting to an analytical expression, or by fitting a system model to the input-output measurements and solving this system model for the modal parameters.

A typical modal test can be divided into the following general steps:

### 1. Test Setup-Pretest Analysis

- a. reason for performing test
  - i. trouble shooting
  - ii. finite element model (FEM) verification
  - iii. finite element model updating
  - iv. generating an analytical model based upon experimental modal parameters
  - v. condition monitoring
- b. pretest analysis
  - i. Finite Element Analysis
    - (1) target modes
  - ii. excitation locations
  - iii. response locations
  - iv. frequency range
    - (1) frequency resolution
- c. pretest testing
  - i. nonlinear testing
    - (1) is the structure nonlinear
    - (2) what kinds of nonlinearities
    - (3) can nonlinearity be removed
  - ii. impact testing to check pretest analysis
    - (1) are exciter locations good?
    - (2) initial estimates of eigenvalues
- d. testing methods
  - i. normal mode method
  - ii. parameter estimation method
  - iii. operating inputs (ambient testing)
  - iv. free decays

- e. excitation methods
    - i. roving input
    - ii. fixed inputs
    - iii. operating inputs
  - f. boundary conditions
    - i. fixed
    - ii. free-free
    - iii. hybrid
    - iv. operating
  - g. calibration of sensors
  - h. cable management
- 2. Data Acquisition
  - a. real time
  - b. post processed
  - c. forces
    - i. single inputs
    - ii. multiple inputs
  - d. responses
    - i. sequential
    - ii. simultaneous
  - e. type of inputs
    - i. periodic
    - ii. random
    - iii. transient
    - iv. operating
- 3. Signal Processing and computations of measurements
  - a. FFT analysis
    - i. windows
    - ii. frequency response estimation
      - (1)  $H_1$
      - (2)  $H_2$
      - (3)  $H_v$
  - b. time domain
    - i. windows
    - ii. correlation matrices
    - iii. random decrement
- 4. Parameter Estimation
  - a. curve fitting analytical expression
    - i. time domain
    - ii. frequency domain
    - iii. SDOF
    - iv. MDOF
  - b. estimating system model
    - i. time domain
    - ii. frequency domain
    - iii. spatial domain

5. Modal Model Assimilation and Validation
  - a. assembling modal model from various estimation processes
  - b. validation of modal model

The above list addresses many, but not all, of the aspects of performing a modal test. These are the items that would normally be covered in a survey or complete review of modal testing. Obviously, several complete books could be written on this subject, for example, a book on just the parameter estimation aspects of modal testing. There have been hundreds of papers written exclusively on the subject of modal parameter estimation. In fact, the best reference for the complete modal testing may be the proceedings from the International Modal Analysis Conference (IMAC) where most of the modal analysis technology has been published in recent years. In the following sections, selected items taken from the above list will be discussed.

### ***Test Setup***

The first question is, What is the purpose of the test? The test purpose could be to solve a simple trouble shooting problem. In this case, the test could simply be grabbing a few accelerometers, an instrumented hammer and a simple portable multiple channel analyzer off the shelf, mounting the accelerometers on a test object, beating the test object with a hammer to measure a set frequency response functions, and using simple parameter estimation procedure to get a quick estimate of the modal parameters. In other words, performing a multiple reference impact test to solve the trouble shooting problem. In this case, very little pretest analysis is required if the test object is already a known item. This contrasts with a very large modal test for finite element verification of updating. For this type of test, pretest planning can be extensive. Sometimes these tests are scheduled a year or more in advance. FEMs are built of these test structures to determine which modes are important to identify, the best locations to excite these modes, and the best response locations to identify the target modes. For the largest of these tests, 10 to 20 exciter positions and up to 1500 response positions may be used and a number of configurations tested.

Afterwards, the pretest analysis is performed. A pretest testing program can be performed to determine the degree of nonlinear response of the system. If the system has significant nonlinear response, then additional testing is performed to better understand the location and the characteristics of the nonlinearities. Can the system be linearized? The pretest testing can also be used to determine the excitation and response locations and to check on the modal density and give an initial estimate of the system eigenvalues. These initial measurements are useful in selecting a test method.

### ***Modal Analysis Applications***

Experimental Modal Analysis has been used for a number of important applications:

## Troubleshooting

Historically, troubleshooting has been by far the biggest application of experimental modal analysis. Solving vibration, noise, and to a lesser extent controls problems have been the major trouble shooting applications. A typical application might be solving a self-excited vibration or chatter problem on a machine tool. This application is a field testing application because the machine tool is normally tested in the manufacturing plant. Another application may be testing a bridge or another type of infrastructure to determine if it has deteriorated or suffered damage. Again, this is a field testing situation.

For troubleshooting applications, a quick and dirty testing procedure can very often be used. Multiple reference impact testing is one of the more modern testing methods for troubleshooting. In this testing method, a number of accelerometers are mounted on the test specimen and a roving instrumented hammer is used for input. This method can easily be set up in field testing applications. The preferred data acquisition system is a small portable multi-channel analyzer connected to a notebook computer. Multiple reference data is obtained with this method and the modal parameters can be extracted using any of the commercial multiple reference parameter estimation algorithms. However, Complex Mode Indicator Function (CMIF), which is one of the newer spatial domain algorithms, is preferred. This algorithm is very simple to use and minimizes the problems of sorting computational modes from real modes. The parameter estimation algorithms will be covered in more detail in later sections of this review.

Testing methods which use operating input as the excitation are also good for trouble shooting. In these methods the actual operating forces excite modes of interest. If the inputs are periodic, then order tracking can be used to extract operating mode shapes at problem frequencies. If the operating inputs can be measured or estimated indirectly, then ordinary frequency response functions can be measured. These frequency response functions can be used to estimate modal parameters. For large structures such as bridges, the operating inputs are often broadband random inputs caused by the wind or traffic flow. In these cases, power spectra, cross spectra, correlation matrices, or random decrement methods can be used as inputs into parameter estimation algorithms for extracting the modal parameters.

## Finite Element Verification

Finite Element Modeling (FEM) is becoming a commonly used design tool for analyzing the static and dynamic characteristics of mechanical systems. FEM is used to develop an initial design of a structural component. After a prototype of the component is manufactured, it is important to verify the FEM model experimentally so that FEM can be used to make design modifications to the initial design. Also, the verification process builds up experience so that the FEM can be used with confidence on future design projects.

Finite element verification requires the most sophisticated modal testing procedures. There have been two general methods for performing these types of tests. The first method is the normal mode testing method, and the second method is the broadband multiple exciter testing method.



The first method has been used since the early '60s for testing aerospace structures. The second method has been the standard testing method over the past ten years.

In both of these methods, the number and location of the exciter positions are determined in the pretest analysis. Likewise, the number and location of the response points are also determined. The finite element model is used to define these points and to identify target modes. The pretest testing program is used to identify potential testing problems such as nonlinearities.

For the more standard broadband excitation method, typically 200 to 500 accelerometers are mounted on the test structure and two to four exciters are used to excite the structure. A multiple input data acquisition system is used to simultaneously measure the input and response matrix. Uncorrelated random excitation signals are used to excite the structure. Multiple Input Multiple Output (MIMO) signal processing is used to compute the frequency response matrix. Advanced parameter estimation algorithms (PTD, ERA, etc) are used to extract the modal parameters.

### Finite Element Updating

The updating of a FEM model is an important process. This involves locating and correcting errors which may exist in the original FEM model. This is a difficult process since the FEM has a large number of Degrees Of Freedom (DOF). The experimental database is normally limited to the first 10 to 100 eigenvalues, depending upon the nature of the test article. Often this is not enough information to locate and characterize the errors in the FEM model. Another point that complicates the testing is that often the test object is a component of a larger system which connects into the system at a number of points. It is normally impossible to experimentally match the boundary conditions at these connection points. This means that the 10 to 100 modes measured may not include the important information at these connection points. Auxiliary measurements need to be taken at these connection points and this data has to be incorporated into the updating process.

The testing methods used for the finite element updating are the same as those used for the verification process.

Several new techniques are being investigated for the updating cases. The purpose of these methods is to increase or supplement the experimental data base. The first method supplements the modal data with driving point frequency response measurements at the connection points. This data is used to update the model at the connection points. The second method is the Perturbed Boundary Condition (PBC) testing method. The PBC test procedure measures a larger database for verification and/or updating of Finite Element models. The test article is tested in a number of configurations where the boundary conditions are drastically perturbed. These additional configurations expand the database that is used for the updating process. Each configuration can generate 10 to 100 additional eigenvectors that can be used in the updating process. Since all of the transducers are premounted on the test article, the data for each configuration can be taken in approximately 30 minutes.

## Experimental Modeling

Very often a FEM model for a component does not exist, but the hardware for the component does exist. An example of this might be an air conditioner compressor used to cool an automobile. This air compressor may contribute to a noise problem. If a model of the engine-accessory package is developed, an analytical model of the air compressor can be developed by experimentally testing the existing air compressor and determining either an impedance model or a modal model of the compressor. This model can be used as a sub-component in the engine-accessory system model.

There are two common types of analytical models based on experimental data. The first is an impedance model where the measured frequency response matrix is used to model the relationships between the input and outputs of the system. The second is a modal model. The testing methods for the component could use an impacting or an exciter test method. This depends on the size and complexity of the test item. For the impedance model, measurements have to be made only at the connection points, but a complete impedance matrix must be measured. In other words, measurements must be made between every input and output point of the structure. This would be an unimaginable task if the number of points was large. However, for cases where the number of connection points is small, this is a viable method. The impedance functions contain all of the modal information for the frequency range of interest. This method is frequently used to design mass dampers that can be applied to a structure to reduce the influence of troublesome modes of vibration.

The second method is to measure the eigenvalues and eigenvectors of the component and to generate a modal model. This model is only as good as the measured modal database. Very often, modes that are not in the frequency range of interest are important. It is important to include the influence of these modes in the model. This is done by measuring residual terms at the connection points or by using a testing method such as the PBC method.

## Condition Monitoring

In more recent years there has been increasing interest in using modal parameters for evaluating the health of infrastructures. In general, these are large structures that are very often located in remote locations. Two testing techniques are currently being pursued. The first is the Multiple Reference Impact Testing (MRIT) method and the second is ambient testing methods. For smaller structures such as overpass bridges, wind turbines, large industrial fans, etc., the MRIT impact testing has been used with success. For very large structures such as dams, large bridges and large buildings, ambient testing is being used. In both of these testing methods, it is almost a requirement that the transducers be premounted.

For some of the infrastructure testing applications, it is important not to interfere with the function of the structure. Generally, this is true for the testing of bridges since interruptions of traffic flow are undesirable. This means that the MRIT has to be performed in a short period of time and that any ambient testing must be done in the presence of the operating forcing functions.

## **Testing Methods**

There are four basic testing methods:

### **Normal Mode Testing**

The normal mode testing method is one of the earlier testing procedures. It dates back to the '50s. In this time period, computing capabilities were very limited. Measurements were performed in an analog sense. ADCs were nonexistent or very expensive. The testing was performed with sine testing and analog filters to exclude noise or harmonic distortion components. For this time period, about the only choice for performing a large scale modal test was to use modal forcing vectors. These forcing vectors would excite one modal characteristic function at a time. The deformation vector at the excitation frequency corresponded to an estimate of the eigenvector. The frequency and damping of the free decay, after the forcing function terminated, corresponded to an estimate of the eigenvalues. The forcing vector that was applied was also designed to remove the damping or to make the damping proportional to the mass or stiffness distribution so that a pure normal mode could be measured. Over time, more sophisticated methods were developed to estimate the eigenvalues. The modal forcing function(vector) was swept over a small frequency band around the eigenvalue which corresponded to the eigenvector being measured. The amplitude of the response of selected points were plotted as a function of frequency. The eigenvalues could be determined from the half power points measured in this plot. Later, parameter estimation algorithms were used to estimate the eigenvalues.

The problem of using this method was that it took great skill in being able to adjust the forcing function so that a single normal mode was excited. The adjustment included controlling the frequency and distribution of forces.

Over the years, this method has been refined by incorporating some of the parameter estimation technology into methods for tuning the modes and estimating the eigenvalues.

If used properly, this method gives excellent results.

### **Parameter Estimation Testing Methods**

In the late '50s and early '60s, the only practical way to perform a big modal test was to use the normal mode testing method. This method required extensive equipment which was very expensive at the time. The only practical method for performing a normal mode test was to premount a large number of sensors to the structure. These sensors and the signal conditioning required by the sensors were very expensive. An approximate method that was used which required minimum equipment was to rove a single sensor or a small number of sensors over the structure. One or more exciters were used with a sinusoidal signal which were tuned to one of

the eigenvalues. If the sensors were accelerometers, the 90 degree out of phase response of the transducers was measured, or if they were velocity sensors, the in phase response was measured as an estimate of the eigenvector at the point on the structure where the sensor was mounted. The transducers had to be roved over the structure for each frequency. This was a very time consuming procedure.

In the early '60s, a technological breakthrough occurred with the development of the tracking filter. With this development, frequency response functions could be measured with swept sine testing in a timely fashion (timely for that period of time; it took 15 minutes or more to measure a single frequency response function). The frequency response functions included all of the significant eigenvalues. This development led to the idea of using a parameter estimation process to determine the eigenvalues and eigenvectors from the measured frequency response functions. Analytically, the frequency response functions could be formulated in terms of modal parameters. This led to a great deal of research on how to estimate modal parameters from frequency response functions. The formulation for solving for the modal parameters generated a set of nonlinear equations which were very difficult to solve. Remember that in the early '60s, computers had limited capabilities and were expensive to use.

The second technology breakthrough occurred in the middle '60s with the advent of the Fast Fourier Transform (FFT). Up till the time this breakthrough occurred, computing the Fourier coefficients using a Discrete Fourier Transform (DFT) took approximately 15 minutes on a mainframe computer of that time. Computer time cost about \$1000 per hour at that time, so it was costly. After the implementation of the FFT, this time dropped to several seconds. Using broadband excitation signals, frequency response measurements could be made in seconds instead of minutes.

Impact testing was developed in the late '60s based on the FFT. Broadband random, pseudo random, and fast sine sweeps were all being investigated as possible excitation signals in the period of the late '60s and early '70s. This was a time of extensive research on signal processing techniques using the FFT, excitation methods, and parameter estimation methods for extracting modal parameters.

The Fourier Analyzer systems which were developed at this time were limited to a maximum of 4 channels and 10 bit ADCs at a cost of approximately \$100,000. The computers which were built into these analyzers were limited to between 16K and 64K bytes of memory. FFT times, programmed in microcode on these machines, took approximately 1 second for 1024 FFT block sizes. In comparison, today using a \$2000 PC and a FFT algorithm programmed in a high level language takes a fraction of a millisecond to compute.

Modal testing using the FFT machines gradually replaced the swept sine testing during the early to mid '70s. Modal tests were performed by either impact testing or roving a single triaxial accelerometer over the structure. Frequency response measurements were made between the exciter location and accelerometer locations. These frequency response functions were curve fit using parameter estimation algorithms.

The cost of the Fourier Analyzer dropped and more channels were added during the late '70s. During the late '70s, Multiple-Input-Multiple-Output (MIMO) methods were developed. It became obvious that it was important to have consistent sets of data. Multiple measurement parameter estimation algorithms were being developed. These algorithms were sensitive to inconsistent data. This led to the idea that all the data should be taken simultaneously in order to get the most consistent data. The problem was that the cost of these systems would be too expensive. At the time, it appeared that the cost of the data acquisition could not be significantly reduced, but the cost of the response sensors could. In the early '80s, this led to the development of the Structcell<sup>TM</sup> system, which was a very inexpensive transducer system. The characteristics of the sensor were compromised in order to get more consistent data (errors of 20 to 30 percent were common in modal parameters determined from inconsistent data sets). The Structcells' sensitivities could fluctuate by 5 percent or more. The net result was an improvement in the modal parameter estimates. At the time, this was a "the chicken or the egg" problem. In order to get an inexpensive system with many channels, the cost of the transducer system was compromised. This led to the development of a multiple channel data acquisition system. Once the multiple channel data acquisition system had been purchased, then the transducer system could be updated. It ultimately led to the development of inexpensive data acquisition systems.

The test setup for a large scale modal test today is to premount several hundred response sensors (a large test may have over a thousand) and to excite the system with a small number of exciters using uncorrelated broadband input signals. A randomizing input signal tends to result in data sets which can be better processed using the current MIMO parameter estimation algorithms.

For these large tests, cable management is very important in order to keep track of the transducers and to minimize bookkeeping errors.

### Operating Inputs (Ambient Testing)

There is an increased interest in using ambient inputs to estimate modal parameters of a structure. There was an initial interest in this technology in the early '70s, primarily in the civil engineering area. There was a great deal of work on the development of using Auto Regressive Moving Average (ARMA) algorithms to process this ambient data. One of the major assumptions is that the inputs are white random noise, or at least have a smooth input spectrum. There should be no peaks or valleys in the input spectrum because these peaks can be identified as resonances of the system. In general, these applications were of minimum interest to the modal analysis community as a whole in the early '70s. There was also some interest in the aerospace industry for processing flight data in the mid '70s using ambient inputs.

In the '90s, a tremendous interest has developed in using modal parameters for the monitoring of infrastructures. In the '90s, there have been an increasing number of researchers looking at the measurement and parameter estimation aspects of this problem.

This area of research is also driving one of the next advances in data acquisition systems. Since these systems are large and located in remote sites, transducers which have digital outputs and can be located and synchronized over a network are being developed. This is going to significantly impact the data acquisition aspects of ordinary modal testing in the near future.

## Free Decay Testing

Free Decay Testing is a special case of ambient testing. For this case, the system is given an initial condition (displacement, velocity, etc.) and the free decay response of the system is measured. It should be noted that the unit impulse response of the system is a free decay. Therefore, the data can be processed using any of the parameter estimation algorithms used for processing frequency response or unit impulse measurements. This is a technique which can be used to test large or very small structures. It is a good field testing method. The data has to be measured simultaneously, or has to be measured relative to a reference transducer which is included in every free decay data set. Again, the advent of inexpensive multiple channels makes this an attractive method.

## ***Sensors-Calibration-Cable Management***

Over the past 30 years there has been significant advances in sensors, signal conditioning and cable management systems. The most popular sensing systems used over the past 30 years have used piezoelectric sensing elements. Both charge type and those with integrated electronics have been used. The integrated electronics have dominated since the mid '70s. The sensitivity of these sensors has increased, the size has been reduced, the dynamic range has significantly improved, and the cost has been reduced. In the '70s, data acquisition systems were limited to just a few channels, and the volume of sensors supplied to the modal analysis community was small. In the '80s, multiple channel data acquisition systems became popular, and less expensive sensing systems were developed. The standard large channel system used two to four load cells and 128 to 256 response channels (accelerometers). Improved cabling systems were developed so that large numbers of channels could easily be managed.

### Sensors

The most popular response sensor has been the accelerometer, but microphones, strain gages, and scanning laser systems are also used. For smaller light-weight systems, the laser system is often the sensor of choice. There is currently research into making the laser more adaptable for making multiple axis measurements at a point, including rotations. However, the laser is still limited to line-of-sight, which limits its applications.

Ambient testing of large structures is becoming more popular, and smart sensors and digital sensors which can be distributed along a network are being developed. These transducers can be accessed over the Internet, making remote sensing a possibility. For large structures, the wiring cost can be significantly reduced. For example, for a large bridge, the wiring cost can easily exceed the cost of the sensors and signal conditioning. For this case, a single network cable or a wireless network can be used which significantly reduces the wiring cost.

## Calibration

For large channel count systems, calibration has become an more important part of the pretest activity. Every sensor should be calibrated before and after every modal test. This can be a formidable task, since there are large numbers of sensors. For these cases, array calibrators are useful. There are commercial array calibrators, which can calibrate up to 120 channels simultaneously.

For the newly developed smart sensors, the calibration is stored in the sensor. If the sensor is moved during the test to a different acquisition channel, the calibration moves with sensor.

There are small hand held end to end calibrators which can be used with the modal test system to check the calibration or to calibrate a sensor during the test. Remember, the test data is only as good as the calibration.

## Cable Management

For large channel count tests, the cable management system becomes a very important part of the acquisition system. The bookkeeping can become one of the more difficult parts of a test. Cable management systems were developed in the mid '80s where keeping track of the cables was not necessary. The cable was arbitrarily connected from a location on the test structure to the data acquisition system. When the transducer was installed, it would self identify. Bar-coded labels were used to read the serial number of the sensor and the location of the sensor on the structure. This tremendously reduced the test setup time and minimized many of the bookkeeping problems.

The smart transducers which are currently being developed will take this a step further. Digital network transducers will make this possible over the Internet.

## ***Data Acquisition***

Over the years, there has been a continuing and accelerating level of development of data acquisition systems. The data acquisition systems became important with the advent of the FFT algorithm in the mid '60s. The initial ADCs used with the FFT system were 10 bits resolution and sampled at 50K samples per second. These ADCs were \$3000-\$5000 per channel. A low pass filter was required for each acquisition channel to eliminate aliasing problems. The original filters had adjustable cutoff frequencies and typically had a 48 dB per octave roll-off after the cutoff frequency.

The cost of the data acquisition systems changed slowly and the resolution had increased to 12 bits by the late '70s. The big change occurred when the anti-aliasing filter was incorporated into the data acquisition channel. A fixed frequency anti-aliasing filter (or possibly several cut-off fixed frequencies) was used and the data was digitally filtered and resampled to get the right

effective sampling rate. This also allowed the acquisition channel to perform a zoom transform. By the early to mid '80s, the cost of an acquisition channel was a little over \$1000.

The big change in the resolution occurred in the mid '80s, with the commercialization of digital music (CD players). The digital music was 16 bits and 16 DACs and ADCs were developed for this market. Since the CD players were a consumer product, the cost of these ADCs dropped significantly. These were delta sigma ADCs which provided basic anti-aliasing protection at the chip level. A small analog anti-aliasing filter would normally be included in the designed ADC to guarantee anti-aliasing protection. 16 bits is approximately 96 dB of dynamic range in the ADC. This reduced the requirements on the transducer signal conditioning. The number of input gain settings could be reduced, which in turn reduced the cost of the front end. However, the cost of the acquisition channel was only slightly reduced, but the resolution had increased and the signal condition requirements were reduced.

Over the period of time from the mid '60s, the transducers also evolved significantly. During the '60s, the accelerometers were either force balance (servo accelerometers) or charge type. The servos were very good accelerometers but were fragile, large and expensive. The charge transducers were smaller but required a charge amplifier, and the cables had to be protected to guarantee a good impedance balance. If the cables became dirty, this could significantly effect the low end frequency response. Also, for the smaller transducers, the capacitance of the cable was important. Simply flexing the cable could change the capacitance and change the sensitivity of the sensor. Low noise cables were required which were expensive and fragile. A third type of transducer was becoming popular in the mid '60s. These were piezo-electric sensors with built in electronics. This transducer operated over a two wire system and required inexpensive conditioning. The major limitation was that these transducers had a relatively high noise floor, and as a result, limited dynamic range. By the early '80s, these transducers were the most popular transducer for modal testing. Most of the commercial data acquisition systems had built-in conditioning for these sensors. Over the years, the noise floor of the built-in electronics improved dramatically, thereby reducing one of the big limitations of these sensors. These transducers also have a very low output impedance, which means that unshielded cabling can be used, thereby reducing cabling cost.

In the mid '80s, a very inexpensive transducer system was developed. This transducer used a sensing element which was essentially a microphone element. This transducer could use very inexpensive cabling. The cabling system was made modular so that patch panels and ribbon cables could be used in an integrated cable management system. The signal conditioning amplifiers for these sensors included a self-identifying feature. This meant that the system could determine which transducer was plugged into which data acquisition channel. Bar-coded labels were put on the sensors so that the calibration factors could be accessed. The points on the structure were also bar-coded, so that the location and direction of the sensor could easily be input into the modal analysis software. This cable management capability was very important for instrumenting structures with many sensors. The initial sensors were very inexpensive. The sensitivity of these sensors would vary approximately five percent due to environmental conditions. Even with this variation, improved modal data was obtained due to improved data consistency. In the late '80s and early '90s, this system was updated with inexpensive piezo-



electric shear element transducers which matched the characteristics of existing modal sensors. The sensitivities of these sensors are stable.

Currently, there are efforts to build smart sensors which have digital data stored in the sensor. This data includes the serial number, calibration data, location, and direction of the sensor. This is part of the IEEE 1451 standard, which will specify the communications protocol for digital sensors. This includes sensors with built-in ADCs and IP addresses so that they can communicate over a network. The network interface, or NCAP, can also handle the synchronization of these transducers.

### ***Signal Processing***

The signal processing area and the parameter estimation area are two areas where the most effort has been spent in the past thirty years in developing the experimental modal analysis area. A complete review of this work would be extensive. There are hundreds of references in the literature and there are a number of good text books on signal processing. In this review, only the high points will be addressed. This review is the equivalent of one day short course relative to a complete university course in signal processing. In order to have a good understanding of the signal processing aspects of modal analysis, it is important to understand the properties of the Fourier transform and the properties of the Laplace transform. These transformations explain the time-frequency-Laplace domain relationships. In terms of sampled data, the  $z$  transform is also important.

The input and output data is digitized and processed in most cases using the Fast Fourier Transform (FFT) algorithm to compute power spectra, cross spectra, frequency response functions, coherence functions, etc. This processed data is used to estimate modal parameters using various curve fitting or system identification processes.

The FFT is essentially a Fourier series approximation of the Fourier transform integral equation. As a result of this approximation, there are serious potential errors in the computation of frequency response functions using the FFT algorithm. A Fourier series assumes that the time function is a periodic function. If the time function is periodic and matches the period of the Fourier series, the results are a good approximation of the Fourier transform. It is also a good approximation if the time function is a completely observed transient. For this case, the Fourier series coefficients fall on the envelope of the Fourier transform. If the time signal is nonperiodic and is not completely observed in the window of the FFT, then serious errors can exist. These errors are referred to as 'leakage errors'. In general, there is no way to completely eliminate this leakage error. The effects of this error can be minimized using various types of sampling windows. The type of window used depends on the characteristics of the time signal and, to a lesser extent, on what the data is being used for. There are many good references on window functions and their effects on controlling leakage errors. All books on FFTs discuss windows and leakage in great detail. Leakage is a major error and has caused much grief in processing measurement data.

If the measurement data is sampled, there are also errors associated with sampling. The maximum frequency that can be computed from sampled data is determined by the sampling

frequency. This maximum frequency is equal to one half the sampling frequency. The maximum frequency is independent of the algorithm used to process the time data. If the data contains information greater than one half the sampling frequency, this information will fold back into the computed frequency range and cause an aliasing error. To eliminate this error, the time data should be passed through a low-pass filter to remove this high frequency content before sampling. All commercial Fourier Analyzer systems have anti-aliasing filters to remove this high frequency data. However, this can be a problem if a system is built up from components. For example, when using a PC and a plug-in data acquisition board, if the data acquisition board does not have anti-aliasing protection, then filters will have to be included.

The data used in most parameter estimation algorithms is either frequency response functions or unit impulse functions. The frequency response functions are simply the Fourier transforms of the unit impulse functions, and vice versa. In other words, they contain the same information about the system, and which one is used depends on the parameter estimation algorithm that is used to extract the modal parameters.

The frequency response function of a system is the Fourier transform of its output divided by the Fourier transform of its input. In terms of sampled data, the frequency responses are estimated by taking the FFT of the input and outputs and processing this data to get an estimate. The FFT operates on a rather small block of time data. The FFT of the input and output blocks of data contain leakage errors if the data is not completely observed (periodic functions which are periodic in the window or completely observed transients). This leakage error, as mentioned above, can be a serious error. In order to reduce these errors, different input signals can be used. In fact, there has been considerable research on the types of input signals and the types of windows which should be used to measure frequency response signals. The choice of the signal type and windows depends on the kind of measurement noise that is present in the data. The noise can be broken up into three types of noise:

- a. Non-Coherent Noise - The noise is due to electrical noise on the transducer signals, due to unmeasured excitation sources, etc., which are non-coherent with respect to the measured input signals or to some other signal which is used in the averaging process. Zero mean non-coherent noise can be eliminated by averaging with respect to a reference signal. This reference signal could be the input signal in terms of a spectrum averaging process, or could be a synchronization or trigger signal in terms of cyclic averaging or a random decrement process.
- b. Signal Processing Noise - The signal processing itself may generate noise. For example, "leakage" is a classic source of noise when using FFTs for computing frequency domain measurements. This type of noise is reduced or eliminated by using completely observed time signals (periodic or transient), by using various types of windows, or by increasing the frequency resolution.
- c. Nonlinear Noise - If the system is nonlinear, then free decay, frequency response, or unit impulse function measurements may be distorted, which consequentially causes problems when estimating modal parameters. Nonlinear distortion noise can be eliminated by linearizing the test structure before testing, or by randomizing the input signals to the structure. This will cause the nonlinear distortion noise to become non-coherent with respect

to the input signal. The nonlinear noise can then be averaged out of the data in the same manner as ordinary non-coherent noise.

In the early to mid '70s, there was a significant amount of research done on excitation techniques. To summarize this research in a sentence or two, if the system is lightly damped and leakage is a problem, then a periodic or a completely observed transient signal should be used. If there is nonlinear noise, then a randomizing signal should be used. Straight averaging then takes care of the non-coherent noise. For heavily damped systems, pure random is normally used, and for lightly damped systems, burst random or periodic random signals are used. Most systems have some degree of nonlinearities, and random signals are best suited for these cases. For field testing, the transient methods are convenient (impact testing, step relaxation). In recent years there has been renewed interest in excitation methods that use cyclic averaging and other techniques for reducing leakage.

Frequency response estimation algorithms are another area where there has been a significant amount of research. A least squares technique is used to estimate the frequency response from the measured input output data. In general, a great deal of time data is used in the estimation of frequency response functions. This amount of data is necessary to average out the non-coherent and nonlinear noise. Like all pseudo inverse solutions, there is no unique answer. The answer depends on the error object function and any weighting functions applied to the data. There are several frequency response estimators:

1.  $H_1$  which assumes all the noise is on the output.
2.  $H_2$  which assumes all the noise is on the input
3.  $H_v$  which assumes the noise is on both the input and output, in a manner where the noise vector is perpendicular to the solution vector.

At a particular frequency, if all the noise is on the output, then  $H_1$  will estimate the true frequency response of the system in the limit of the averaging process. In other words, if the objective function is satisfied, then the estimator will give the right answer if enough averages are taken to average the noise to zero.

It is important to make good measurements, since the parameter estimation does not compensate for bad measurements.

### ***Parameter Estimation***

Modal parameters can be estimated from a variety of different measurements. These measurements can include free decays, forced responses, frequency responses, and unit impulse responses. These measurements can be processed one at a time, or in partial or complete sets simultaneously. The measurements can be generated with no measured inputs, a single measured input, or multiple measured inputs. The data can be measured individually or simultaneously. In other words, there is tremendous variation in the types of measurements and in the types of constraints that can be placed on the testing procedures.

There has been at least a hundred different modal parameter estimation algorithms developed in the past 30 years. The ones covered in this review are the ones that have become commercially

available. Many users have developed their own algorithms, or have some sort of special adaptation of a standard algorithm.

Modal parameter estimation algorithms became important in the mid '60s when it became convenient to measure frequency response functions. The frequency response functions can be expressed in terms of modal parameters as follows:

$$H_{pq}(\omega) = \sum_{r=0}^N \frac{A_{pq_r}}{j\omega + \lambda_r} \quad \text{where } A_{pq_r} \approx \Psi_{p_r} \Psi_{q_r} \quad 5$$

In this equation, the eigenvalues ( $\lambda$ ) and residues ( $A$ ) (function of eigenvectors( $\Psi$ )) are the modal parameters. The frequency response is measured and is known, and the modal parameters are unknown. The modal parameters are estimated from this expression by a parameter estimation process.

The frequency response function is measured between a given input point and a given output point as function of frequency. The temporal information (frequency) is a function of the eigenvalue, and the spatial information is a function of the eigenvector. In order to measure the eigenvector, a number of frequency responses must be measured as a function of response point or input point in order to sample the spatial information.

Unfortunately, a set of nonlinear equations must be solved in order to estimate the modal parameters from the measured frequency response functions. In the mid '60s, this was a difficult problem since these nonlinear equations are very ill-conditioned. It took very good starting values to have the solution converge. Approximate solutions based on Single-Degree-of-Freedom (SDOF) algorithms were used to obtain the starting values. These SDOF algorithms proved to be useful tools for getting quick estimates of the modal parameters. As a result, these algorithms are useful for troubleshooting applications and are still used. Quadrature response method, circle fitting, and SDOF polynomial method are the more useful of these algorithms.

In the late '60s and early '70s, an algorithm which was being used to analyze noise signatures from submarines turned out to be the first really successful parameter estimation algorithm for fitting measured frequency response functions. The algorithm was the Complex Exponential Algorithm (CEA). The complex exponential algorithm has historically been derived from the Prony algorithm, which was developed over one hundred years ago. The method simply takes a time history, which is composed of a summation of damped exponentials, and estimates the amplitude, frequency and damping of the individual terms in the summation. Of course, this corresponds to computing the eigenvalues and residues for a measured initial condition or impulse response of a system. The Prony algorithm is in reality a very clever algebraic manipulation for solving a nonlinear set of equations for the eigenvalues. When this algorithm was first used in modal analysis to process unit impulse functions, it was a revelation. Over the next several years, this method was extended to handle multiple impulse measurements in a least squares sense. This method was referred to as the Least Squares Complex Exponential (LSCE) algorithm. At the same time that the least squares complex exponential algorithm was being

developed, the Ibrahim Time Domain (ITD) method was also being developed. Both of these methods were designed to take multiple impulse or initial condition measurements from a single initial condition or a single reference input and compute a set of eigenvalues and eigenvectors. The mathematical derivations were completely different, and as a result these methods were thought of as being completely different methods. In reality, both methods are similar and can be derived from a common starting point which will be discussed later in this review. All of these techniques fit unit impulse response functions which are computed by taking the inverse Fourier transform of the frequency response functions.

In the early '80s when MIMO testing became popular, the LSCE method was extended to handle multiple references. This was a major step forward since the multiple references allowed repeated roots to be estimated, and in general did a better job of uncoupling closely coupled modes. Again, the basic philosophy was one of fitting the response of the system. This was the Polyreference Time Domain (PTD) method. Again, algebraic manipulation was used in deriving this method. It was very difficult to see any common threads between the ITD method and CEA methods (CEA, LSCE and PTD). The next major development occurred in the mid '80s, and this was the development of the Eigenvalue Realization Algorithm (ERA). This algorithm was developed from a viewpoint used by control engineers and represented a breakthrough view point. Instead of fitting a model to the solution, the concept of fitting a model to determine the underlying system equations was developed (generalized equations of motion). This was the approach used by the engineers looking at large civil engineering structures in the early '70s, who were trying to determine modal parameters of civil engineering from unmeasured ambient inputs. Unfortunately, the modal analysis community did not pick up on this method as a general parameter estimation procedure in the early '70s. The approach used an Auto-Regressive Moving Average (ARMA) modeling method. The ARMA consisted of two types of terms, auto-regressive terms which were simply regression coefficients, and moving average terms which in general handled the unmeasured inputs and noise. The AR terms are linear but the MA terms are nonlinear. This complicated the solution of the set of equations. In the ERA case, the responses were assumed to be free decays and the MA could be neglected. Therefore, a simple set of linear equations could be solved to determine the AR terms.

Historically, ARMA models were developed as a set of finite difference equations and were specific to the time domain. However, with the general use of both the Fourier transform and the Laplace transform, the frequency and Laplace domain information are used in an interchangeable manner. The time, frequency and the Laplace variables are simply transforms of each other and correspond to the temporal information. The definition of an ARMA model has been expanded from the finite difference formulation to include the time, frequency and Laplace domains. One of the problems is that the definition of the ARMA model has been expanded by many researchers in different areas of science, to the point where there is general confusion over what is an ARMA model. In the context of the applications described in this paper, only the matrix polynomial formulation of the ARMA model will be used and expressed as a function of the time, frequency or the Laplace variable with only the AR (ARX model) terms being used. This method will simply be referred to as the Unified Matrix Polynomial Approach (UMPA). The basic form of the UMPA model is given in the discrete time and frequency domains below:

## Time Domain

$$\sum_{i=0}^N [A_i] \{x_{m+i}\} = \sum_{j=0}^M [B_j] \{f_{m+j}\} \quad 6$$

## Frequency Domain

$$\sum_{i=0}^N (j\omega_k)^i [A_i] \{X(\omega_k)\} = \sum_{j=0}^M (j\omega_k)^j [B_j] \{F(\omega_k)\} \quad 7$$

where  $N$  is the order of the UMPA model. The number of eigenvalues are determined by the order of the model times the size of the  $A$  matrices. The eigenvalues are computed from a companion matrix constructed from the  $A$  coefficients. The  $\{X\}$  vector is the measured responses and the  $\{F\}$  vector is the measured forcing functions.

The  $A$  and  $B$  coefficients are both auto-regressive terms, and can be determined by solving a set of linear equations from the measured response and input forces. Once the  $A$ 's and  $B$ 's are known, then an eigenvalue problem can be formulated to determine the eigenvalues. For the ITD and ERA methods, which are formulated as low order models, the eigenvector is also obtained from the eigenvalue solution. For PTD, which uses a high order model, the eigenvector is only estimated at the reference points (exciter locations). The complete eigenvector is estimated in a second phase.

In the time domain, if free decays (unit impulse functions) are used, then there are no external forces acting on the system during the free decay, and the  $A$  terms are the only important terms for determining the responses of the system. The CEA, LSCE, PTD, ITD and ERA methods can all be derived using the UMPA model formulation based on a model with just  $A$  terms. These methods are all related, and the relationships are made clear. These methods are also the time domain algorithms which have been commercialized and are currently the most popular time domain methods.

There are frequency domain equivalents to all of the above techniques, and they can easily be derived. For the frequency domain case, the  $B$  terms have to be included. In the frequency domain, the force always acts at the frequency of the response. The Polyreference Frequency Domain (PFD) algorithm is a commercial example. The PFD algorithm is actually a frequency domain implementation of a low order model closer to ERA than to PTD.

The differences between all of the above-mentioned methods are simply the order of the polynomial and the number of references. CEA is a high order model with a single reference. PTD is a high order model with the number of references equal to the number of inputs. ITD is a low order model with a single reference, and ERA is a low order model with multiple references. PFD is a low order frequency domain model with multiple references. All of these techniques are commercially available.

The procedure for using these parameter estimation algorithms is as follows:

1. Measure Input-Output Responses

- a. Can be time histories, Fourier spectra
- b. Averaged data: power spectra, cross spectra, correlation functions, etc.
- c. Frequency Response Functions, Unit Impulse Responses, etc.
- d. Random decrement measurements

2. Determine type of UMPA model to use (domain, model order-matrix size)

3. Determine [A]'s and [B]'s of the UMPA model using a pseudo-inverse procedure.

Important

- a. Condensation procedures
  - i. Least Squares
  - ii. Total Least Squares
  - iii. Singular Value Decomposition (SVD)
  - iv. Coherent Averaging

4. Solve for eigenvalues from UMPA model

5. Solve for the eigenvectors

Determination of the model order-matrix size is important, as indicated above. As a result, there have been a number of techniques which have been developed to aid in the determination of the model order-matrix size. For example, the rank of the matrices used in computing the pseudo-inverse can be used as an indicator for determining the model order. Singular Value Decomposition (SVD) is another technique used to evaluate the rank of the pseudo-inverse problem. For the multiple reference cases, various mode indicator functions can help (Multiple Variant Mode Indicator function, Complex Mode Indicator Function(CMIF), to name several). These techniques establish a starting point for the model order. In general, with the time domain techniques, the model order-matrix size should be over specified by a factor of 1.5 to 2. Over specifying allows the technique to have other degrees-of-freedom which are useful in reducing the influence of measurement noise, distortion errors, data inconsistencies, etc. The over specifying gives better answers for the system eigenvalues, but generates computational or noise eigenvalues which are difficult to sort out.

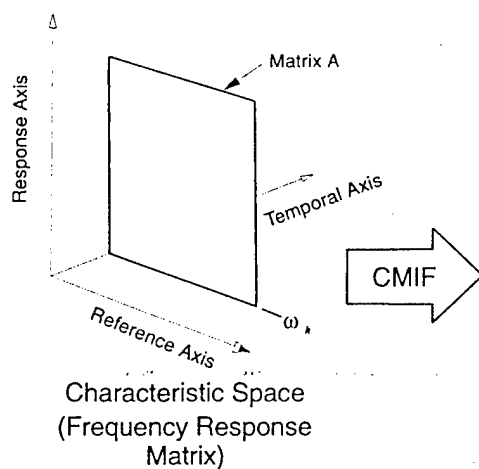
There are a number of techniques which are useful in sorting out these computational eigenvalues (modes). Stability diagrams, modal indicator functions, Modal Assurance Criteria (MAC) tests, and jack knife procedures, are some of the more important techniques. The commercial software vendors have tried to make this sorting process as simple as possible. However, sorting out computational modes is still the biggest problem with using the multiple reference time domain methods. It requires experience and judgement to use the sorting tools effectively. Novice users have great difficulty using these tools on difficult data.

There is currently renewed research activity on other parameter estimation strategies which do not have the computational mode problems. As mentioned in the introduction, technology often folds back on itself. Methods which were used earlier are being rediscovered and modified to take advantage of newer computational and measurement technologies. One of the more successful new parameter estimation methods is the CMIF parameter estimation method, which

is based on a SDOF method which has been used since the late '50s. The CMIF method can process multiple reference frequency response measurements using the peak picking or quadrature response technique for measuring the eigenvector. If there are  $n$  references, then  $n$  quadrature estimates of the eigenvector can be obtained for a given frequency. The  $n$  estimates of the eigenvector can be loaded as the columns of a matrix. The matrix size is the number of response points by the number of references. A SVD can be performed on the matrix. If the quadrature eigenvector is a good approximation of the true eigenvector, there should only be one significant Singular Value (SV). The SVD vector associated with the largest SV is the best least squares estimate of the dominant vector that spans the space of the  $n$  quadrature estimates. This SVD vector is a good estimate of one of the system eigenvectors if the quadrature vectors are picked at one of the natural frequencies of the system.

The CMIF algorithm is implemented by performing a SVD on the frequency response matrix at each spectral line. The SVs of this SVD operation are plotted as a function of frequency. See Figure 2 for a graphical description of the CMIF Algorithm. The peaks in the largest SV curve correspond to the eigenvalues of the system. In fact, the plot of the largest SV curve corresponds to information similar to the information contained in a power spectrum plot generated from all the measured frequency response functions. Repeated or closely coupled eigenvalues are indicated when two or more of the SV curves reach peaks at the same frequency. The data processing is independent of frequency or temporal information. This model corresponds to a zero order UMPA model. This type of model is a *spatial domain* model. It does not estimate the eigenvalues directly. The eigenvalues are estimated in a second phase. The eigenvectors are used as modal filters to filter the frequency response data and generate a series of Enhanced Frequency Response Functions (eFRFs). These eFRFs can be fit with a

The CMIF plot is constructed by performing a Singular Value Decomposition (SVD) of the Frequency Response Matrix as a function of spectral line (frequency). The resulting SV's are plotted as a function of frequency.



$$[A] = [H(\omega_k)]$$

$$SVD([A]) = [U][sv][V]'$$

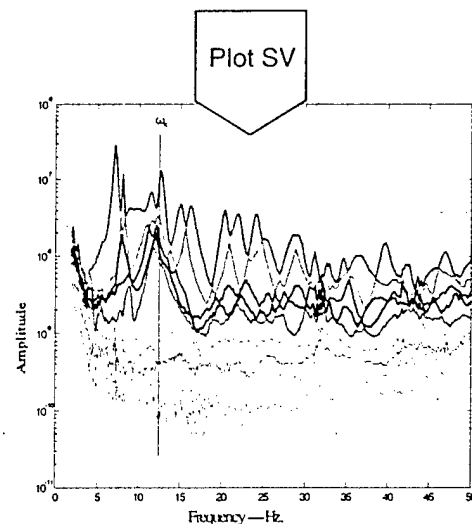


Figure 2 -- CMIF Schematic



simple SDOF polynomial method to get the eigenvalues and modal scale factors.

CMIF is a relatively simple multiple reference algorithm which works well for cases where many references are available. The CMIF does an excellent job of uncoupling closely coupled modes, but at some point it starts to collapse. In the early '70s when the quadrature method was no longer useful, a more sophisticated SDOF polynomial curve fitting method was used. This method assumed a single mode but included higher order B coefficients to handle important out of band eigenvalues. This same approach has been used to extend the CMIF method. It is called the Enhanced Mode Indicator Function (EMIF). It uses a simplified version of the PFD algorithm. Instead of adding extra eigenvalue terms to handle noise and distortion, extra B terms are added. CMIF is used to estimate the number of eigenvalues in a small frequency band and the PTD method, set for a fixed number of eigenvalues with extra B terms, is used to estimate the modal parameters for the frequency band in question. This method eliminates the difficult step of sorting out computational modes. The EMIF method is meant to supplement the CMIF method. CMIF is the method of choice and EMIF is only used in the difficult areas.

Circle fitting, another of the early SDOF methods, is currently being used to understand and characterize modes that exhibit non-linear behavior. This is an important application when solving real problems.

As mentioned previously, there is increasing interest in extracting modal parameters from ambient inputs. Currently, there is considerable research in developing and refining modal parameter estimation algorithms for ambient testing. All of the standard methods will work with ambient response data if assumptions about the input forces can be made. When exciters are used to excite the structure, the resulting measurements are relatively good. Adding extra terms to the AR model to handle the noise is a reasonable strategy to get a linear solution. For ambient testing where there is significantly more noise, then including the Moving Average (MA) terms is useful. When the MA terms are included, the solution is nonlinear and an iterative solution may be necessary.

There have been hundreds of papers written on modal parameter estimation over the past 30 years. Instead of referring to individual papers, the best reference is the IMAC proceedings where most of the work has been documented for the past 15 years. The first 5 or 6 proceedings of IMAC included a rather complete biography of modal analysis up to the time of those proceedings. The IMAC proceedings are available on CDs from SEM (Society for Experimental Mechanics, Inc.). These CDs make it relatively easy to search for developments in the modal analysis area.

## **Modal Model Assimilation and Validation**

A typical modal test consists of a number of individual tests for different exciter configurations and test setups. A modal model has to be assembled from all of these tests. From one exciter configuration, a fraction of the modal parameters can be determined. Other exciter configurations supply additional information. More than one estimate of each modal parameter is normally available from a complete modal test. Averaging and sorting the modal database

generated during the testing is required to obtain the best modal model. Once the modal model is generated, then it can be validated using a number of techniques:

- 1) Synthesis of arbitrary FRF
- 2) Reciprocity-Curve fitting
- 3) Perturbation
  - a) Mass Additive
  - b) Constraints
- 4) Forced Response
- 5) Orthogonality
- 6) Model Correlation

The first four items use the modal model to predict measurements or system performance which can be experimentally checked using data taken during the modal test. Frequency response measurements can be synthesized from the modal model and checked against measurements made during the testing programs. These could be measurements used in extracting the modal parameters, or measurements made specifically to validate the model.

Most of the parameter estimations algorithms do not enforce reciprocity, so this can be used to check the model. If the modal model demonstrates reciprocity, then input and output predictions should obey reciprocity.

The modal model should be able to predict simple modifications to the structure, such as simple mass modifications. Masses can be added at important connection points during the test and frequency response measurements made on the modified structure and checked against results predicted by the modal model.

One of the more severe tests is to predict responses due to inputs to the model, and to compare these against the measured responses. Inputs signals can be input at selected points and the response time histories can be predicted at other points. The predicted time histories can be compared against the measured time histories.

The last two cases are checks against FEM models. The first is to check orthogonality with respect to the analytical mass matrix, to determine if the measured eigenvectors satisfy orthogonality. The second is to correlate the model with FEM results. Of course, these last two methods are making assumptions about the validity of the analytical model. These tests primarily provide a check on the analytical model.

## Concluding Remarks

As mentioned throughout this review, this is only a brief overview of the high points of experimental modal testing. No specific references are cited in the review, since it covers the complete area of modal testing, and there are hundreds of reference articles. A complete overview of the complete area of modal analysis can be found in the IMAC proceedings.

# VIBRATION-BASED DAMAGE DETECTION

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## Abstract

Many aerospace, civil, and mechanical systems continue to be used despite aging and the associated potential for damage accumulation. Therefore, the ability to monitor the structural health of these systems is becoming increasingly important. A wide variety of highly effective local non-destructive evaluation tools are available. However, damage identification based upon changes in vibration characteristics is one of the few methods that monitor changes in the structure on a global basis. The material presented herein will summarize developments in vibration-based damage detection that have taken place over the last thirty years. The process of vibration-based structural health monitoring will first be formulated as a problem in statistical pattern recognition. This process is composed of four parts: 1.) Operational evaluation; 2.) Data acquisition and cleansing; 3.) Feature selection and data compression, and 4.) Statistical model development. Throughout this summary of the statistical pattern recognition paradigm, applications of this technology to rotating machinery and civil engineering infrastructure will be used to demonstrate various aspects regarding the implementation of the health monitoring process. The rotating machinery application represents a mature technology where vibration-based damage detection has made the transition from a research topic to industry practice. The civil engineering applications represent a field that is still primarily the focus of research efforts. This summary identifies technical challenges that must be addressed if vibration-based structural health monitoring is to gain wider application. Finally, a discussion of future directions for this technology is presented.

## 1. INTRODUCTION

In the most general terms damage can be defined as changes introduced into a system that adversely effect its current or future performance. Implicit in this definition is the concept that damage is not meaningful without a comparison between two different states of the system, one of which is assumed to represent the initial, and often undamaged, state. This discussion is focused on the study of damage identification in structural and mechanical systems. Therefore, the definition of damage will be limited to changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely effect the current or future performance of that system.

The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical and aerospace engineering communities. Current damage-detection methods are either visual or localized experimental methods such as acoustic or ultrasonic methods, magnetic field methods, radiograph, eddy-current methods and thermal

field methods (Doherty, 1987). All of these experimental techniques require that the vicinity of the damage is known *a priori* and that the portion of the structure being inspected is readily accessible. Subjected to these limitations, such experimental methods can detect damage on or near the surface of the structure. The need for quantitative **global** damage detection methods that can be applied to complex structures has led to the development and continued research into methods that examine changes in the vibration characteristics of the structure.

The basic premise of vibration-based damage detection is that damage will significantly alter the stiffness, mass or energy dissipation properties of a system, which, in turn, alter the measured dynamic response of that system. Although the basis for vibration-based damage detection appears intuitive, its actual application poses many significant technical challenges. The most fundamental challenge is the fact that damage is typically a local phenomenon and may not significantly influence the lower-frequency global response of structures that is normally measured during vibration tests. Stated another way, this fundamental challenge is similar to that in many engineering fields where the ability to capture the system response on widely varying length scales, as is needed to model turbulence, has proven difficult. Another fundamental challenge is that in many situations vibration-based damage detection must be performed in an *unsupervised learning* mode. Here, the term *unsupervised learning* implies that data from damaged systems are not available. These challenges are supplemented by many practical issues associated with making accurate and repeatable vibration measurements at a limited number of locations on complex structures often operating in adverse environments.

In an effort to emphasize the extent of the research efforts to date in vibration-based damage detection, a brief summary of applications that have driven developments in this field over the last thirty years is first presented. Recent research has begun to recognize that the vibration-based damage detection problem is fundamentally one of statistical pattern recognition and this paradigm is described in detail. Current damage detection methods are then summarized in the context of this paradigm. This summary is supplemented with examples of structural health monitoring applied to rotating machinery and bridge structures. The rotating machinery application represents a mature technology where vibration-based damage detection has made the transition from a research topic to industry practice. The bridge applications represent a field that is still primarily the focus of individual research efforts with only recent and limited marketing of this technology on a commercial basis. Concluding comments will compare and contrast the rotating machinery applications with the bridge application. These comments will focus on the future research directions for this technology and some of the economic considerations that are driving this research.

## 2. HISTORICAL PERSPECTIVE

It is the authors' speculation that damage or fault detection, as determined by changes in the dynamic properties or response of systems, has been practiced in a qualitative manner, using acoustic techniques, since modern man has used tools. More recently, this subject has received considerable attention in the technical literature and a brief summary of the developments in this technology over the last thirty years is presented below. Specific references are not cited; instead the reader is referred to (Doebeling, et al. 1996, 1998) for a recent review of literature on this subject.

The development of vibration-based damage detection technology has been closely coupled with the evolution, miniaturization and cost reductions of Fast Fourier Transform (FFT) analyzers and digital computing hardware. To date, the most successful application of vibration-based damage detection technology has been for monitoring rotating machinery. The rotating machinery application has taken an almost exclusive non-model based approach to damage detection. The detection process is based on pattern recognition applied to time histories or spectra generally measured at a single point on the housing of the machinery during normal operating conditions. Often this pattern recognition is performed only in a qualitative manner. Databases have been developed that allow specific types of damage to be identified from particular features of the vibration signature. For these systems the approximate damage location is generally known making a single channel FFT analyzer sufficient for most periodic monitoring activities. Typical damage that can be identified includes loose or damaged bearings, misaligned shafts, and chipped gear teeth. Today, commercial software integrated with measurement hardware is marketed to help the user systematically apply this technology to the operating equipment.

During the 1970s and 1980s the oil industry made considerable efforts to develop vibration-based damage detection methods for offshore platforms. This damage detection problem is fundamentally different from that of rotating machinery because the damage location is unknown and because the majority of the structure is not readily accessible for measurement. To circumvent these difficulties, a common methodology adopted by this industry was to simulate candidate damage scenarios with numerical models, examine the changes in resonant frequencies that were produced by these simulated changes, and correlate these changes with those measured on a platform. A number of very practical problems were encountered including measurement difficulties caused by platform machine noise, instrumentation difficulties in hostile environments, changing mass caused by marine growth and varying fluid storage levels, temporal variability of foundation conditions, and the inability of wave motion to excite higher vibration modes. These issues prevented adaptation of this technology, and efforts at further developing this technology for offshore platforms were largely abandoned in the early 1980s.

The aerospace community began to study the use of vibration-based damage detection during the late 1970's and early 1980's in conjunction with the development of the space shuttle. This work has continued with current applications being investigated for the National Aeronautics and Space Administration's space station and reusable launch vehicle. The Shuttle Modal Inspection System (SMIS) was developed to identify fatigue damage in components such as control surfaces, fuselage panels and lifting surfaces. These areas were covered with a thermal protection system making them inaccessible and, hence, impractical for conventional local non-destructive examination methods. This system has been successful in locating damaged components that are covered by the thermal protection system. All orbiter vehicles have been periodically subjected to SMIS testing since 1987. Space station applications have primarily driven the development of experimental/analytical damage detection methods. These approaches are based on correlating analytical models of the undamaged structure with measured modal properties from both the undamaged and damaged structure. Changes in stiffness indices as assessed from the two model updates are used to locate and quantify the damage. Since the mid 1990's, studies of damage detection for composite materials have been motivated by the development of a composite fuel tank for a reusable launch vehicle.

The civil engineering community has studied vibration based damage assessment of bridge structures since the early 1980's. Modal properties and quantities derived from these properties such as mode-shape curvature and dynamic flexibility matrix indices have been the primary features used to identify damage in bridge structures. Environmental and operating condition variability present significant challenges to the bridge monitoring application. The physical size of the structure also presents many practical challenges for vibration-based damage assessment. Regulatory requirements in eastern Asian countries, which mandate the companies that construct the bridges periodically certify their structural health, are driving current research and commercial development of vibration-based bridge monitoring systems.

In summary, the review of the technical literature presented by (Doebling et al. 1996, 1998) shows an increasing number of research studies related to vibration-based damage detection. These studies identify many technical challenges to the adaptation of vibration-based damage detection that are common to all applications of this technology. These challenges include better utilization of the nonlinear response characteristics of the damaged system, development of methods to optimally define the number and location of the sensors, identification of the features sensitive to small damage levels, the ability to discriminate changes in features caused by damage from those caused by changing environmental and/or test conditions, the development of statistical methods to discriminate features from undamaged and damaged structures, and performance of comparative studies of different damage detection methods applied to common data sets. These topics are currently the focus of various research efforts by many industries including defense, civil infrastructure, automotive, and semiconductor manufacturing where multi-disciplinary approaches are being used to advance the current capabilities of vibration-based damage detection.

### **3. VIBRATION-BASED DAMAGE DETECTION AND STRUCTURAL HEALTH MONITORING**

The process of implementing a damage detection strategy is referred to as *structural health monitoring*. This process involves the definition of potential damage scenarios for the system, the observation of the system over a period of time using periodically spaced measurements, the extraction of features from these measurements, and the analysis of these features to determine the current state of health of the system. The output of this process is periodically updated information regarding the ability of the system to continue to perform its desired function in light of the inevitable aging and degradation resulting from the operational environments. Figure 1 shows a chart summarizing the structural health-monitoring process. The topics summarized in this figure are discussed below. Examples of structural health monitoring applied to rotating machinery and to bridges are used to further illustrate the topics summarized in Fig. 1.

#### **3. 1. Operational Evaluation**

Operational evaluation answers three questions in the implementation of a structural health monitoring system:

1. How is damage defined for the system being investigated and, for multiple damage possibilities, which are of the most concern?

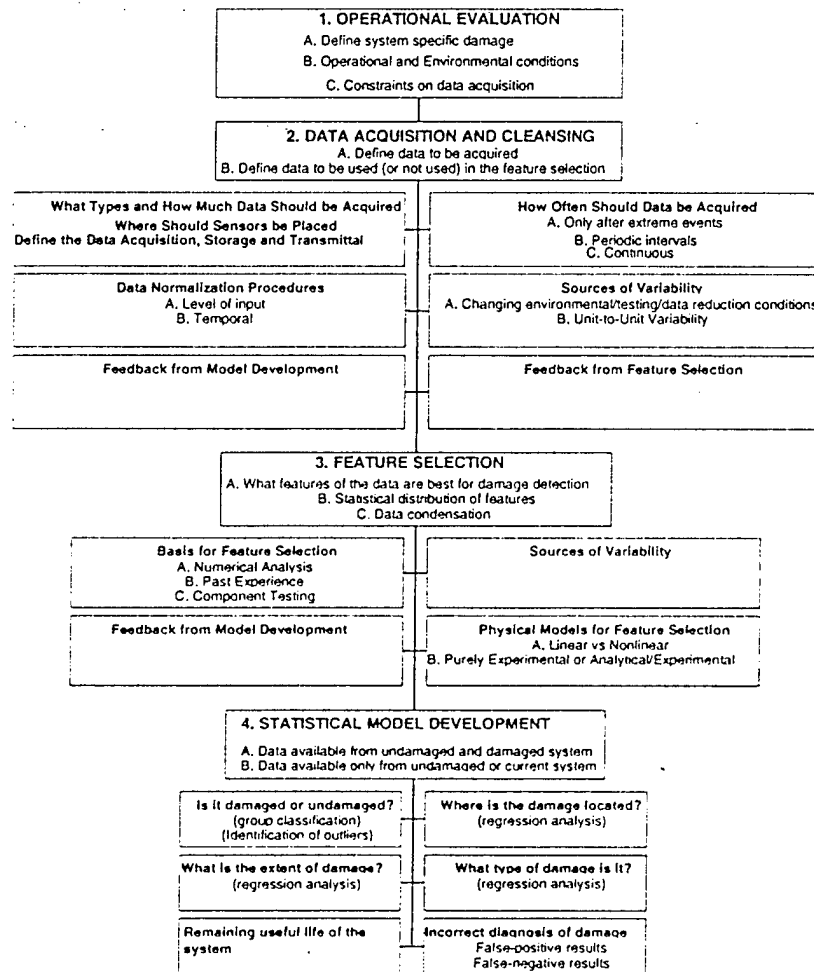


Fig. 1. Flow chart for implementing a structural health monitoring program.

2. What are the conditions, both operational and environmental, under which the system to be monitored functions?
3. What are the limitations on acquiring data in the operational environment?

Operational evaluation begins to set the limitations on what will be monitored and how the monitoring will be accomplished. This evaluation starts to tailor the health monitoring process to features that are unique to the system being monitored and tries to take advantage of unique features of the postulated damage that is to be detected.

### 3.1.1. Operational evaluation for rotating machinery

The definition of damage is often very straightforward for rotating machinery. Many times there are a limited number of damage scenarios and the possible damage locations are known *a priori*. The primary operational limitation on acquiring data is that the machine will typically be in operation and performing its normal function or will be in a transient start-up or shutdown

mode. In its *in situ* environment many other machines will most likely produce additional vibration sources that must be accounted for in the damage detection process. Limitations to acquiring vibration data can vary widely. For many applications the limitations will be based on administrative criteria such as the availability of personnel to make the necessary measurements. In other applications the machine may be located in hazardous environments that limit access time.

### 3.1.2. Operational evaluation for bridges

The definition of damage for bridges can vary widely as many bridges are one-of-a-kind structures. In many cases it is difficult to specifically define the damage to be monitored because there has not been enough experience with failures of the particular structure being studied or similar structures. For large bridges it is imperative that the damage definition be restricted so as to make the monitoring possible with a cost-effective number of sensors. The need to perform the monitoring in a manner that does not impede traffic flow dictates that the monitoring occurs when the bridge is subjected to changing traffic patterns coupled with typical environmental variability. These operational and environmental constraints pose a formidable challenge to bridge health monitoring. Traffic flow can vary on a 24-hr cycle, and a weekly cycle. Environmental variability can occur over a 24-hr cycle as shown in Fig. 2, over a seasonal cycles as discussed in (Askegaard and Mossing, 1988), and intermittent cycles caused by varying rainfall conditions. Portions of the bridge may not be accessible for instrumentation during normal operating conditions. Traffic usually prevents the topside of the deck from being instrumented. For large bridges many of the structural elements are difficult to instrument because it is impractical to access them in a safe and economic manner. Finally, the sensors and associated data transmission hardware are subjected to harsh environments that make equipment reliability a serious issue (Nigbor, and Diehl, 1997).

## 3. 2. Data Acquisition and Cleansing

The data acquisition portion of the structural health monitoring process involves selecting the types of sensors to be used, selecting the location where the sensors should be placed, determining the number of sensors to be used, and defining the data acquisition/storage/transmittal hardware. This process is application specific. Economic considerations play a major role in these decisions. Another consideration is how often the data should be collected. In some cases it is adequate to collect data immediately before and at periodic intervals after a severe event. However, if fatigue crack growth is the failure mode of concern, it is necessary to collect data almost continuously at relatively short time intervals.

Because data can be measured under varying conditions, the ability to normalize the data becomes very important to the damage detection process. One of the most common procedures is to normalize the measured responses by the measured inputs. When environmental or operating condition variability is an issue, the need can arise to normalize the data in some temporal fashion to facilitate the comparison of data measured at similar times of an environmental or operational cycle. Sources of variability in the data acquisition process and with the system being monitored need to be identified and minimized to the extent possible. In



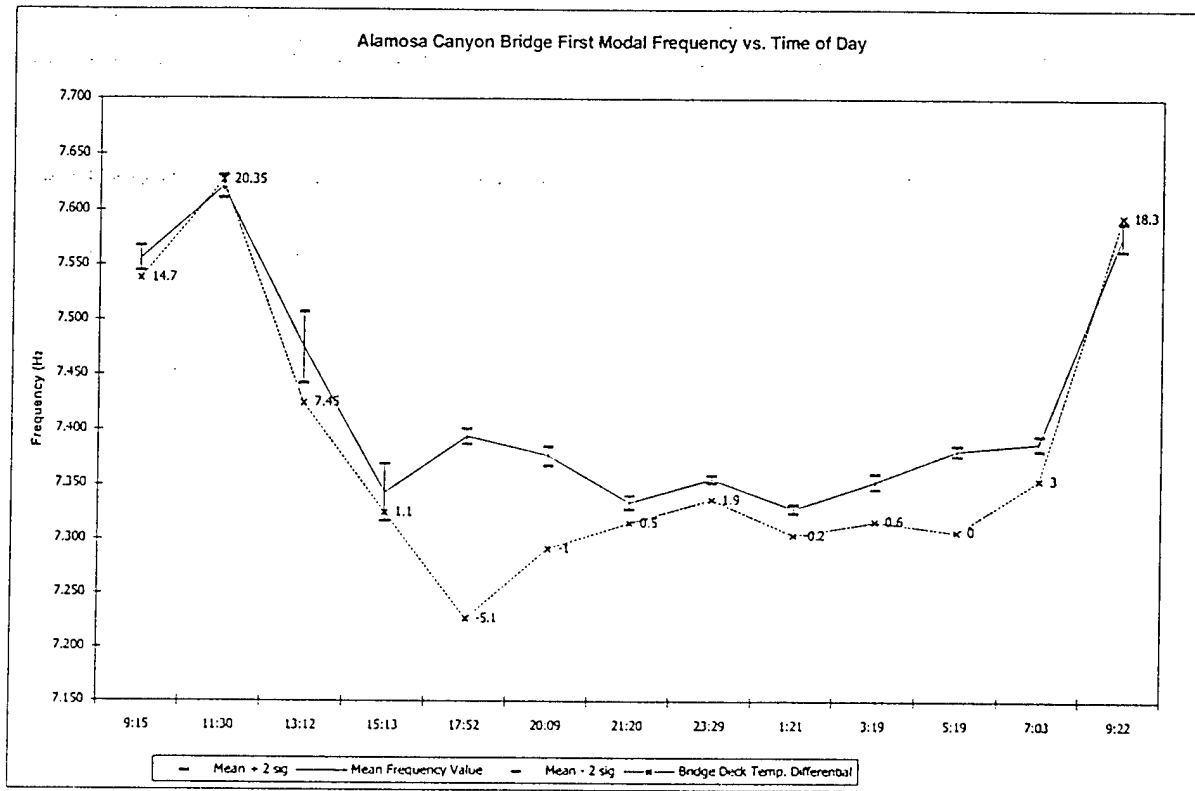


Fig. 2. The change in the first mode resonant frequency (solid line) over a 24-hr period measured on the Alamosa Canyon Bridge in southern New Mexico. The dashed line is the temperature differential (in degrees F) measured across the deck. Testing procedures and the statistical methods used to generate the error bars can be found in (Farrar, et al., 1997).

In general, all sources of variability cannot be eliminated. Therefore, it is necessary to make the appropriate measurements such that these sources can be statistically quantified.

Data cleansing is the process of selectively choosing data to accept for, or reject from, the feature selection process. The data cleansing process is usually based on knowledge gained by individuals directly involved with the data acquisition. Finally, it is noted that the data acquisition and cleansing portion of a structural health-monitoring process should not be static. Insight gained from the feature selection process and the statistical model development process provides information regarding changes that can improve the data acquisition process.

### 3.2.1. General data acquisition issues common to rotating machinery and bridges

Data acquisition issues for rotating machinery and bridge applications include the sensor type and the number of sensors, sensor location, sensor mounting, environmental effects on the sensors, signals recording, and record duration. Averaging, windowing and similar data processing parameters must also be determined. Another issue is determining the steps that could be taken to make the data acquisition as repeatable as possible.

### 3.2.2. Data acquisition for rotating machinery

Accelerometers are the primary vibration transducer used for damage detection and condition monitoring of rotating machinery. Piezoelectric accelerometers have a broad operating frequency range and are well suited to monitoring of roller bearings and gear trains. Accelerometers are typically used in conjunction with single-channel signal analyzers so that the machinery vibration output signal can be viewed in the frequency domain as well as a function of time, i.e., amplitude-frequency, amplitude-time, and waterfall plots. Velocity transducers and non-contact displacement transducers are also widely used. Non-contact (Eddy current) displacement transducers find application for monitoring shaft motion and position relative to fluid-film bearings. A set of two transducers, mounted at right angles, is often used to determine the orbit of the shaft in its bearing. Data acquisition recording equipment and transducers used to monitor rotating machinery are discussed in detail in (Mitchell, 1992; Crawford, 1992; Eisenmann and Eisenmann, 1997; Hewlett-Packard Application Note 243-1, 1997; Taylor, 1994; and Wouk, 1991). The selection and placement of appropriate transducers depends upon the type of machinery and its construction. A detailed discussion of sensor placement for rotating machinery applications is given in (Eisenmann and Eisenmann, 1997).

### 3.2.3. Data acquisition for bridges

As with the rotating machinery the primary sensors used for bridge health monitoring are piezoelectric accelerometers. Force-balance accelerometers and electric resistance and vibrating-wire strain gages are also widely used. More recently, fiber-optic sensors utilizing Bragg grating (Todd, et al., 1999) have been studied as a means of increasing channel counts for bridge monitoring in a cost-effective manner. In addition to the motion measuring devices, anemometers and temperature sensors such as thermocouples are used to characterize the environmental variability. Data transmission and recording are done in manners similar to most mechanical vibration applications as summarized in (McConnell, 1995). Recent developments in wireless data acquisition systems have shown promise for large civil engineering structures (Straser, 1998). When ambient excitation sources are used, all channels of data are typically recorded simultaneously. The damage to be monitored and the number of available sensors typically dictate sensor placement. Most bridge health monitoring studies reported to date utilize between 15 and 50 sensors. At the extreme is the Tsing Ma Bridge in Hong Kong that has been instrumented with 600 sensors at an approximate cost of US\$16 million. Data acquisition intervals are not well defined for bridges as most studies have been conducted in a research mode where long term monitoring has not been the focus of the study.

## 3. 3. Feature Selection

The data features used to distinguish the damaged structures from undamaged ones receives the much attention in the technical literature. Inherent in the feature selection process is the condensation of the data. The operational implementation and diagnostic measurement technologies needed to perform structural health monitoring typically produce a large amount of data. Condensation of the data is advantageous and necessary, particularly if comparisons of many data sets over the lifetime of the structure are envisioned. Also, because data may be acquired from a structure over an extended period of time and in an operational environment,

robust data reduction techniques must retain sensitivity of the chosen features to the structural changes of interest in the presence of environmental noise.

The best features for damage detection are typically application specific. Numerous features are often identified for a structure and assembled into a feature vector. In general, a low dimensional feature vector is desirable. It is also desirable to obtain many samples of the feature vectors. There are no restrictions on the types or combinations of data contained in the feature vector. As an example, a feature vector may contain the first three resonant frequencies of the system, the time when the measurements were made, and a temperature reading from the system. A variety of methods are employed to identify features for damage detection. Past experience with measured data from a system, particularly if damaging events have been previously observed for that system, is often the basis for feature selection. Numerical simulation of the damaged system's response to simulated inputs is another means of identifying features. The application of engineered flaws, similar to ones expected in actual operating conditions, to laboratory specimens can identify parameters that are sensitive to the expected damage. Damage accumulation testing, during which significant structural components of the system under study are subjected to a realistic accumulation of damage, can also be used to identify appropriate features. Fitting linear or nonlinear, physical-based or non-physical-based models of the structural response to measured data can also help identify damage-sensitive features. Common features used in vibration-based damage detection studies are briefly summarized below. A more detailed summary can be found in (Doebling, et al., 1996, 1998).

Basic modal properties, mode shape curvature changes, dynamic flexibility matrices and stiffness indices from updated finite element models are features most commonly used for damage detection in bridges. Features derived from the time-histories, the spectral pattern or from nonlinear response of the system are primarily used for rotating machinery damage detection studies.

### **3.3.1. Basic modal properties**

The most common features that are used in damage detection, and that represent a significant amount of data condensation from the actual measured quantities, are resonant frequencies and mode-shape vectors. These features are identified from measured response time-histories, most often absolute acceleration, or spectra of these time-histories. The technology required to accurately make these measurements is summarized in (McConnell, 1995). Often these spectra are normalized by spectra of the measured force input to form frequency response functions. Well-developed experimental modal analysis procedures are applied to these functions or to the measured-response spectra to estimate the system's modal properties (Ewins, 1995, and Maia and Silva, 1997).

The amount of literature that uses resonant frequency shifts as a data feature for damage detection is quite large. The observation that changes in structural properties cause changes in vibration frequencies was a primary impetus for developing vibration-based damage identification technology. In general, changes in frequencies cannot provide spatial information about structural changes. For applications to large civil engineering structures the low sensitivity of frequency shifts to damage requires either very precise measurements of frequency change or

large levels of damage. An exception occurs at higher modal frequencies, where the modes are associated with local responses. However, the practical limitations involved with the excitation and identification of the resonant frequencies associated with these local modes, caused in part by high modal density and low participation factors, can make them difficult to identify.

Figure 3 shows incremental damage made by torch cuts in the main girder of a bridge as part of a controlled damage detection study (Farrar, et al., 1994). Also shown in Fig. 3 is the first mode resonant frequency of the undamaged structure (dam 0) and changes in the first mode frequencies for each subsequent damage case. Although difficult to see, error bars corresponding to the 95 percent confidence intervals on the identified resonant frequencies are also shown (Doebling and Farrar, 1998). The increases in frequency associated with the first two damage cases are the result of systematic errors in the testing procedure. In terms of absolute value, these changes are more significant than those produced by the third damage case. This study indicates some of the difficulties in using resonant frequencies as a damage indicator for large civil engineering structures.

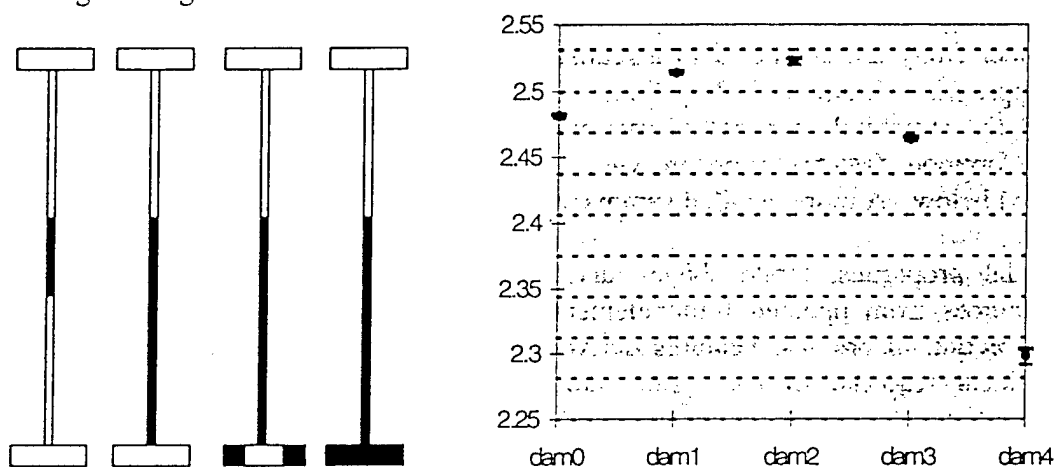


Fig. 3. Bridge damage cases and corresponding changes in the first mode resonant frequency.

Damage detection methods using mode shape vectors as a feature generally analyze differences between the measured modal vectors before and after damage. Mode shape vectors are spatially distributed quantities; therefore, they provide information that can be used to locate damage. However, a large number of measurement locations can be required to accurately characterize mode shape vectors and to provide sufficient resolution for determining the damage location.

Figure 4 shows the first three modes shapes and corresponding resonant frequencies measured on the undamaged bridge and similar modes and resonant frequencies after the introduction of the final level . After the final damage was introduced, the location of the damage is clearly indicated when Mode 1 in Fig. 4 (final stage of damage) is compared with the corresponding damaged mode in Fig. 4 (undamaged). Intermediate damage levels produced mode-shape changes in that were not statistically significant from the undamaged case (Doebling and Farrar, 1998). It should also be noted that when damage is located at a node for a particular mode, as it is for Mode 3, even the most severe damage case produces no significant change in that mode shape or corresponding resonant frequency. Again, this example illustrates the relative

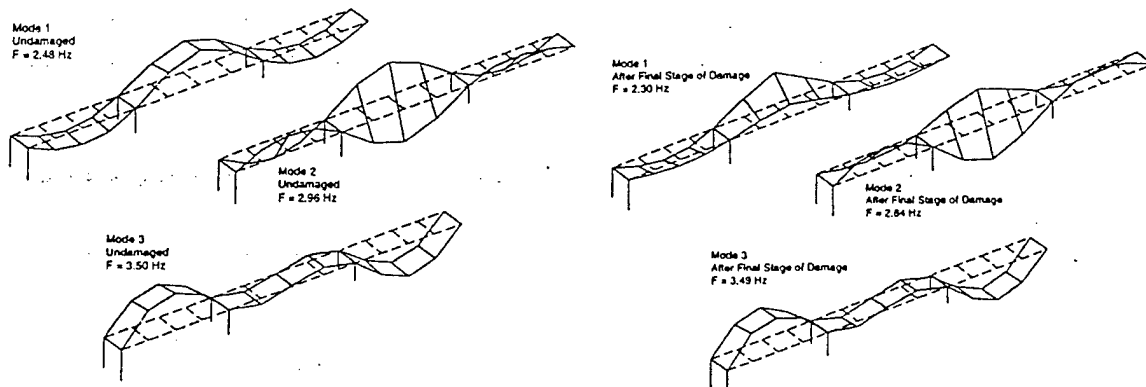


Fig. 4. First three modes identified on the undamaged bridge (left) compared with the first three modes identified after the final level of damage (right).

insensitivity of the lower-frequency global modes and resonant frequencies of a large civil engineering structure to local damage

### 3.3.2. Mode shape curvature changes

An alternative to using mode shapes to obtain spatially distributed features sensitive to damage is to use mode shape derivatives, such as curvature. Mode shape curvature can be computed by numerically differentiating the identified mode shape vectors twice to obtain an estimate of the curvature. These methods are motivated by the fact that the second derivative of the mode shape is much more sensitive to small perturbations in the system than is the mode shape itself. Also, for beam- and plate-like structures changes in curvature can be related to changes in strain energy, which has been shown to be a sensitive indicator of damage. A comparison of the relative statistical uncertainty associated with estimates of mode shape curvature, mode shape vectors and resonant frequencies showed that the largest variability is associated with estimates of mode shape curvature followed by estimates of the mode shape vector. Resonant frequencies could be estimated with least uncertainty (Doebeling, et al., 1997).

### 3.3.3. Dynamically measured flexibility

Changes in the dynamically measured flexibility matrix indices have also been used as damage sensitive features. The dynamically measured flexibility matrix is estimated from the mass-normalized measured mode shapes and measured eigenvalue matrix (diagonal matrix of squared modal frequencies). The formulation of the flexibility matrix is approximate because in most cases all of the structure's modes are not measured. Typically, damage is detected using flexibility matrices by comparing the flexibility matrix indices computed using the modes of the damaged structure to the flexibility matrix indices computed using the modes of the undamaged structure. Because of the inverse relationship to the square of the modal frequencies, the measured flexibility matrix is most sensitive to changes in the lower-frequency modes of the structure.

### 3.3.4. Updating structural model parameters

Another class of damage identification methods is based on features related to changes in mass, stiffness and damping matrix indices that have been correlated so the numerical model predicts as closely as possible the identified dynamic properties (resonant frequencies, modal damping and mode shape vectors) of the undamaged and damaged structures. These methods solve for the updated matrices (or perturbations to the nominal model that produce the updated matrices) by forming a constrained optimization problem based on the structural equations of motion, the nominal model, and the identified modal properties (Friswell and Mottershead, 1995). Comparisons of the matrix indices that have been correlated with modal properties identified from the damaged structure to the original correlated matrix indices provide an indication of damage that can be used to quantify the location and extent of damage. Degree of freedom mismatch between the numerical model and the measurement locations can be a severe limitation on performing the required matrix updates.

### 3.3.5. Time-history and spectral pattern methods

Non-model based approaches that examine changes in the features derived directly from measured time histories or their corresponding spectra have been used extensively by the rotating machinery industry. There exist numerous detailed charts of anticipated characteristic faults of a variety of machines and machine elements and corresponding features in the measured time histories or spectra (Mitchell, 1992; Crawford, 1992). These features have been widely used to successfully detect the presence, location and type of fault, and the degree of damage. Commercially available software specifically designed for the isolation of faults based on vibration signatures is readily available.

Qualitative features include, for example, the presence of peaks in acceleration spectra at certain multiples of shaft rotational frequency and their growth or change with time. The important qualitative features are quite distinct to the type of machine element, the specific fault, and in some cases to the level of damage. Therefore, it may be possible to locate the defective machine element, isolate the specific fault in the element, and determine the level of damage based purely on these qualitative features. As an example, Figure 5 shows that changes in a power spectral density function can be used to monitor the deterioration in a vacuum blower's bearings. Early studies using qualitative features were based on the concept that detection of each fault is fundamentally different. Recent progress has, however, been reported on generalized failure prediction indices capable of monitoring the condition of a wide variety of manufacturing equipment (Roth and Pandit, 1999).

Quantitative features fall into the following categories: time-domain methods, transformed-domain methods, and time-frequency methods. Included in transformed-domain methods are the well-known frequency-domain methods as well as the cepstrum (the inverse Fourier transform of the logarithm of the Fourier spectra magnitude squared) techniques. Briefly, frequency domain methods characterize features in machine vibrations over a given time window. Time domain and time-frequency methods have application to non-stationary faults, i.e., those associated with machines that exhibit different phenomena in different phases of the machine cycle.

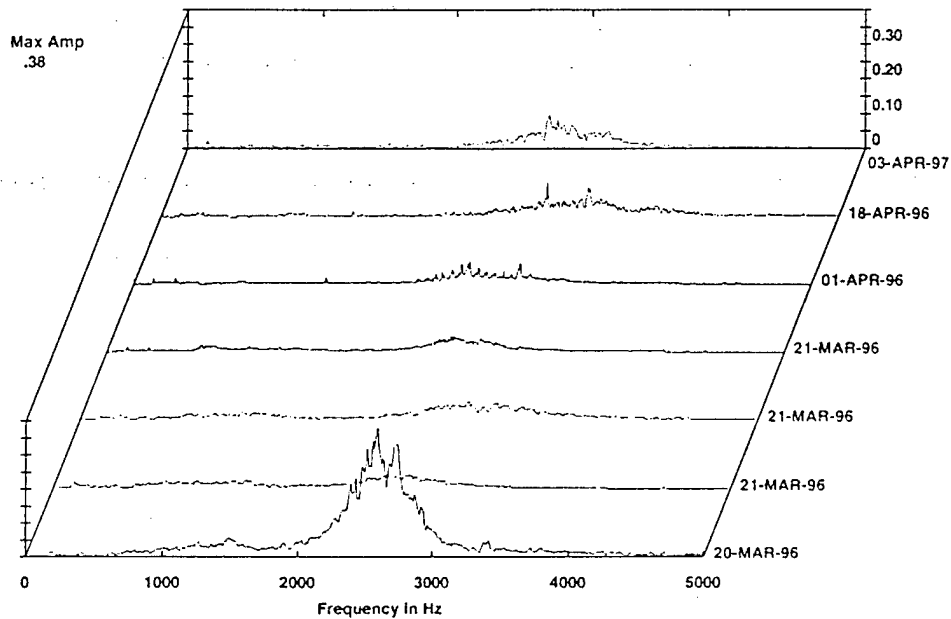


Fig. 5. Power spectral density measured on a vacuum blower. The March 20<sup>th</sup> measurement was made when maintenance personnel felt that the equipment was running at a hotter temperature than normal. Bearings were replaced on March 21<sup>st</sup>. Subsequent plots show spectra corresponding to the degrading bearings. (Courtesy of Intel Corp.)

### 3.3.5.1. Time domain methods

These methods have particular application to roller bearings because roller bearings typically fail by localized defects caused by fatigue cracking and the associated removal of a piece of material on one of the bearing contact surfaces. (Ma and Li, 1993) summarize these methods (particularly for roller bearing analysis) as: peak amplitude, rms amplitude, crest factor analysis, kurtosis analysis, and shock pulse counting. (Martin, 1989) utilizes Kurtosis measurements of the acceleration amplitudes for detection of surface damage in roller bearings. If surface roughness attributes are used as an indicator of damage, then for a good surface, the profile is random corresponding to a Gaussian distribution with an infinite-sample theoretical value of 3.0. A Kurtosis value other than 3.0 denotes that the profile is no longer Gaussian, thus indicating the presence of damage. Proprietary time-domain methods and associated instrumentation are commercially available for the detection of defects involving repetitive mechanical impacts, primarily associated with roller bearings (Le Bleu and Xu, 1995)

### 3.3.5.2. Frequency domain methods

Approaches summarized in (Ma and Li, 1993) for roller bearings include Fourier spectra of synchronized-averaged time histories, cepstrum analysis, sum and difference frequency analysis, the high frequency resonance technique, and short-time signal processing. Quantitative evaluation gear faults using cepstrum peaks as a harmonic indicator is proposed in (Tang, et al., 1991). Thresholds distinguishing normal, moderate and serious wear in gears are determined quantitatively. Other cepstral approaches for spectral-based fault detection applied to helicopter gearboxes are presented in (Kemerait, 1987).

### 3.3.5.3. Time-frequency methods

These methods have their application in the investigation of rotating machinery faults exhibiting non-stationary vibration effects. Non-stationary effects are associated with machinery in which the dynamic response differs in the various phases associated with a machine cycle. Examples include reciprocating machines, localized faults in gears, and cam mechanisms. The wavelet transform is discussed in (Chui, 1992) and is applied to fault detection and diagnosis of cam mechanisms in (Dalpiaz and Rivola, 1997) and to a helicopter gearbox in (Wang and McFadden, 1996). An application to fault detection utilizing three widely differing methods falling in the above categories (Fourier transform, power cepstrum, and wavelet transform) as applied to two meshing spur gears with an induced local fault on one gear is shown in (Petrilli, et al., 1995). A comparative study of various quantitative features that fall into the time-domain and frequency-domain categories is presented in (Elbestawi and Tait, 1986).

### 3.3.6. Nonlinear Methods

Identification of the basic modal properties, mode shape curvature changes and dynamic flexibility are based on the assumption that a linear model represents the structural response before and after damage. However, in many cases the damage will cause the structure to exhibit nonlinear response. Therefore, the identification of features indicative of nonlinear response can be a very effective means of identifying damage in a structure that originally exhibited linear response. The specific features that indicate a system is responding in a nonlinear manner vary widely. Figure 6 shows the Wigner-Ville transform applied to vibration response data from both an uncracked and cracked cantilever beam. The time-frequency plots in Fig. 6 show the generation of resonant frequency harmonics in the freely-vibrating, cracked cantilever beam as well as the change in stiffness state as the crack opens and closes (Prime and Shevitz, 1996). For an extreme event such as an earthquake, the normalized arias intensity provides an estimate of the structure's kinetic energy as a function of time and has been successfully used to identify the onset of nonlinear building response subjected to damaging earthquake excitations (Straser, 1998). Deviations from a Gaussian probability distribution function of acceleration response amplitudes for a system subjected to a Gaussian input have been used successfully to identify that loose parts are present in a system. Temporal variation in resonant frequencies identified using canonical variate analysis is another method to identify the onset of damage (Hunter, 1999). In general, features based on the nonlinear response of a system have only been used to identify that damage has occurred. Few methods have been described that locate the source of the nonlinearity. Because all systems exhibit some degree of nonlinearity, it is a challenge to establish a threshold for which changes in the nonlinear response features are indicative of damage. The statistical model building portion of the structural health monitoring process is essential for establishing such thresholds. Note that the previously discussed features based on time-history and spectral pattern changes are often the result of nonlinear response caused by the damage.



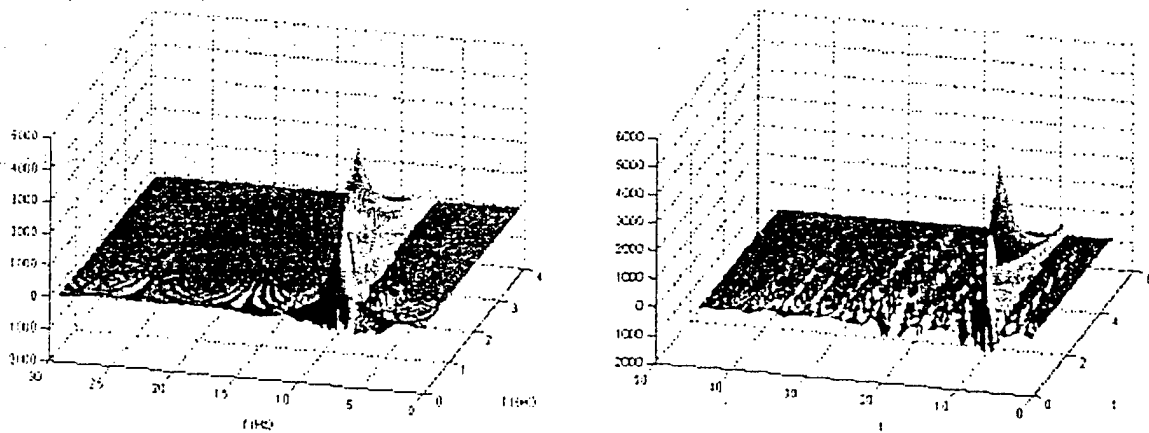


Fig. 6. Wigner-Ville transforms of the free-vibration acceleration-time histories measured on an uncracked cantilever beam (left) and a cracked cantilever beam (right).

### 3. 4. Statistical Model Development

The portion of the structural health monitoring process that has received the least attention in the technical literature is the development of statistical models to enhance the damage detection. Almost none of the hundreds of studies summarized in (Doebling, et al, 1996, 1998) make use of any statistical methods to assess if the changes in the selected features used to identify damaged systems are statistically significant. (Doebling and Farrar, 1998) present one of the few applications of statistical analysis to a bridge structure damage detection study. However, there are many reported studies for rotating machinery damage detection applications where statistical models have been used to enhance the damage detection process (e.g. Petrilli, et al., 1995; Chin and Danai, 1991; and Stevenson, et al., 1991).

Statistical model development is concerned with the implementation of the algorithms that operate on the extracted features to quantify the damage state of the structure. The algorithms used in statistical model development usually fall into three categories. When data are available from both the undamaged and damaged structure, the statistical pattern recognition algorithms fall into the general classification referred to as *supervised learning*. *Group classification* and *regression analysis* are supervised learning algorithms. *Unsupervised learning* refers to algorithms that are applied to data not containing examples from the damaged structure. All of the algorithms produce statistical distributions of the measured or derived features to enhance the damage detection process.

The damage state of a system can be described as a five-step process along the lines of the process discussed in (Rytter, 1993) to answers the following questions: 1. Is there damage in the system (existence)?; 2. Where is the damage in the system (location)?; 3. What kind of damage is present (type)?; 4. How severe is the damage (extent)?; and 5. How much useful life remains (prediction)? Answers to these questions in the order presented represents increasing knowledge of the damage state. The statistical models are used to answer these questions in an unambiguous and quantifiable manner. Experimental structural dynamics techniques can be used to address the first two questions. To identify the type of damage, data from structures with

the specific types of damage must be available for correlation with the measured features. Analytical models are usually needed to answer the fourth question unless examples of data are available from the system (or a similar system) when it exhibits varying damage levels.

Finally, an important part of the statistical model development process is the testing of these models on actual data to establish the sensitivity of the selected features to damage and to study the possibility of false indications of damage. False indications of damage fall into two categories: 1.) False-positive damage indication (indication of damage when none is present), and 2.) False-negative damage indications (no indication of damage when damage is present). Although the second category is detrimental to the damage detection process and can have serious implications, false-positive readings also erode confidence in the damage detection process.

### 3.4.1 Supervised learning: group classification

Group classification attempts to place the features into respective "undamaged" or "damaged" categories in a statistically quantifiable manner. Informally, skilled individuals can use their experience with previous undamaged and damaged systems and the changes in the features associated with previously observed damage cases to deduce the presence, type and level of damage. This is an example of informal *supervised learning*. For example, it is possible to examine acceleration signals in the frequency or time domain and deduce in some cases, from the presence and location of peaks, the type, location, and extent of damage of a rotating machinery component. As previously cited, extensive tables are commercially available to facilitate this process.

More formal methods founded in machine learning have been applied to damage detection. These methods place features into either an undamaged category or one or more damaged categories (Chin and Danai, 1991). The classification techniques fall into three general categories: Bayesian Classification,  $K^{\text{th}}$ -nearest neighbor rules, and artificial neural network classifiers (Lin and H.-P. Wang, 1993).

A specific example of a Bayesian group-classification statistical model applied to damage detection process is shown in Fig. 7. Here, a linear discriminant operator, "Fisher's Discriminant" (Bishop, 1995), is applied to the problem of identifying structural deterioration in progressively damaged concrete columns (Farrar, et al., 1999). Accelerometer time histories are recorded from sensors attached to the columns while the columns are excited with an electrodynamic shaker. Linear prediction coding (LPC) coefficients convert the accelerometer time-series data into multi-dimensional samples representing the dynamic response of the system during a brief segment of the time series. Fisher's discriminant is then used to find the linear projection of the LPC data distributions that best separates data from undamaged and damaged systems. Fisher's discriminant is defined such that the separation of features from the undamaged columns and the columns at incipient damage is maximized. The same projection is then used with subsequent damage cases. For the cases shown, the method captures a clear distinction between undamaged and damaged vibration features. Data from subsequent tests can then be classified using Bayesian methods as belonging to either the undamaged or damaged group.

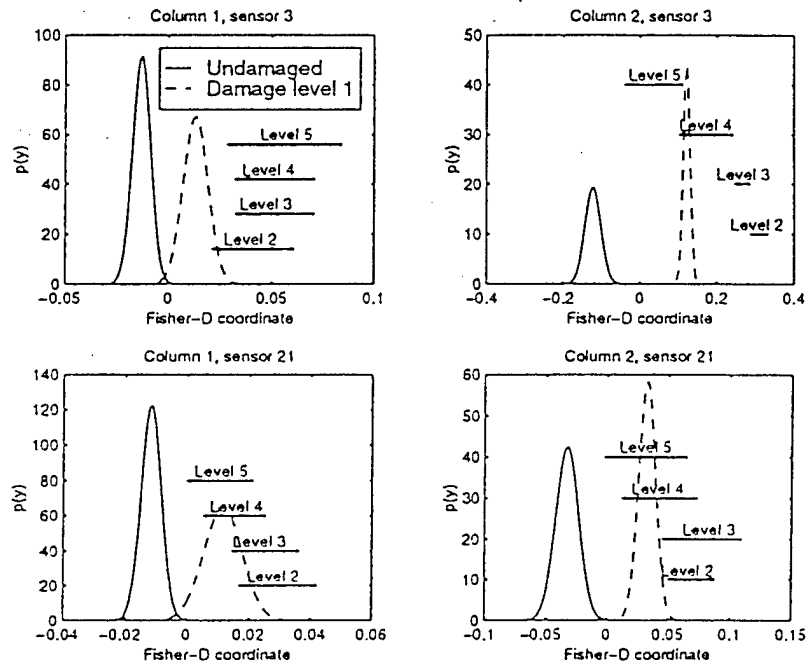


Fig. 7. Distribution of LPC-generated feature vectors projected onto the Fisher coordinate. Solid horizontal lines represent the widths of the distributions for higher damage levels.

### 3.4.2. Supervised learning: regression analysis

Another category of statistical modeling that can be employed in the damage detection process is regression analysis. Typically, this analysis refers to the process of correlating data features with particular locations or extents of damage. The features are mapped to a continuous parameter, such as spatial location or a remaining-useful-life temporal parameter, as opposed to group classification where the features correspond to discrete categories such as “damaged” or “undamaged”. Regression analysis for damage detection applications requires that features from the undamaged structure and from the varying damage cases or levels are available to generate the mapping

### 3.4.3. Unsupervised learning: density estimation

Finally, analysis of outliers is employed when data are not available from a damaged structure. This type of analysis attempts to answer the question: When data from a damaged structure are not available for comparison, do the observed features indicate a significant change from the previously observed features that can not be explained by extrapolation of the feature distribution? Multivariate probability density function estimation is one of the primary statistical tools employed in this type of analysis. A particular difficulty with performing an analysis of outliers is that as the feature vectors increase in dimension, large amounts of data are needed to properly define the density function. The trigger level indicators of anomalous response that have been incorporated in commercial bridge monitoring systems can be thought

of as providing indicators of outliers (Nignor and Diehl, 1997). However, to date, these indicators have not been based on a statistical analysis of the measured data.

#### 4. COMPARISON OF BRIDGE AND ROTATING MACHINERY APPLICATIONS

Throughout this summary, applications of vibration-based damage detection to rotating machinery and bridge structures have been used to emphasize various issues and current limitations of vibration-based damage detection schemes. These bridge and rotating machinery applications represent the extremes in successful applications of the damage detection technology. In general, the application of vibration-based damage detection to rotating machinery has made the transition from a research topic to successful implementation by practicing engineers. In contrast, vibration-based damage detection in larger structures, such as bridges, has been studied for many years, but this application has, in most cases, not progressed beyond the research phase. In conclusion, a summary directly comparing these two applications is presented. This comparison further emphasizes some of the research directions that must be followed if vibration-based damage detection for large structural systems is to gain the same acceptance that it has in the rotating machinery industry.

1. Motivation: Damage detection in bridges has been primarily motivated by the prevention of loss of life; damage detection in rotating machinery is motivated largely by economic considerations often related to minimizing production downtime. Clearly, there are exceptions where bridges are being monitored to facilitate timely and cost-effective maintenance and where failure of rotating machinery can have life-safety implications, as for example, fracture of jet engine turbine blades.
2. Availability: Highway bridges are generally one-of-a-kind items with little or no data available from the damaged structure. Rotating machines are often available in large inventories with data available from both undamaged and damaged systems. It is much easier to build databases of damage-sensitive features from these inventories and, hence, supervised machine learning can be much more readily accomplished for rotating machinery.
3. Definition of Damage: For rotating machinery there are a finite number of well-defined damage scenarios and the possible locations of that damage are limited to a fairly small spatial region. Many bridge damage detection studies do not specifically define either the damage type or location.
4. Operational Evaluation: In practical health-monitoring applications, measured vibration inputs are not applied to either class of system. Rotating machinery typically exhibits response to a harmonic-like input, while traffic tends to produce inputs that are typically assumed to be random in nature.
5. Data Acquisition: Because the approximate location of the damage is generally known, vibration test equipment for rotating machinery can consist of but a single sensor and a single-channel FFT analyzer. Monitoring of bridges is normally performed with few channels distributed over a relatively large spatial region. For damage ID on a highway bridge, 30-50 data acquisition channels represent a sparsely instrumented bridge. A permanent *in situ* data acquisition system for bridge structures can be represent a significant capital outlay and further funds would be needed to maintain such a system over extended periods of time.
6. Feature Selection: A well developed database of features corresponding to various types of damage has been developed by the rotating machinery community. Many of these features

are qualitative in nature and have been developed by comparing vibration signatures from undamaged systems to signatures from systems with known types, locations and levels of damage. Many of the features observed in the vibration signatures of rotating machinery result from nonlinear behavior exhibited by the damaged system. Features used to identify damage in bridge structures are most often derived from linear modal properties such as resonant frequencies and mode shapes. These features are identified before and after damage and require a distributed system of sensors. Few studies report the development of damage-sensitive features for bridge structures based on nonlinear response characteristics.

7. Statistical Model Building: The rotating machinery literature reports many more studies that investigate the application of statistical pattern classifiers to the damage detection process than have been reported for civil engineering infrastructure applications. Rotating machinery is often sited in a relatively protected environment and operates under relatively consistent conditions. The primary sources of extraneous vibration inputs are other rotating machinery in the vicinity. Changes in damage-sensitive features caused by environmental and operational variability are significant and must be accounted for in bridge applications through statistical pattern classifiers. However, the literature shows little application of this technology to bridge damage detection studies.

Clearly, the application of vibration-based damage detection to rotating machinery is a much more mature technology than that associated with large engineering infrastructure. This comparison shows that a pressing need for the large system applications is to define a limited number of damage scenarios to be monitored. Such a limitation will reduce the requirement for an expensive and difficult to maintain distributed sensing system. Advances in low-cost, wireless instrumentation and data acquisition systems can make a major contribution to the large structures applications. Further developments of sensitive analog-to-digital converter technology will reduce the noise floor in measurements allowing for better assessment of the high-frequency structural response. It is postulated that this high frequency response is more sensitive to local damage. Also, to account for variability in ambient loading conditions and environmental variability, it is imperative that the statistical pattern classifier technology must be adopted and further developed by researchers in this field. Without this technology it will be difficult to determine if changes in the identified features are caused by damage or are caused by varying operational/environmental conditions. Identifying new damage-sensitive features, particularly those that are based on nonlinear, time-varying response, should always be a focus of research efforts. Finally, there is a pressing need to make measurements on large one-of-a-kind structures such as bridges and buildings. Experience gained from analyzing data from *in situ* structures will be instrumental in developing new damage sensitive features as well as defining new and improved hardware for the vibration measurements.

## 5. CONCLUDING COMMENTS

The development of robust vibration-based damage detection technology has many elements that make it a potential "Grand Challenge" for the structural dynamics community. First, almost every industry wants to detect damage in its structural and mechanical infrastructure at the earliest possible time. Industries' desire to perform such monitoring is based on the tremendous economic and life-safety benefits that this technology has the potential to offer. The semiconductor manufacturing industry is adopting vibration-based damage detection to help

minimize the redundancy in rotating machinery required to prevent inadvertent downtime in their fabrication plants. Such downtime can cost these companies on the order of US\$10 million per hour. Driven by life-safety concerns resulting from recent structural failures, regulatory agencies in Asia are requiring the builders of large infrastructure to instrument the structures and periodically certify their health.

Significant future developments of this technology will, in all likelihood, come by way of multi-disciplinary research efforts encompassing fields such as structural dynamics, signal processing, motion and environmental sensing hardware, computational hardware, data telemetry, smart materials, and statistical pattern recognition, as well as other fields yet to be defined. Finally, the problem of global structural health monitoring is significantly complex and diverse that it will not be solved in the immediate future. Like so many other technology fields, advancements in vibration-based structural health monitoring will most likely come in small increments requiring diligent, focused and coordinated research efforts over long periods of time.

Finally, a web site that is dedicated to vibration-based damage detection and that contains many of the papers and reports referenced in this study, including links to other damage detection web sites, can be accessed at [http://ext.lanl.gov/projects/damage\\_id/](http://ext.lanl.gov/projects/damage_id/).

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# Updating of Analytical Models - Review of Numerical Procedures and Application Aspects \*

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## Abstract

In the paper basic procedures for computational updating of analytical model parameters are presented. The procedures reviewed include

- the numerical estimation techniques for solving the updating equations and
- the type of the residuals formed by the test/analysis differences to be minimised.

The residuals presented are formed by force and response equation errors, by eigenfrequency (resonance and antiresonance), mode shape and frequency response errors. All the procedures allow to handle incomplete test vectors, where the number of measured degrees of freedom (DOF) is much less than the DOF no. of the computational model which is a prerequisite for computational updating of large order finite element models.

Application aspects are also addressed including

- the influence of different parametrisations defining the type and the location of the erroneous model parameters,
- the requirements to be posed on the initial analysis model and
- the assessment of the final model quality.

The application aspects are supported using an industrial application example.

## 1. Introduction

The validation of analytical models in practise is mainly based on comparing experimental modal analysis results with the analytical predictions. Despite the high sophistication of analytical (Finite Element) modelling practical applications often reveal considerable discrepancies between analytical and test results. In recent times some effort has therefore been spent in the development of mathematical procedures for updating analytical mass and stiffness matrices using dynamic test data. The books of Natke [1] and Friswell/ Mottershead [2] represent comprehensive monographs containing the most relevant techniques.

The requirement for updating design parameters selected by the analyst like local mass and stiffness parameters results in iterative methods using non-linear optimisation in conjunction with least square procedures. The success of these methods is governed not only by the skill of the analyst to assume an appropriate initial analysis model but also the source and the location of the erroneous parameters to be corrected. The mathematical procedures presented below assume that these requirements are met.

In practical applications the source and location of the errors can be manifold resulting in non-unique updated matrices all of them fulfilling the mathematical minimisation criteria. For example, using a physical design parameter like a bending stiffness to update a discretisation error caused by a coarse finite element mesh would not be consistent with the real error source and would therefore destroy the

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physical significance of the design parameters. The updated model plays the role of a substitute model which at least has to fulfil the requirement of reproducing the test data used for updating. Generally we call all updating approaches where the real error source and location is not consistent with the assumed error source and location an inconsistent updating approach. The practical applicability of any localisation and updating procedure requires its ability and robustness with respect to handle

- (1) incomplete test data where the no. of measurement DOF's is less than the no. analytical DOF,
- (2) local and global physical modelling errors related to parameters like the stiffness or the mass of a single or a group of finite elements,
- (3) inconsistent assumptions with respect to location and type of modelling error (model structure error)
- (4) measured data polluted with random noise and unavoidable (small) systematic errors.

## 2. Mathematical Background of Model Parameter Estimation

Parameter estimation techniques aim at fitting the parameters of a given initial analytical model in such a way that the model behaviour corresponds as close as possible to the measured behaviour. The resulting parameters represent estimated values rather than true values since the test data are unavoidably polluted by unknown random and systematic errors. Also the mathematical structure of the initial analysis is not unique depending on the idealisations made by the analyst for the real structure. The method of extended weighted least squares is summarised in the following since it represents the most important estimation technique.

The first step in parameter estimation is the definition of a residual containing the difference between analytical and measured structural behaviour, for example the difference between analytical and measured eigenfrequencies. The weighted least squares technique requires to define a weighting matrix  $W_v$  accounting for the importance of each individual term in the residual vector  $\epsilon$  :

$$\epsilon_w = W_v \epsilon = W_v (v_M - v(p_i)) \quad (1)$$

$v_M$  represents the measured and  $v(p)$  the corresponding analytical vector which is a function of the parameters  $p_i$  ( $i = 1, \dots, n_p$  = number of correction parameters). The weighted squared sum of the residual vector yields the objective function

$$J = \epsilon_w^T \epsilon_w = \epsilon^T W \epsilon \rightarrow \min, \quad W = W_v^T W_v \quad (2)$$

whose minimisation yields the unknown parameters. In general the model vector  $v$  represents a non-linear function of the parameters resulting in a non-linear minimisation problem. One of the techniques to solve this non-linear optimisation problem is to expand the model vector into a Taylor series truncated after the linear term according

$$v(p) = v_a + G \Delta p \quad (3)$$

where  $v_a = v|_{p=p_a}$  represents the model vector at the linearisation point  $p = p_a$ .  $G = \frac{\partial v}{\partial p}|_{p=p_a}$

represents the sensitivity matrix (order  $m, n_p$  with  $m$  = no. of measurements and  $n_p$  = no. of parameters) and  $\Delta p = p - p_a$  represents the vector of the parameter changes. Eq.(3) introduced into eq.(2) yields the linear residual

$$\varepsilon_w = W_v \varepsilon = W_v (v_M - v_a - G\Delta p) = W_v (r_a - G\Delta p) \quad (4)$$

where  $r_a = v_M - v_a$  contains the residual at the linearisation point. Of course, this formulation includes the special case when the model vector is a linear function of the parameters resulting in a constant sensitivity matrix. The stepwise calculated minimum of the objective function with respect

to the parameter changes is obtained from the derivative of the objection function  $\partial J / \partial \Delta p = 0$  yielding the linear system of equations

$$W_v G \Delta p = W_v r_a \quad (5)$$

with the solution

$$\Delta p = \underbrace{(G^T W G)^{-1} G^T W}_{Z^T} r_a = Z^T r_a \quad (6)$$

The condition of the sensitivity matrix  $G$  plays an important role for the accuracy and the uniqueness of the solution. It is clear that in the case when less measurements than parameters ( $m < n_p$ ) are available eq.(4) leads to an underdetermined system whose solution is not unique. Even if a minimum norm or a minimum parameter change solution is selected the resulting parameters will in general not retain their physical meaning. In parameter updating the number of measurements should always be made larger than the number of parameters ( $m > n_p$ ) which yields overdetermined equation systems. If in practical applications it is not possible to increase the number of measurements it is recommended to reduce the number of parameters by applying parameter localisation techniques like those described in [3]- [5] in order to retain only the most erroneous and sensitive parameters.

Instead of decomposing  $(G^T W G)$  in eq.(6) the numerical solution of the overdetermined system is preferably done via QR or singular value decomposition in order to check the condition of  $G$ . The singular value decomposition (SVD) of the matrix  $G$  is defined by (see e.g.[6])

$$G_{(m,n_p)} = U_{(m,m)} \Sigma_{(m,n_p)} V_{(n_p,n_p)}^T \quad (7)$$

$U$  and  $V$  represent orthogonal matrices with the properties  $U^T U = V^T V = V V^T = I_{n_p}$  and  $\Sigma = \text{diag}(s_1 \dots s_r \dots s_{n_p})$ . The singular values  $s_r$  are the roots of the eigenvalues of  $G^T G$ .

$$\text{If } \text{rank}(G) = R: \quad \Sigma = \text{diag}(s_1 \dots s_r \dots s_R, s_{R+1} = 0 \dots s_{n_p} = 0) \quad (8)$$

The pseudo inverse is calculated from:

$$G^+ = V \Sigma^+ U^T \quad (9)$$

$$\text{with } \Sigma^+ = \text{diag}(1/s_1 \dots 1/s_R, 1/s_{R+1} = 0 \dots 1/s_{n_p} = 0)$$

In practical cases a clear rank defect represented by zero singular values and caused by linear dependent parameter sensitivities can often not be detected, for example due to the influence of measurement noise. In this case the singular values can be truncated below a certain threshold of the ration  $s_R / s_1$ . In any case it is recommended to consider a priori only parameters exhibiting linear independent sensitivities.

With these definitions the solution to Eq.(4) can be expressed by

$$\Delta p = G_v^+ r_{av} \quad (10)$$

with  $G_v = W_v G$  and  $r_{av} = W_v r_a$

The statistical properties of the solution are calculated from the mean values and the covariance matrix of the estimate. After substituting the unknown true vector  $\Delta p^0$  into the eq.(4) we obtain

$$\varepsilon^0 = v_M - v_a - G\Delta p^0 = r_a - G\Delta p^0$$

which is a measure for the random measurement error. With  $r_a = G\Delta p^0 + \varepsilon^0$  an estimate vector is calculated from

$$\Delta \hat{p} = Z^T r_a = Z^T (G\Delta p^0 + \varepsilon^0) = \Delta p^0 + Z^T \varepsilon^0 \quad (11)$$

The mean values of this estimate is calculated by the expectation operation

$$E(\Delta \hat{p}) = \Delta p^0 + E(Z^T \varepsilon^0) \quad (12)$$

If the matrix  $Z$  defined in eq.(6) is statistically independent of  $\varepsilon^0$  and if it is assumed that the mean of the measurements error vector equals zero,  $E(\varepsilon^0) = 0$ , then  $E(Z^T \varepsilon^0)$  also equals zero and

$$E(\Delta \hat{p}) = \Delta p^0 \quad (13)$$

The mean values of the estimate in this case are equal to the true values. Such an estimate is called an unbiased estimate. Often the above assumptions are not valid in particular when the sensitivity matrix is corrupted by measurement errors or when the model vector used to calculate  $Z$  is a non-linear function of the parameters, i.e. the estimate is biased. Procedures like the instrumental variable technique [7] directed to reduce the bias are useful in model updating, however, updating methods should be preferred where the sensitivity matrix is not directly corrupted by measurement errors.

The covariance matrix  $\text{cov}(\Delta \hat{p})$  represents a measure for the deviation of the estimate depending on the covariance matrix of the measurement vector  $\text{cov}(\varepsilon^0) = E(\varepsilon^0 \varepsilon^{0T})$ . With the assumption of an unbiased estimate

$$\text{cov}(\Delta \hat{p}) = E[(\Delta \hat{p} - \Delta p^0)(\Delta \hat{p} - \Delta p^0)^T] = Z^T E(\varepsilon^0 \varepsilon^{0T}) Z \quad (14)$$

Substituting the inverse of the measurement covariance matrix into eq.(14) as a weighting matrix, i.e.  $W = [E(\varepsilon^0 \varepsilon^{0T})]^{-1}$ , yields the covariance matrix of the estimate in the form

$$\text{cov}(\Delta \hat{p}) = (G^T W G)^{-1} \quad (15)$$

This result shows the importance of the numerical condition of the sensitivity matrix  $G$  as well as the influence of the measurement errors on the error of the parameter estimate. In the special case  $E(\varepsilon^0 \varepsilon^{0T}) = \sigma_v^2 I$ , i.e. when it is assumed that all measured quantities have the same standard deviation  $\sigma_v$  the covariance matrix of the parameter estimate is

$$\text{cov}(\Delta \hat{p}) = \sigma_v^2 (G^T G)^{-1} \quad (16)$$

In [7] it is shown that the covariance matrix of the estimate is always a minimum when the above assumption for the weighting matrix is used, i.e. the covariance matrix is always smaller than the covariance matrix resulting from the standard unweighted LS with  $W = I$ . Such estimation procedures are called Markov estimates in estimation theory, they yield the best linear unbiased estimate.

The classical weighted LS method described above can be extended in cases where it is difficult to obtain a convergent solution because of an ill-conditioned sensitivity matrix. The objective function (2) is extended by the requirement that the parameter changes  $\Delta p$  shall be kept minimal

$$J(p) = \varepsilon^T W \varepsilon + \Delta p^T W_p \Delta p \rightarrow \min \quad (17)$$

When the parameters are unbounded the minimisation (17), now with respect to the parameter changes  $\Delta p$ , yields the following linear problem to be solved within each iteration step which represents the linearisation point a:

$$(G^T W G + W_p) \Delta p = G^T W r_a \quad (18)$$

Of course, any other mathematical minimisation technique could also be applied, in particular when the parameters shall be constrained by upper and lower bounds. In case of  $W_p = 0$  the solution of eq. (17) represents the standard weighted least squares solution, otherwise the solution is affected by the choice of the weighting matrix  $W_p$ . In [8] this matrix was related to the inverse of the squared sensitivity matrix acc. to

$$W_p = w_p B \quad \text{where} \quad B = \frac{\text{mean}(g)}{\text{mean}(g^{-1})} g^{-1} \quad \text{and} \quad g = \text{diag}(G^T W G) \quad (19)$$

This definition allows to constrain  $\Delta p$  according to the sensitivity of the parameters. In consequence the parameters  $p_k$  remain unchanged if their sensitivity approaches zero ( $W_{pk} \rightarrow 0$ ). The weighting factor  $w_p$  allows to scale  $W_p$  with respect to  $B$ .  $B = I$  represents the classical Tikonov regularisation [9] used to solve ill conditioned systems of equations. The question remains who to choose the regularisation parameter  $w_p$ . Several proposals for the selection of  $W_p$  have been proposed, e.g. in refs. [10] - [12]. Hansen [13] proposed to balance the norms  $n_\varepsilon = \varepsilon^T W \varepsilon$  and  $n_p = w_p \Delta p^T B \Delta p$  of the two terms in the extended objective function (17) with respect to minimising  $n_\varepsilon + n_p$  as a function of  $w_p$ . This minimum is localised as the corner of the so-called L-curve obtained from plotting  $n_\varepsilon$  versus  $n_p$ . In [14] model updating applications of this and other regularisation methods were investigated. MATLAB algorithms for the different procedures have been developed and described by Hansen [15].

### 3. Definition of updating parameters

Starting point for updating are the assumptions on those model parameters defining the type and the location of the erroneous parameters to be updated in the equation motion of the finite element elastodynamic model :

$$(-\omega^2 M + K + j\omega D)y = \hat{K} y = f \quad (20a)$$

where the system matrices  $M$ ,  $K$  and  $D$  represent the mass, the stiffness and the damping matrix,  $\omega$  the excitation frequency,  $f$  the excitation force vector and  $y(j\omega)$  the complex frequency response vector ( $j = \sqrt{-1}$ ).

The special case of the undamped eigenequation is also considered in this paper:

$$(-\omega_0^2 \mathbf{M} + \mathbf{K}) \mathbf{y}_0 = 0 \quad (20b)$$

with  $\omega_0$  and  $\mathbf{y}_0$  denoting the undamped eigenfrequency and eigenvector, respectively.

In the most popular approach first introduced by Natke [16] the system matrices are updated by substructure matrices according to

$$\mathbf{K} = \mathbf{K}_A + \sum \alpha_i \mathbf{K}_i \quad (21a)$$

$$\mathbf{M} = \mathbf{M}_A + \sum \beta_j \mathbf{M}_j \quad (21b)$$

$$\mathbf{D} = \mathbf{D}_A + \sum \gamma_k \mathbf{D}_k \quad (21c)$$

where

$[\alpha_i \ \beta_j \ \gamma_k] = [p_s]$  = unknown correction (design) parameters,

( $i = 1, 2, \dots, I$ ;  $j = 1, 2, \dots, J$ ;  $k = 1, 2, \dots, K$ ;  $s = 1, 2, \dots, S$ )

where

$S = I + J + K$  = no. of correction parameters

$\mathbf{K}_A, \mathbf{M}_A, \mathbf{D}_A = (n, n)$  analytical (initial) stiffness, mass and damping matrix

$\mathbf{K}_i, \mathbf{M}_j, \mathbf{D}_k$  = assumed correction substructure matrices (elements or element groups)

defining source and location of modelling error.

The correction sub-matrices defined above can be considered as the first derivative of the updated matrices with respect to a physical or geometrical model parameter :

$$\mathbf{K}_i = \alpha_A \partial \mathbf{K} / \partial \alpha_i, \quad \mathbf{M}_j = \beta_A \partial \mathbf{M} / \partial \beta_j \quad \text{and} \quad \mathbf{D}_k = \gamma_A \partial \mathbf{D} / \partial \gamma_k \quad (22)$$

These derivatives are constant like in the case of a beam element with  $\alpha$  representing Young's modulus where the stiffness matrix is a linear function of the modulus. The derivatives must not be constant like in the case when  $\alpha$  represents the shear modulus of a Timoshenko beam where the stiffness matrix terms are non-linear functions of the shear modulus.  $\alpha_A, \beta_A$  and  $\gamma_A$  denote the initial parameters used to make the parameter changes dimensionless. Other parametrisations related to generalised elements or substructures have been proposed in [23] and [37].

The success of parameter updating is governed not only by the skill of the analyst to assume an appropriate initial analysis model but also to assume the right source and location of the erroneous parameters to be corrected. In practical applications the source and location of the errors can be manifold resulting in non-unique updated matrices all of them fulfilling the mathematical minimisation criteria. For example, using a physical design parameter like a bending stiffness to update a discretisation error caused by a coarse finite element mesh would not be consistent with the real error source and would therefore destroy the physical significance of the design parameters. The updated model plays the role of a substitute model which at least has to fulfil the requirement of reproducing the test data used for updating. Generally we call all updating approaches where the real error source and location is not consistent with the assumed error source and location an inconsistent updating approach.

#### 4. Definition of test/ analysis residuals

Another important assumption the analyst has to make is the choice of the residuals formed by the differences of the predicted analytical and the measured behaviour. In the present investigation we have considered the following residuals which have most often been applied in the past:

- eigenvalues and antiresonances
- mode shapes
- ( weighted) input forces and
- frequency response functions (FRF )

In [17] the authors present a comprehensive selection of these and other residuals with special consideration of statistically based weighting and the statistical properties of the parameter estimates. In the following we concentrate on those residuals which allow to update individually selected structural design parameters. Other residuals exist ( e.g. [26],[46], [47] ) allowing to update directly all the terms of the system matrices under physical constraints like conservation of matrix symmetry and connectivity. These methods are not included here since past experience with such methods was restricted to small order systems.

The linearised undamped **eigenvalue residuals** are defined by the differences between measured (index M ) and analytical undamped eigenvalues at the linearisation point a:

$$\varepsilon_L = \lambda_M - \lambda = r_{\lambda_a} - G_{\lambda} \Delta p \quad (\lambda = \omega_a^2) \quad (23a)$$

where

$$r_{\lambda} = \lambda_M - \lambda_a \quad (23b)$$

= residual vector containing test/ analysis differences of eigenvalues and

$$G_{\lambda} = [\dots \partial \lambda / \partial p_s \dots] = [\dots \partial \lambda / \partial \alpha_i \dots \partial \lambda / \partial \beta_j \dots]_{\alpha=\alpha_a, \beta=\beta_a} \quad (23c)$$

= sensitivity matrix at point a.

If the undamped problem is considered  $G_{\lambda}$  can be calculated by differentiation of the undamped eigenvalue equation and by substituting the parametrisation of eqs. (21a-b) which results in

$$\partial \lambda / \partial \alpha_i = y_o^T K_i y_o \quad \text{and} \quad \partial \lambda / \partial \beta_j = -\lambda y_o^T M_j y_o \quad (23d-e)$$

for the sensitivities with respect to the i-th stiffness and the j-th mass parameter. ( $y_o$  = real mode shape normalised to unit modal mass).

In [41] and [42] the authors proposed to expand the residuals by including the **antiresonances** which appear as zeros in the (undamped) frequency response functions thus enlarging the measured information. The zeros are calculated from the eigenvalue problem  $(-\omega_o^2 M + K)_{ij} z = 0$  where the indices i and j denote the system with row i and column j deleted. For  $i = j$  (driving point FRF) the system can physically be interpreted as being grounded at DOF  $i = j$  which means that the above sensitivity expressions of eqs.(23) can directly be applied for the grounded system matrices. It can be shown that in this case the antiresonances are located between the resonances (interlacing property) . In the general case  $i \neq j$  the reduced matrices become non- symmetric resulting in negative and/or complex antiresonances in conjunction with the loss of the interlacing properties and the loss of physical interpretation. In [43] the author found that the antiresonance sensitivities can be expressed by a sum of eigenvalue and mode shape sensitivities. In cases where the influence of out-of-frequency range modes is not negligible the antiresonances thus contain additional independent information compared to the conventional eigenvalue and mode shape residuals taken from a limited frequency range. However, it should be noted that even in the case where the influence of the out of range modes is negligible the addition of even redundant information is favourable since it allows to average out unavoidable measurement errors in the parameter estimation process. The study on updating a model using only eigenvalues and antiresonances ( i.e. no mode shape residuals) was conducted in [44] in which case the influence of the mode shape sensitivities is only implicitly contained in the antiresonance

information. Special attention was directed to problems when  $i \neq j$  antiresonances are used in addition to the  $i = j$  antiresonances.

The linearised real **mode shape residuals** are obtained from the differences of the measured modes at the reduced set of  $n_M < n$  measured DOF's denoted by the index  $c$ :

$$\varepsilon_y = y_{oM} - y_{oc} = r_{ya} - G_y \Delta p \quad (24a)$$

where

$$r_{ya} = y_{oM} - y_{oca} \quad (24b)$$

= residual vector with test/ analysis differences of eigenvectors at point a and

$y_{oca} := (y_{oc1} \dots y_{ocr} \dots y_{ocR})_a$  = real model modes ( $r = 1, 2 \dots R$  = no. of measured modes)

$y_{oM}$  = corresponding measured modes

$$G_y = [\dots \partial y_{oc} / \partial p_s \dots] = \left[ \dots \partial y_{oc} / \partial \alpha_i \dots \partial y_{oc} / \partial \beta_j \dots \right]_{\alpha=\alpha_s, \beta=\beta_s} \quad (24c)$$

= sensitivity matrix at point a.

The calculation of the mode shape sensitivity matrix involves a major numerical effort. The modal method of Fox and Kapoor [18] is widely used due to its simplicity of implementation. It is based on expanding the gradients by a weighted sum of the eigenvectors

$$\partial y / \partial p_s = \sum_r y_{or} c_r \quad (r = 1, \dots, R \leq n = \text{model order}) \quad (25a)$$

which yields after substitution into the derivative of the eigenequation the gradients with respect to the stiffness parameters:

$$\frac{\partial y_o}{\partial \alpha_i} = - \sum_{s=1}^R y_{os} y_{os}^T K_i y_o / (\lambda_s - \lambda) \quad \text{for } \lambda_s \neq \lambda, y_{os} \neq y_o$$

$$\frac{\partial y_o}{\partial \alpha_i} = 0 \quad \text{for } \lambda_{os} = \lambda, y_{os} = y_o \quad (25b)$$

The gradients with respect to the mass parameters follow from:

$$\frac{\partial y_o}{\partial \beta_j} = - \sum_{s=1}^R y_{os} y_{os}^T M_j y_o / (\lambda_s - \lambda) \quad \text{für } \lambda_s \neq \lambda, \lambda_s \neq \lambda$$

$$\frac{\partial y_o}{\partial \beta_j} = -0.5 y_{os} y_{os}^T M_j y_o \quad \text{für } \lambda_s = \lambda, \lambda_s = \lambda \quad (25c)$$

This expansion is exact if  $R = n$  modes are used. For  $R < n$  the expansion represents an approximation depending on the number of modal terms. Corrections to this approach have been investigated by several authors [19]. Eqs.(25b,c) also show that the convergence of the expansion will decrease for neighboured eigenvalues  $\lambda_s \approx \lambda$ . Lallement [20] proposed a procedure to overcome this difficulty. A recent investigation of several procedures developed in the past was given by Balmes [21] with respect to using other reduced projection bases than the modal basis in eq.(25a).

Since the sensitivity matrices are derived from the ( updated) analytical model they do not contain measurement errors which is an essential prerequisite for an unbiased estimate. However it should be kept in mind that due to the iterative process the sensitivity matrices depend on the parameters from the previous iteration step calculated from the noise polluted residuals in eqs.(23b) and (24b). Another advantage stems from the fact that the mode shape residuals and the sensitivities need only to be calculated for the measured DOF's, i.e. the analytical model must neither be condensed nor must the measured mode shapes be expanded to the unmeasured DOF's. It must be noted that the modal residual



have to be formed between paired mode shapes. Most often the correct mode shape correlation is checked using the modal assurance criterion  $MAC = (y_M^T y_A)^2 / (y_M^T y_M y_A^T y_A)$  which approaches one if the measured mode  $y_M$  and the analytical mode  $y_A$  are fully correlated. Mode pairs with MAC values smaller than a certain threshold ( for example  $MAC < 0.7$ ) should not be included in the residuals.

Using the deviations between the analytical and experimental mode shapes in the residual vector suffers from the disadvantage that the updated model is fitted to the test data in a least square's sense as close as possible although the test data are uncertain . In [22] and [23] we presented a technique of relaxing this requirement by applying a model based smoothing procedure to the experimental modes. This derivation is repeated here for completeness. The result is that the modal data of the updated model are not fitted as exactly as possible to the original experimental data but more to the smoothed data which allows to cancel not only random but also systematic measurement errors . The idea behind this approach is to bring simultaneously together both sides, the test data and the analytical parameters. Of course, both types of modifications must be kept within realistic bounds.

The smoothed  $(n, R_M)$  modal matrix  $\Phi = [\phi_1 \dots \phi_R]$  is expressed by a linear combination of the expanded measured modal matrix  $Y_M$  via an unknown transformation matrix  $Q$

$$\Phi = Y_M Q \quad (26)$$

where  $\Phi \in \mathbb{R}^{n, R_M}$ ,  $Y_M \in \mathbb{R}^{n, R_M}$  and  $Q \in \mathbb{R}^{R_M, R_M}$ .

We now require that the smoothed expanded shapes shall satisfy the orthonormality condition with respect to the updated analytical mass matrix in the form:

$$\Phi^T M \Phi = I \quad (27)$$

In order to carry out the product in equation (27) the measured modes are expanded with respect to the unmeasured DOFs. The expansion is done by the so-called modal co-ordinate method [45] is done by expressing the expanded measured modal matrix  $Y_M$  by the analytical modal matrix  $Y_A = [y_{o1} \dots y_{oR}]_A$  according to

$$Y_M = Y_A C \quad (28)$$

After partitioning the modal matrices with respect to the measured DOFs (index c) and the unmeasured DOFs (index u) equation (28) yields

$$\begin{bmatrix} Y_{Mc} \\ Y_{Mu} \end{bmatrix} = \begin{bmatrix} Y_{Ac} \\ Y_{Au} \end{bmatrix} C \quad \text{where } C \in \mathbb{R}^{R_A, R_M}. \quad (29)$$

The first row of equation (29) allows to solve for the unknown transformation matrix by minimising the norm  $\|Y_{Mc} - Y_{Ac} C\|$  which results in

$$C = Y_{Ac}^+ Y_{Mc} \quad (30)$$

("+" indicates the pseudoinverse of the analytical  $(c, R_A)$  modal sub-matrix  $Y_{Ac}$ ,  $c$  = no. of measured DOFs,  $R_A$  = no. of analytical modes). Using equation (30) the full measured modal matrix is expressed by equation (28). After introducing (26) and (28) and using the orthonormality condition,  $Y_A^T M Y_A = I$ , eq.(27) yields:

$$Q^T C^T C Q = I \quad (31)$$

Providing that the transformation matrix is symmetric  $Q^T=Q$  then:

$$Q = (C^T C)^{-1/2} \quad (32)$$

Equation (32) is identical with the optimal orthogonalisation introduced by Baruch [25] which also includes the method of Targoff [24]. The smoothed shapes are finally obtained from eqs.(26) and (32) by

$$\Phi = \begin{bmatrix} \Phi_c \\ \Phi_u \end{bmatrix} = \begin{bmatrix} Y_{Ac} \\ Y_{Au} \end{bmatrix} C(C^T C)^{-1/2} \quad (33)$$

This method was used in [25] and [26] to orthogonalise the mode shapes, and to update all terms of the stiffness matrix, in a global manner. We use this technique here after a given number of iteration steps, which allows to reduce the influence of the measurement errors and to build a test data set that is more consistent with the analytical model. It should be noted that the presented technique combines both, the mode shape smoothing and the expansion on the basis of the orthonormality condition. Numerically it does not require the use of the full mass matrix. The original measured mode shapes in the residual in eq.(24b) can now be replaced by the smoothed shapes taken at the measured DOF's:

$$Y_{Mc} = [y_{o,1} \dots y_{o,R_M}]_M \Rightarrow \Phi_c = [\varphi_1 \dots \varphi_{R_M}]_c$$

The **input error** is given by substituting the measured frequency response into the equation of motion. Since the number of measured DOF's is generally much smaller than the number of analytical DOF's it is necessary to expand the measured vector to full model size or to condense the model order down to the number of measured DOF's.

$$\varepsilon_F = f_M - \hat{K}_c(j\omega_M, p)y_M \quad (34a)$$

where

$$\hat{K}_c(j\omega_M) = \hat{K}_{ca}(j\omega_M) + \sum_s p_s S_s \quad (34b)$$

= updated dynamic stiffness matrix dynamically condensed to  $N_M < N$  measured DOF's

$$\hat{K}_{ca}(j\omega_M) = -\omega_M^2 M_{ca} + K_{ca} + j\omega_M D_{ca} \quad (34c)$$

= condensed dynamic stiffness matrix at point a.

$p_s = \alpha_i, \beta_j$  or  $\gamma_k$  correction parameters

$S_s = K_i, M_j$  or  $D_k$  correction submatrices

$y_M(j\omega_M)$  = complex frequency response vector measured at c measurement DOF's

$f_M$  = measured harmonic exciter forces, if  $f_{Mi} = 1$  at exciter DOF i,  $y_M$  represents the i-th column of the frequency response matrix

With eqs.(34b,c) introduced the force residual at the measured DOF's can be expressed by

$$\varepsilon_F = r_F - G_F \Delta p \quad (35a)$$

where

$$r_F = f_M - \hat{K}_{ca}(j\omega_M, p_a)y_M = \text{residual vector at point a} \quad (35b)$$

$$\mathbf{G}_F = \left[ \dots - \omega_M^2 \mathbf{M}_{cj} \mathbf{y}_M \dots \mathbf{K}_{ci} \mathbf{y}_M \dots j\omega_M \mathbf{D}_{ck} \mathbf{y}_M \dots \right]_a = \text{gradient matrix} \quad (35c)$$

Two condensation methods have most extensively been investigated in the past and are presented here. One is the dynamic condensation which includes the static condensation (also called the Guyan reduction) as a special case, the other is the modal expansion technique already presented in eqs.(29) and (30). Other techniques have been investigated like those described in [40].

The dynamically condensed matrices (index c) in the above equations are obtained by first introducing the parameterisations of eqs. (21) into the equation of motion and by partitioning the matrices with respect to the measured (index M) and unmeasured DOF's (index U):

$$\left( \begin{bmatrix} \hat{\mathbf{K}}_{MM} & \hat{\mathbf{K}}_{MU} \\ \hat{\mathbf{K}}_{UM} & \hat{\mathbf{K}}_{UU} \end{bmatrix}_A + \sum_s p_s \begin{bmatrix} \hat{\mathbf{K}}_{MM} & \hat{\mathbf{K}}_{MU} \\ \hat{\mathbf{K}}_{UM} & \hat{\mathbf{K}}_{UU} \end{bmatrix}_s \right) \begin{bmatrix} \mathbf{y}_M \\ \mathbf{y}_U \end{bmatrix} = \begin{bmatrix} \mathbf{f}_M \\ \mathbf{f}_U = 0 \end{bmatrix} \quad (36)$$

where the vector  $\mathbf{y}_u$  contains the unknown response vector components at the unmeasured DOF's. The second row of this equation can be used to express the unmeasured vector  $\mathbf{y}_u$  as a function of the correction parameters and the measured vector  $\mathbf{y}_M$ :

$$\mathbf{y}_U = \mathbf{T}(j\omega_M, p) \mathbf{y}_M = -\hat{\mathbf{K}}_{UU}^{-1} \hat{\mathbf{K}}_{UM} \mathbf{y}_M \quad (37)$$

The transformation matrix  $\mathbf{T}$  is an function of the excitation frequency  $\omega_M$ . For  $\omega_M = 0$  this matrix represents the static condensation matrix (also called Guyan condensation).

The expanded full vector is given by

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_M \\ \mathbf{y}_U \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{T}(j\omega_M, p) \end{bmatrix} \mathbf{y}_M = \tilde{\mathbf{T}}(j\omega_M, p) \mathbf{y}_M \quad (38)$$

where the transformation matrix  $\tilde{\mathbf{T}}$  can be used to condense the system and the correction matrices according to

$$\mathbf{S}_c = \tilde{\mathbf{T}}^T (\mathbf{S}_A + \sum_s p_s \mathbf{S}_s) \tilde{\mathbf{T}} = \mathbf{S}_{Ac} + \sum_s p_s \mathbf{S}_{cs} \quad (39)$$

$\mathbf{S}_c = \mathbf{K}_c, \mathbf{M}_c$  or  $\mathbf{D}_c$  = condensed system matrices

Note: in the general damped case  $\tilde{\mathbf{T}}$  is complex and  $\tilde{\mathbf{T}}^T$  denotes complex- conjugate transpose.

This type of model reduction has several drawbacks. Looking at large order model typical for industrial applications the number of measured DOF's is much smaller than the model order:  $n_M \ll n$ . Since the transformation matrix depends on the current erroneous parameter estimate it becomes apparent that the sensitivity matrix in eq.(35c) will be polluted by a systematic error which can not be compensated by the LS method. In addition the numerical effort to decompose the  $\hat{\mathbf{K}}_{UU}$  matrix in eq.(31) makes its application prohibitive for updating of large order models.

The second technique of order reduction also called the modal reduction technique has already been presented in eqs.(29) and (30). When applied to the frequency response vectors the transformation between the response at the unmeasured and the measured DOF's is given by:

$$\mathbf{y}_U = \mathbf{T} \mathbf{y}_M = \mathbf{Y}_{Au} \mathbf{Y}_{Ac}^+ \mathbf{y}_M \quad (40)$$

It should be noted that using this transformation to condense the system matrices according to eq.(39) the eigendata of the original and the condensed system are the same.

Another disadvantage results from the fact that the gradient matrix is polluted not only by the systematic condensation errors but also by measurement errors which both result in biased parameter estimates.

The **pseudo response** error residual is obtained from transforming the input force error to an output error by multiplying the force residual of eq.(34a) with the FRF matrix of initial model (index A). This is equivalent with exciting the initial model (assumed to exhibit modal damping for simplicity) with the arbitrary frequency  $\omega_F$ :

$$\varepsilon_{PR} = \hat{K}_{Ac}^{-1}(\omega_F) \varepsilon_F \quad (41)$$

where

$$\begin{aligned} \hat{K}_{Ac}^{-1} &= (-\omega_F^2 \mathbf{M}_{Ac} + j\omega_F \mathbf{D}_{Ac} + \mathbf{K}_{Ac})^{-1} = \sum_i \mathbf{y}_{oAi} \mathbf{y}_{oAi}^T / (\omega_{Ai}^2 - \omega_F^2 + j2\omega_{Ai} \omega_F \xi_{Fi}) \\ &= \text{condensed FRF matrix expressed by modal data of initial model (index A)} \end{aligned} \quad (42)$$

$\omega_{Ai}$  = filter frequency

$\hat{K}_{Ac}^{-1}$  can also be interpreted as a dynamic filter. Applying the force residual vector on the initial system vibrating with the arbitrary filter frequency  $\omega_F$  controls the magnification of the pseudo response error  $\varepsilon_{PR}$ . The idea behind this filtering is to reduce the bias of the estimate.

A special case is obtained by applying the residual force vector on the analytical system linearised at point 'a' using the measured excitation frequencies:

With  $\hat{K}_{Ac}^{-1} = \hat{K}_{ca}^{-1}$  and  $\omega_F = \omega_M$  introduced into eqs.(41) and (35a-c) the pseudo response residuals are calculated by

$$\varepsilon_{PR} = \hat{K}_{ca}^{-1}(\omega_F) \varepsilon_F = \mathbf{r}_{PR} - \mathbf{G}_{PR} \Delta \mathbf{p} \quad (43a)$$

where

$$\mathbf{r}_{PR} = \mathbf{y}_{ca}(j\omega_M) - \mathbf{y}_M(j\omega_M) \quad (43b)$$

= residual response at point 'a'

$$\mathbf{G}_{PR} = \hat{K}_{ca}^{-1} \left[ \dots - \omega_M^2 \mathbf{M}_{cj} \mathbf{y}_M \dots \mathbf{K}_{ci} \mathbf{y}_M \dots j\omega_M \mathbf{D}_{ck} \mathbf{y}_M \dots \right]_a \quad (43c)$$

= gradient matrix at measured DOF's

The force residual and the pseudo response technique have been investigated by several authors with respect to the bias and ill-conditioning problems mentioned and with respect to the optimal choice of the excitation frequencies above, e.g. in refs. [27]- [34] and [48].

Another special case investigated in [38] is obtained when the input force error is transformed to a static admissible displacement by replacing the dynamic stiffness matrix in eq.(41) by the static stiffness

matrix  $\hat{\mathbf{K}}_{Ac}^{-1} \Rightarrow \mathbf{K}_{ca}^{-1}$  and  $\omega_F \Rightarrow 0$ . The dynamic condensation matrix  $\mathbf{T}$  of eq.(37) thus reduces to the static (Guyan) transformation matrix  $\mathbf{T} = -\mathbf{K}_{UU}^{-1} \mathbf{K}_{UM}$ .

The above **input error** and **pseudo response** equations also hold when measured modal data shall be used instead of the measured frequency response data. In this case the excitation force  $\mathbf{f}_M$  is set to zero, the excitation frequency (Fourier variable)  $-j\omega_M$  is replaced by the complex eigenvalue  $\lambda_{Mi}$  (Laplace variable) and the frequency response vector  $\mathbf{y}_{Mi}$  is replaced by the complex mode vector. Undamped real modes can also be used when the damping matrices in the above equations are set to zero, the excitation frequency  $\omega_M$  is replaced by the real eigenfrequency  $\omega_{xi}$  and when the frequency response vector  $\mathbf{y}_M$  is replaced by the measured real mode vector  $\mathbf{y}_{0Mi}$ . When in the latter case the input error residual (34a) is multiplied by  $\mathbf{y}_{0Mj}^T$  we arrive at the **orthogonality residual**  $\mathbf{y}_{0Mj}^T \boldsymbol{\varepsilon}_F = \mathbf{y}_{0Mj}^T \hat{\mathbf{K}}_c \mathbf{y}_{0Mi}$  allowing to enforce the orthogonality of the experimental modal matrix with the updated condensed mass and stiffness matrix.

The linearised **frequency response (FR) residuals** are obtained from the differences of the measured and the analytical FR at the reduced set of  $n_M < n$  measured DOF's denoted by the index c. The sensitivity matrix of the **FR** residual is derived from differentiating the equation of motion (20a) with respect to the parameters.

$$\boldsymbol{\varepsilon}_R = \mathbf{y}_M - \mathbf{y}_c = \mathbf{r}_R - \mathbf{G}_R \Delta \mathbf{p} \quad (44a)$$

where

$$\mathbf{r}_R = \mathbf{y}_M(j\omega_M) - \mathbf{y}_{ca}(j\omega_A) \quad (44b)$$

= residual vector with test/ analysis differences of frequency response at linearisation point a and at excitation frequency  $\omega_A = \omega_M$ .

$\mathbf{y}_{ca}(j\omega_A)$  := analytical complex frequency response vector at excitation frequency  $\omega_A = \omega_M$

$\mathbf{y}_M(j\omega_M)$  = frequency response vector measured at the same DOF's at excitation frequency  $\omega_M$

$$\begin{aligned} \mathbf{G}_R &= [\dots \partial \mathbf{y}_c / \partial p_s \dots] = -\hat{\mathbf{K}}^{-1} \left[ \dots \frac{\partial \hat{\mathbf{K}}}{\partial p_s} \mathbf{y} \dots \right] \Big|_{p=p_a} \\ &= -\hat{\mathbf{K}}^{-1} \left[ \dots -\omega_M^2 \mathbf{M}_j \mathbf{y} \dots \mathbf{K}_i \mathbf{y} \dots j\omega_M \mathbf{D}_k \mathbf{y} \dots \right] \Big|_{p=p_a} \end{aligned} \quad (44c)$$

= sensitivity matrix at measured DOF's at linearisation point a.

$$\hat{\mathbf{K}}^{-1} = (-\omega_M^2 \mathbf{M} + j\omega_M \mathbf{D} + \mathbf{K})^{-1} \Big|_{p=p_a} = \sum_{i=1}^{R \leq N} \mathbf{y}_{oci} \mathbf{y}_{oci}^T / (\omega_{oi}^2 - \omega_M^2 + j2\omega_{oi} \omega_M \xi_i) \Big|_{p=p_a} \quad (44d)$$

= FRF matrix of analytical model expressed by  $R \leq N$  analytical modal quantities at linearisation point a (proportional modal damping values  $\xi_i$  assumed for simplicity) calculated at the measured DOF's.

Like the sensitivity matrix for the modal sensitivities the FR sensitivity matrix is not directly corrupted by measurement errors allowing (approximately) unbiased parameter estimates in contrast to the force and pseudo response residuals in eqs.(35c) and (43c). In addition there is no need to expand the test vectors to the unmeasured DOF's. However, there is a crucial drawback in this formulation which has prevented its application to other than academic cases. Comparing measured and analytical FR functions like those of fig.(3) it may be noticed that due to the shifts of the resonance peaks caused by the mismatch of the eigenfrequencies the test/analysis differences at a given excitation frequency become extremely large in particular in such cases where the ordering of the test and analysis eigenfrequencies is not the same (also called mode crossing). The consequence is that the first order

theory either fails to predict the updated model response or the rate of convergence is very low unless the test/analysis deviations are very small. Attempts have been made by several authors to overcome the problem, e.g. by using second order derivatives, artificial damping or eliminating or reducing the weight [32] of the data in the range of the resonances. Other approaches deliberately make use of the shift of the resonance peaks e.g. references [33], [34] and [39] and.

It is advocated here to restrict updating to the resonance peaks taken at the analytical and experimental eigenfrequencies which are not identical due to mass and stiffness errors. The residuals therefore are the same as in eq.(44) except that the analytical excitation frequencies are replaced by the analytical eigenfrequencies and the experimental excitation frequencies by the experimental eigenfrequencies:

$$\omega_A \Rightarrow \omega_o \quad \text{and} \quad \omega_M \Rightarrow \omega_{Mo}. \quad (45)$$

Of course, the latter assumption means to extract the experimental eigenfrequencies prior to FRF updating which is in conflict with the motivation behind using FRF's for model updating instead of modal data aimed to avoid experimental modal analysis errors. However, it is believed that in practise when FRF measurements have been taken an experimental modal analysis will be performed anyway. Of course, the experimental modal data would also be useful to validate the FRF updating results. Also it must be expected that in cases when the accuracy of the test data and/or the elastodynamic mode assumptions will not allow a successful modal extraction then FRF updating will even be less successful due to the fact that the physical model includes more error sources than the curve fitting approach used in experimental modal analysis.

One advantage of FRF updating remains: the possibility to identify damping parameters. If it is assumed that the stiffness and mass parameters have been updated before by using the undamped modal residuals of eqs.(23) and (24) ( Note: undamped experimental modal parameters can either be measured directly by phase resonance testing or by phase separation techniques like ISSPA [35] or by techniques transforming complex modes to real modes [36] ) then the FRF residuals of eqs.(44) under consideration of eq.(45) could be applied directly with the damping parameters  $\Delta p := \Delta \gamma$  being the only unknowns in this case. However, this possibility is very restricted because the local physical damping parametrisation of eq.(21c) is not available for most structural applications. This is the reason why the global modal damping approach is widely used in practise and why it is often retained in FRF based parameter updating.

The approach described in the following is directed to update initial estimates of modal damping parameters. Using the mass/stiffness updated models to calculate the frequency response one would of course use at first the damping values resulting from experimental modal analysis. If the fit of the eigenfrequencies and modes of the updated model to their experimental counterparts was perfect then the FRF's calculated with the updated model would be the same as the experimental FRF's used to extract the modal data, i.e. any updating of the modal damping parameters would not be necessary.

Since in practical applications such a perfect fit cannot be obtained it seems desirable to have a procedure allowing to fit an initial modal damping estimate to the experimental FRF's.

Such an approach is described in the following. The residual used is the same as in eqs.(43) and (44), i.e. the excitation frequencies are taken at the eigenfrequencies differing between test and analysis:

$$\epsilon_{\xi} = y_M(\omega_{Mo}) - y_c(\omega_o) = r_{\xi a} - G_{\xi} \Delta \xi \quad (46)$$

where

$$r_{\xi a} = y_M(j\omega_{Mo}) - y_{ca}(j\omega_o)$$

= residual vector with test/ analysis differences of frequency response at linearisation point a

$y_{ca}(j\omega_o) :=$  complex frequency response vectors at excitation frequency = analytical eigenfrequency  $\omega_o$   
 $y_M(j\omega_{Mo}) =$  frequency response vectors measured at the same DOF's  
at experimental excitation frequency = experimental eigenfrequency  $\omega_{Mo}$

To calculate the sensitivity matrix of the modal damping the equation of motion (20a) is first transformed to modal co-ordinates by the superposition of undamped modes

$$y_c = \sum_r^{R \leq N} y_{ocr} q_r \quad (47a)$$

resulting in  $r = 1, \dots, R$  decoupled equations if proportional damping is assumed otherwise the modal equations are coupled by the non-diagonal modal damping matrix  $D_g^T = y_o^T D y_o$  (The proportional damping assumption is not necessary and was made to simplify the derivation):

$$(\omega_{or}^2 - \omega_{os}^2 + j 2 \omega_{os} \omega_{or} \xi_r) q_r = y_o^T f \quad (47b)$$

Eq.(46b,) derived with respect to the viscous modal damping parameter  $\xi_r = D_{gr} / (2\omega_{or})$  yields

$$G_{\xi} = \left[ \dots \partial y_c(\omega_{os}) / \partial \xi_r = y_{ocr} \frac{\partial q_r(\omega_{os})}{\partial \xi_r} \dots \right]_{\xi=\xi_s} \quad (48a)$$

$$\text{where} \quad \frac{\partial q_r(\omega_{os})}{\partial \xi_r} = \frac{-j 2 \omega_{os} \omega_{or} q_r(\omega_{os})}{\hat{K}_{gr}} \quad (48b)$$

( $r, s = 1, \dots, R$  number of active modes  $\leq N$ )

## 5. Application aspects

In the following the different steps of updating industrial type complex structures are discussed when the model order can be as large as 100000 DOF's. Considering the non-unique correction parameter sets caused by the unavoidable idealisation and discretisation errors (not to forget the influence of random and systematic measurement errors which are discussed elsewhere, e.g. [1-2]) the problem of finding criteria to select the best initial model and the best parameter estimate must be solved in conjunction with an the assessment of the final model quality [48]. In the following this procedure is illustrated by an industrial application example. The aim was to validate the models of two aeroengine components (HPT, high pressure turbine casing and RBSS, rear bearing support structure, highlighted in Fig.1) as parts of the carcass of an aeroengine. Due to the complexity of such structures non-consistent model structures and parameterisations as well as experimental errors and incomplete test data can generally not be avoided.

Main goal of this industrial project was to improve the confidence in the predictions for an extremely complex model of the whole engine. The validation concept was based on updating the FE-models of engine components using experimental modal data of the components which allows to restrict the number of uncertain updating parameters of the whole engine model. The aim of model improvement for each individual component was to verify that the updated single models can improve the eigendynamics and the dynamic response predictions of the assembly. The first question to be answered was of how many modes each component must accurately predict in order to predict the assembly dynamics in a given frequency range. If the initial model idealisation and parameterisation was consistent one can expect that updating the parameters using experimental modal data of a limited frequency range would not only improve the analytical modal data in that range but would also improve the out-of-range data. Unfortunately this expectation can only seldom be fulfilled for industrial type structures. From an analytical component mode synthesis we concluded that about 20 component modes were necessary to

predict the assembly dynamics in the desired frequency range, a number which governed the design of the experiments and also the design of the initial FE model.

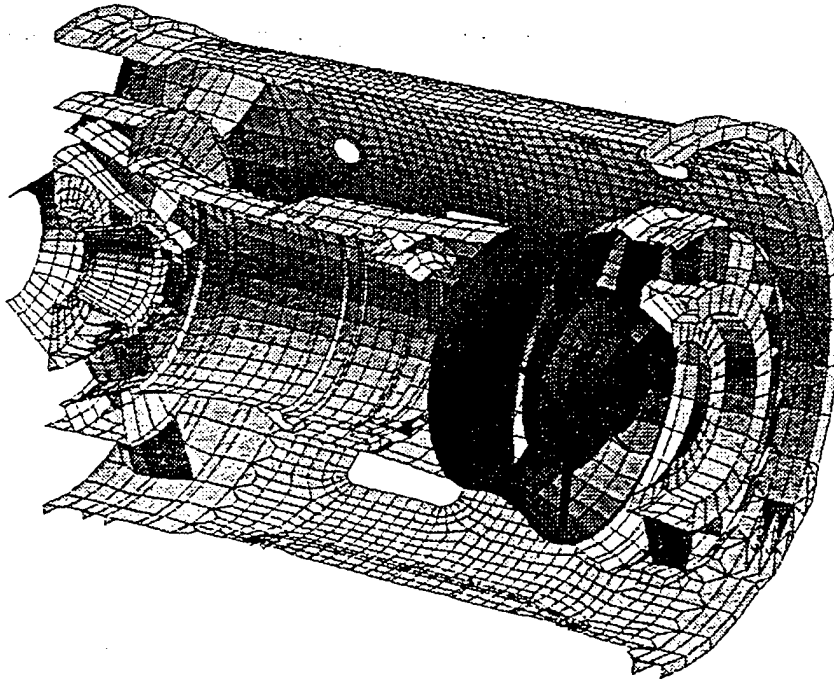


Fig.1: Aeroengine carcass model with high pressure turbine casing (HPT) and rear bearing support structure (RBSS) highlighted

### 5.1 Update of the component models

Fig.2 shows the design and fig.3 the initial FE model of the HPT structure. 20 mode shapes could be correlated with their experimental counterparts with an average MAC - value of 91.1%. The average eigenfrequency error was -4%, the maximum error -6.6% and the error of the fundamental frequency was -2.5%. Usually this degree of accuracy can be considered to be sufficient for practical applications. Since due to the variety of parameters further improvement of the results by manual adjustment is no more feasible the code UPDATE\_N developed at the University of Kassel for updating NASTRAN FE models using eigenfrequency and mode shape residuals was applied. Attempts to use a coarser mesh like that shown in fig.1 for the components failed because the test/analysis distance was too large. The selection of the correction parameters was based on the sensitivity of the mode shapes and the eigenfrequencies with respect to small parameter variations. All parameters which are suspect of being uncertain had to be considered in the sensitivity analysis.

The evolution of the updating process is shown in Fig.4. Finally 21 analytical mode shapes correlated with the experimental shapes with an average MAC value of 92% and an average frequency error of -0.2%. The convergence of the frequency deviations, the parameter changes, the MAC values and the objective function expressed by the mean frequency error and the mean MAC error during the iterations is presented in Fig.4a-d. It should be noted that in contrast to the objective function some parameters in Fig.4b have not yet converged. This means that the diverging parameters may be changed without changing the modal behaviour in the frequency range since a convergence of the objective function can be stated from Fig.4d. The physical significance of such type of parameters must be questioned. In the present case these parameters represent shell thicknesses. Changing a thickness means to increase the membrane stiffness and the mass distribution simultaneously which tends to cancel the effect of the thickness change on the eigenfrequencies.



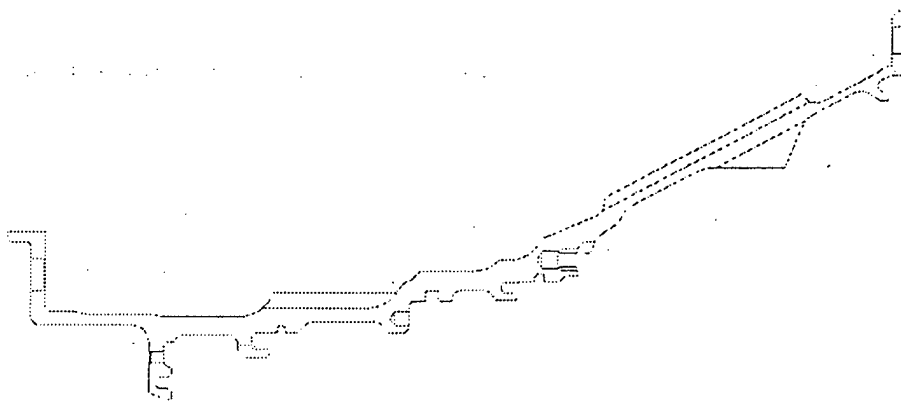


Fig. 2 HPT cross section as designed

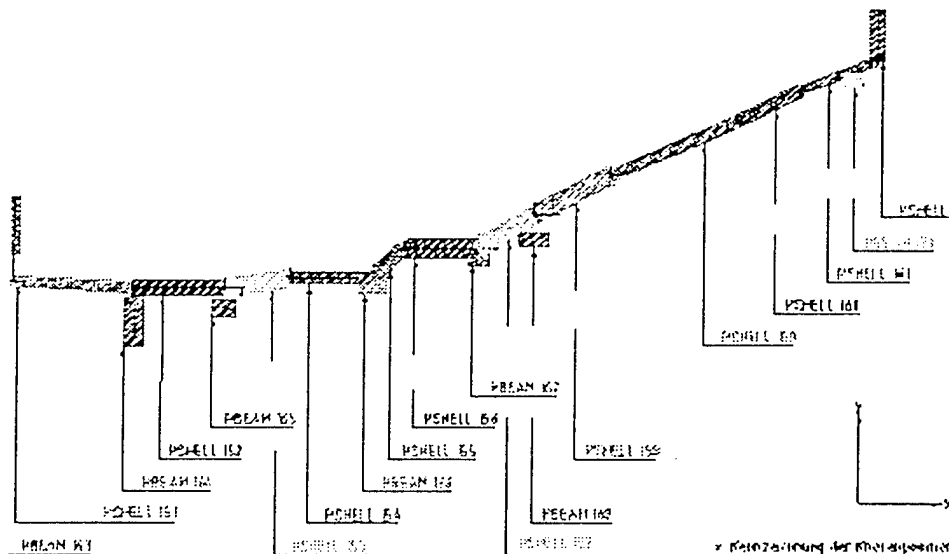


Fig. 3: FE model (NASTRAN) of cross section

It should also be noted that even when the parameters have not converged each parameter set at a given iteration step represents an input data set which yields a valid FE solution. Of course, the solution is not unique in such cases. Other criteria can help to find the best solution. In the present case that solution was selected where the overall model mass and the measured mass was nearly equal.

Figs. 4a,d show the significant reduction of the 21 eigenfrequency errors before and after updating with the mean frequency error centred about zero. The quality of the final correlation results should be considered as above what can be expected from large scale industrial applications. In the present case this quality can be attributed to the quality of the initial modelling and also to the accuracy of the experimental modal data obtained under scientific laboratory conditions.

Computational updating of the second component model, the rear bearing support structure (RBSS) followed the same principles. Initially 14 analytical and experimental modes could be correlated with an average of 81.8% for the MAC and -2.8% for the eigenfrequencies. However, the frequency error range between -20% for the fundamental frequency and a maximum 6.3% was considered too large.

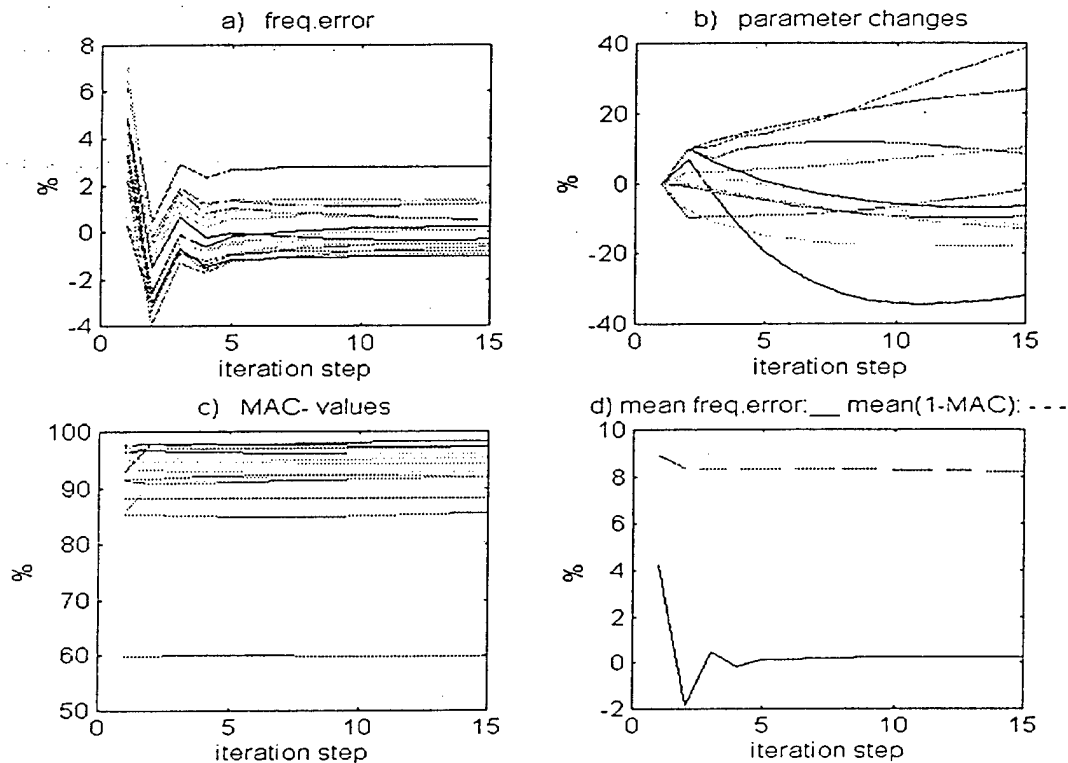


Fig.4 Evolution of eigenfrequency errors, parameter changes, MAC - values and objective function during iteration of HPT update

For computational updating 9 parameters were defined by the area and torsional moments of inertia of the spokes and of the outer ring and of the stiffness of the transition area between the circular plate and the outer cylinder represented by an equivalent Young's modulus of the shell elements in that area.

The correlation of 14 experimental modes could be increased to an average of 88.9% for the MAC and of 0.3% for the eigenfrequencies. The frequency error range was reduced to between -5.9% to 4.9%.

## 5.2 Assembly of HPT & RBSS components

The main goal of the present study was to improve the prediction capability of the assembly model by using *updated component* models. The correlation of the test and analysis results of the assembly model where the initial not updated component models were used showed poor correlation. Under the assumption that the updating parameters were consistent with the source and the location of the real modelling uncertainties and under the assumption that the mathematical model structure was appropriate one has to expect that the assembly of the two updated components would yield an improved model and should improve its prediction capability without further updating of the component parameters except for the uncertain parameters introduced at the bolted connection areas.

The results of using the *updated component* models confirmed that the prediction accuracy has been improved significantly compared with that of using the initial component models. This gave an indication that the initial model structure and also the selected correction parameters fulfilled the consistency requirement. 21 modes were correlated with an average of 85.8% for the MAC and 5.5 % for the eigenfrequencies. However, since the frequency error range between 0.44% for the fundamental frequency and a maximum of 10.15 % still showed a positive bias and also because 3 experimental modes ( nos. 4,7 and 16) could not be correlated at all computational updating was applied.

The correction parameters for updating the assembly model shown in fig.5 could be restricted to 7 parameters located in the bolted joints areas and represented by translational and rotational spring

stiffnesses and the Young modulus of elasticity for the shell elements in the HPT/ spoke connection areas.

Typical results are shown in fig.6. 24 modes , that means 3 more modes compared to the previous case were correlated with an MAC average of 82.9% and an average frequency error of 0.9% in a band between -1.5 % and 4.6 %. The average MAC was a little bit lower than in the previous case which however can be tolerated in view of the fact that 3 more modes could be correlated.

A final model quality check was done by comparing the measured FRF's and those calculated with the updated assembly model together with the experimental damping values.

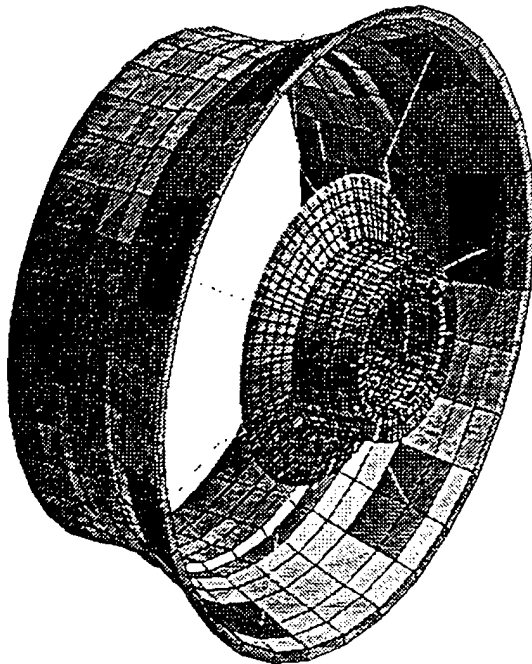


Fig.5 HPT & RBSS Assembly Model

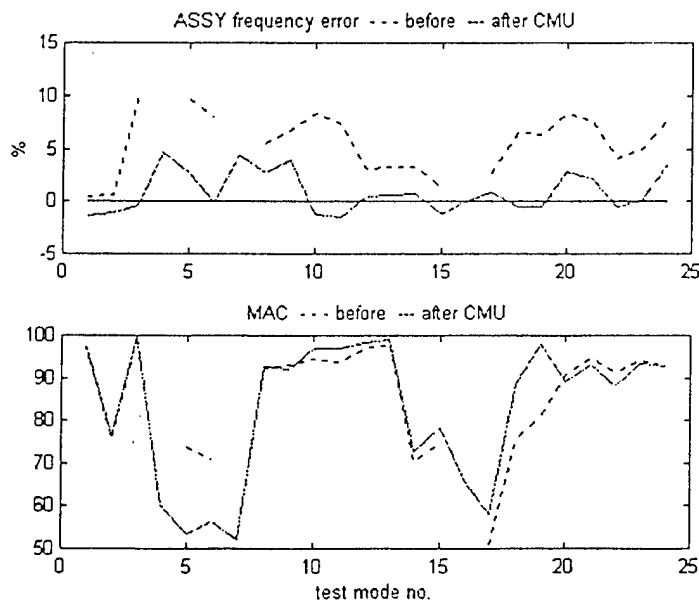


Fig.6: Results before and after computational model updating of HPT& RBSS assembly

The correlation of the envelope over all 82 measured DOF's on top of fig.7 and the correlation of the response plots for three selected DOF's below looks quite satisfactory and meets the industrial requirement to ensure the response prediction up to 1000 Hz for the assembly.

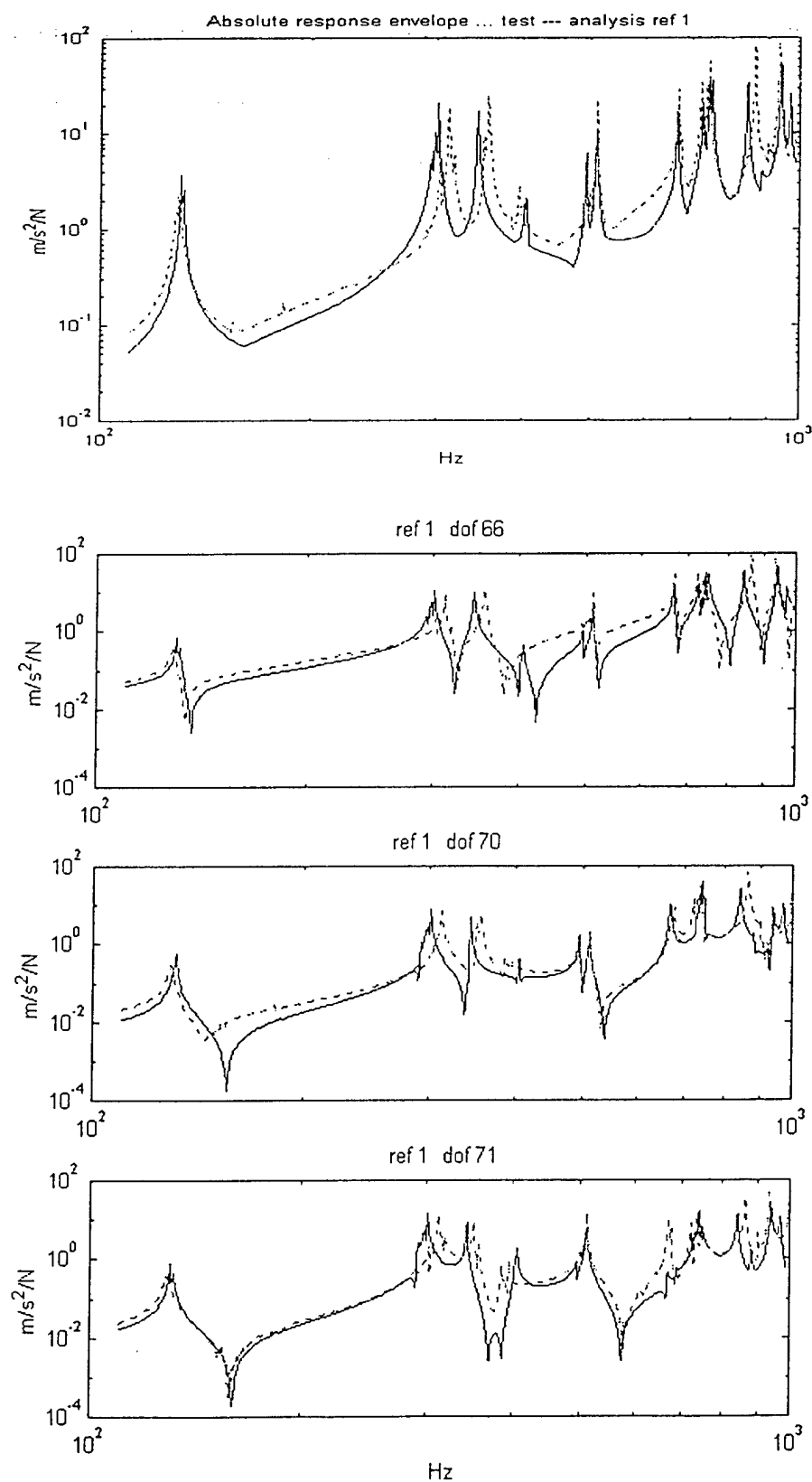


Fig. 7: Experimental (---) and updated model FRF's (—)

## Summary

In the paper a summary of current procedures for model updating is presented. At first the mathematical background of the parameter estimation is given with respect to the numerical estimation techniques for solving the updating equations based on the iterative extended least squares technique. The model parametrisations defining the type and the location of the erroneous parameters are then used to construct the residuals formed by the test/analysis differences to be minimised. The residuals presented are formed by force and pseudo response equation errors, by eigenfrequency, antiresonance and mode shape errors and by frequency response errors. The procedures have been derived to handle incomplete test vectors, where the number of measured degrees of freedom (DOF) is much less than the DOF no. of the computational model. Some recent extensions of the estimation techniques with respect to handle systematic mode shape errors and to updating of modal damping values have also been included. Finally an application of updating the models of an aeroengine structure was used to demonstrate the most important updating steps.

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# Random Vibrations: Assessment of the State of the Art

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## Abstract

Random vibration is the phenomenon wherein random excitation applied to a mechanical system induces random response. We summarize the state of the art in random vibration analysis and testing, commenting on history, linear and nonlinear analysis, the analysis of large-scale systems, and probabilistic structural testing.

## Introduction

Random vibrations deals with the probabilistic analysis of the response of structures with potentially random parameters and initial conditions and with random excitation and potentially random boundary conditions, and it deals with the simulation of random environments in the laboratory for the testing of real structures. Both random vibration analysis and testing are used for the design, optimization, and reliability assessment of structures and for other purposes.

The activities undertaken in analysis and testing in random vibrations can be succinctly described using a simple, representative dynamic equation of equilibrium. Consider the following (scalar or vector) representation.

$$\dot{x} = g(x, y, a) \quad x(0) = x_0, t \geq 0 \quad (1)$$

The quantity  $x$  represents system response;  $y$  represents system excitation;  $a$  represents system parameters; the dot denotes differentiation with respect to time; and  $g(\cdot)$  is the deterministic functional form that relates the former quantities to the response derivative.

Traditional random vibrations specifies the form of  $g(\cdot)$ , takes  $a$  as a scalar or vector of constants, specifies  $y$  as a random process, and analyzes the probabilistic character of the response random process  $x$ . Many aspects of this problem have been solved for the case where  $g(\cdot)$  is a linear function of the excitation, and the excitation is a stationary, Gaussian random process. When the excitation is nonstationary, the expression for the response can be written, but it cannot always be solved easily in closed form for the desired response characteristics. For the case where  $g(\cdot)$  is not a linear function of the excitation, many means have been developed for characterizing the response random process. None is completely general, and practically all involve some form of approximation.

Many large-scale structural analysis computer codes implement the Gaussian excitation/linear response solution mentioned above. However, not much else is typically analyzed in commercial finite element codes. One can always resort to a Monte Carlo approach for the analysis of practically any system including nonlinear systems and systems with random structural parameters,  $a$ . However, depending on the information desired and the complexity of the structure, the number of random quantities or the number of simulations may be significantly limited.

Two techniques for the analysis of large-scale structures with random parameters have been developed: reliability-based techniques and stochastic finite element techniques. The former takes the parameters,  $a$ , to be random variables with known joint probability distributions and uses this information to characterize the probabilistic response of structural systems. It provides solutions that are not traditional random vibration solutions. Stochastic finite element analysis is a relatively new technique. It permits system parameters,  $a$ , to be represented approximately as random fields and approximates the response,  $x$ , as a random field.

The laboratory testing of structural systems is the link between random vibration theory and the practical excitation of structures in the field. Stationary random vibration testing in the laboratory operates in two phases, with the assumption that the electromechanical testing system and the structure under test have parameters,  $a$ , that are either constant or slowly varying functions of time. In the first phase of operation, the random vibration test seeks a preliminary identification of the system parameters,  $a$ , in the linear framework. Having performed this identification, the second phase seeks a random input,  $y$ , to excite a response,  $x$ , with a preestablished spectral density. As the test progresses, the system parameters,  $a$ , are periodically updated, and the spectral density of the input,  $y$ , is modified. Though satisfactory algorithms for the control of single-shaker, stationary random vibration tests have been developed, more development is required, particularly in the control of nonstationary tests, the generation of non-Gaussian environments, the testing of nonlinear systems, and the operation of multi-shaker tests.

In this paper we will briefly summarize the history of analytical technique development in random vibrations; then we will discuss the random vibrations of linear systems, the topic on which the great majority of research and development effort has historically been placed. Next we will discuss the random vibration of nonlinear systems. Following this the state of the art in the random vibration analysis of large-scale structures using numerical codes will be described. Finally, the state of the art in probabilistic experimental structural dynamics will be described. The paper concludes with a summary.

## History

It is difficult to identify precisely the paper or the event that marked the beginning of the field of random vibrations analysis. Lord Rayleigh wrote a paper in the late 1800s (see Rayleigh, 1880) considering a problem that is a very much idealized and specialized case of random structural response. Two later papers (see Rayleigh, 1919a, 1919b) were extensions of the earlier one and treated more practical random vibration subjects, but by that time several other papers that must be considered treatments of random vibrations had been published.

Around the turn of the century, Einstein (1905) constructed a framework for analyzing the Brownian movement, the random oscillation of particles suspended in a fluid medium and caused by the molecular motion postulated by the kinetic theory of matter. His framework is a special form of what was later to become known as the Fokker-Planck equation governing the probability density function (PDF) of particle motion and relating it to mechanical system parameters. Einstein augmented his initial study with more, related investigations (all reprinted in Einstein, 1956) considering, among other things, the problem of parameter identification.

According to Gnedenko (1997), Smoluchowski (1916) generalized Einstein's analysis and performed Brownian motion experiments to verify the predictions of the theory. Smoluchowski was first to write the Fokker-Planck equation for systems in which a displacement-related force restrains the mass. The single-degree-of-freedom system in which linear damping and stiffness are present was called the case of the "harmonically bound" particle. Planck (1927) and Fokker (1913) started with the discrete space/discrete time framework of the random walk and offered arguments regarding the relative and limiting values of various parameters in the model. They found that a limiting form of the random walk model is the partial differential equation developed by Einstein to characterize the Brownian motion.

In a paper written a few years later, Wiener (1923) overcame a problem with previous analyses - namely, that the solution of a differential equation excited by ideal white noise does not normally possess as many derivatives as appear in the equation of dynamic equilibrium. He did this by rewriting the governing equation in stochastic differential form. Doob (1942) later formalized this procedure.

An important paper written in 1930 by Wiener (1930) defined the concept of the spectral density function, and he attributed the fundamental idea to Schuster (1906). He showed that the Fourier transform of the correlation function exists when the underlying random process is stationary and called that quantity the spectral density. These, of course, are concepts that are essential to the theory of random vibration today.

Numerous other investigators extended the theme introduced by Einstein in the following decades. Uhlenbeck and Ornstein (1930) developed the probability distribution of white noise-excited response at small times. Wang and Uhlenbeck (1945) provided a general derivation of the Fokker-Planck equation from the Chapman, Kolmogorov, Smoluchowski equation.

Some papers by S. O. Rice (1944, 1945) appear to be the first to introduce ideas on how to develop probabilistic measures of some special and important characteristics of system response. He developed (1) equations governing the rate of zero crossings and crossings of a preestablished level by a random process; (2) the probability distribution of maxima of a Gaussian random process; (3) the probability distribution of the envelope of a stationary, narrowband Gaussian random process; (4) the probability distribution of the output of many (zero memory) nonlinear devices; and many other subjects.

## Analysis of Linear Systems

Linear system analysis is the single area in random vibrations where a substantial number of problems has been solved and the solutions used in a wide variety of practical applications. In fact, outside of research and development projects where new techniques for solving random vibrations problems are being sought, perhaps 95 percent or more of current efforts in random vibration problem solution involve the analysis of linear systems excited by stationary inputs. The fundamental techniques for analyzing the response of linear systems with constant coefficients to stationary random excitation were given in Crandall (1958). His paper appears to be the first to express structural response moments in the time and frequency domains, in the form used widely today. He cites Laning and Battin (1956), a text on control systems, as the source of the fundamental statistical input/output relations that he presents. They, in turn, refer to an article by Phillips (1947), dealing with servomechanisms. He refers to the classical paper of Wiener (1930) as the source of the fundamental mathematics (autocorrelation/spectral density relation) in his paper, though it appears to be Phillips who derived the spectral input/output relation. Many authors rederived and generalized these results. Among them are Crandall (1963); Crandall and Mark (1963); Robson (1964); Lin (1967); Elishakoff (1983); Nigam (1983); Newland (1984); Bolotin (1984); Augusti, Baratta and Casciati (1984); Ibrahim (1985); Yang (1986); Schueller and Shinozuka (1987); Roberts and Spanos (1990); Soong and Grigoriu (1993); and Wirsching, Paez, and Ortiz (1995).

To summarize these results, consider a stable, linear system that is discretely modeled with  $n$  degrees-of-freedom. Let the system excitation be a stationary, Gaussian, vector random process denoted  $\{Y(t), -\infty < t < \infty\}$ . There are  $n$  elements in the vector random process, each a scalar random process that represents the excitation at a degree of freedom. Because the excitation is assumed stationary, it possesses a spectral density; denote this  $S_{YY}(\omega), -\infty < \omega < \infty$ . The  $i^{\text{th}}$  row  $j^{\text{th}}$  column element of this matrix is the cross-spectral density function between the excitation random processes at the  $i^{\text{th}}$  and  $j^{\text{th}}$  degrees of freedom. The matrix is Hermitian at every frequency,  $\omega$ . The diagonal elements of the matrix are the autospectral density functions of the excitations. These are real, symmetric, non-negative functions of the frequency,  $\omega$ .

Because the system is linear, it possesses a frequency response function. Let  $H(\omega)$ ,  $-\infty < \omega < \infty$ , denote the matrix frequency response function of the system under consideration; this matrix has dimension  $n \times n$ . The  $i^{\text{th}}$  row  $j^{\text{th}}$  column element of this matrix is the frequency response function of motion at the  $i^{\text{th}}$  degree of freedom excited by input at the  $j^{\text{th}}$  degree of freedom.

Let the response be the vector random process denoted  $\{X(t), -\infty < t < \infty\}$ , and let the response have the spectral density  $S_{XX}(\omega)$ ,  $-\infty < \omega < \infty$ . The vector random process contains  $n$  elements, and its spectral density has dimension  $n \times n$ . Then the spectral density of the response can be expressed:

$$S_{XX}(\omega) = H(\omega)S_{YY}(\omega)H'(\omega) \quad -\infty < \omega < \infty, \quad (2)$$

where the prime denotes the operations of transposition and complex conjugation. The response vector random process is Gaussian distributed because the system was assumed linear, and a linear function of a Gaussian random process is also Gaussian. If the excitation has a nonzero mean, then the mean of the response can be easily established. The response mean and the second order statistics provided in Eq. (2) are sufficient to completely specify the response random process.

In the special case where the input excitation is a stationary random process but the initial conditions of the system are specified at a particular time, the system response is nonstationary from the time of initiation to the time when the system achieves the stationary state. The responses at all degrees of freedom approach the stationary state at an exponential rate that is a function of the system damping. The responses to initial and boundary conditions that are random processes can also be written directly.

The moments and probability distributions of many important measures of structural response were derived by Rice (1944, 1945). Among them are the average rate at which a random process crosses a barrier, the probability distribution of the maxima of a random process, and the probability distribution of the envelope of a narrowband random process. The first of these, in combination with a Poisson process assumption, is used to approximate the first passage probability distribution, even now. (See Coleman, 1959.)

When the system under consideration is modeled as continuous, we can write an integral expression equivalent to Eq. (2) for the response spectral density. However, it is typically more difficult to use because it requires knowledge of the system's Green's function and yields information about the response spectral density only through integration of what may be complicated functions. The continuous approach is used infrequently in this age when spatially discrete finite element models are available and widely used.

All real random excitations are nonstationary in nature. The stationarity assumption is an idealization that is used frequently and sometimes quite accurately. When the duration of an input is short with respect to the fundamental period of the structure being excited, then it is appropriate to treat the input and the response it excites as nonstationary. It is relatively simple to write the expression for the random response of a linear system excited by a nonstationary excitation. It is

$$X(t) = \int_0^\infty h(t-\tau)Y(\tau)d\tau \quad X(0) = \dot{X}(0) = 0, t \geq 0, \quad (3)$$

where the excitation and response random processes are assumed to exist in some well-defined sense, and the initial time and initial conditions have arbitrarily been assumed zero. The quantity  $h(t)$  is the system impulse response function;  $\{X(t), t \geq 0\}$  is the response random process; and  $\{Y(t), t \geq 0\}$  is the excitation random process. It is usually a matter of interest to establish the mean and autocorrelation function of the response, and one would hope to be able to do this for realistic excitation random processes. Because of our preoccupation with white noise excitation in the stationary excitation case, we might hope to specify

the input as a filtered and modulated white noise or as a sum of filtered and modulated, band-limited white noise components. For example, we might hope to define a scalar excitation random process as

$$Y(t) = m(t)L[w(t)] \quad t \geq 0, \quad (4)$$

where  $m(t)$  is an appropriately specified modulating function,  $w(t)$  is an ideal or band-limited white noise, and  $L[\cdot]$  is a filter operating on the white noise. However, it turns out to be very difficult to evaluate the required integrals to make this approach work. We can evaluate the integrals numerically, of course, but then we lose the advantage of relating changes in the input parametric model to the effects they cause in the response statistics.

To overcome this problem, an approach has been developed over the past decade, or so, that simplifies (in some sense) the computation of nonstationary response moments. It has been recently summarized by Masri, Smyth, and Traina (1998). They first establish a representation for the excitation nonstationary random process using a truncated Karhunen-Loeve expansion (Karhunen, 1947, Loeve, 1948). This expresses the excitation random process,  $\{Y(t), t \geq 0\}$ , as a finite sum of terms involving the eigenvalues and eigenvectors of the excitation autocorrelation function and a sequence of uncorrelated random variables. Each eigenvector is then represented as a series in Chebychev polynomials. The resulting expression for the excitation random process is used in Eq. (3), and an expression for the response random process can be developed because of the simple form of the Chebychev polynomials. Moments of the response can be computed, and if the probability distribution of the underlying uncorrelated random variables (from the Karhunen-Loeve expansion) has been derived, then the probability distribution of the response and its measures can be established. However, when the probability distribution of the excitation is non-Gaussian, this may be a difficult problem.

An important problem related to both the stationary and nonstationary response of linear systems is the first passage/peak response problem. It is typically treated by modeling a system response (or envelope of the response) crossing process as a Poisson random process. The approach is accurate at high barrier levels but less so at low levels. More work is required in this area. Pandey and Ariaratnam (1996) is an example of a recent paper in this area.

An interesting method for bounding the peak response probability distribution (and, therefore, the first passage probability distribution) of a linear system was introduced by Koopmans, Qualls, and Yao (1973). It is based on an inequality described by Drenick (1970), which relates peak response to input energy and system impulse response function. They use this relation to derive an upper bound on the probability distribution of peak response of a linear system, and they specialize the analysis to the Gaussian excitation case. Their bound on the peak response probability distribution has the advantage that it does not rely upon any assumptions regarding the probability distribution of response random process peaks. A tighter bound on the response of a linear system was proposed by Shinozuka (1970) soon after the one described above. This bound can also be used to establish a bound on the peak response probability distribution (see Rojwithya, 1980).

The techniques described in the previous paragraph can be used in several other contexts. For example, they can be used to bound the peak response probability distribution of large-scale systems, they can be used in the experimental context, and they can be extended to the nonlinear case. Some of these will be discussed later.

The problem of non-Gaussian excitation and response is also important because many real excitations are non-Gaussian, and almost all real structural responses are nonlinear and non-Gaussian, to some extent. The problem of non-Gaussian excitations has not been treated to near the depth of the Gaussian problem. The problem of nonlinear response will be treated in the following section.

## Nonlinear System Analysis and Chaotic Vibrations

Practically all real structural systems display some degree of nonlinear behavior when subjected to realistic environments. When the types of nonlinearities that a system displays are continuous with input level or when the level of response is mostly below the threshold at which nonlinear behavior commences, then, under many circumstances, it may be reasonable to model the system as linear, ignoring the nonlinearity in the system response behavior. However, when it is necessary to understand detailed structural response behavior at full level or near a failure threshold, then a nonlinear analysis of the system must be performed. There are several approaches available for dealing with nonlinear systems. Some of them are summarized in the following.

The response quantities of interest in a random vibration analysis vary according to the application. When only quantities such as the root-mean-square (RMS) structural response levels are required from an analysis, then equivalent linearization approaches might be used to investigate structural behavior. The subject of equivalent linearization has been widely investigated and is discussed, for example, in Roberts and Spanos (1990) and Lin (1967). Another linearization technique that accounts for the potential for non-Gaussian response is given by Iyengar and Roy (1996). The fundamental objective of equivalent linearization techniques is to establish the "best" linear approximation to a nonlinear mathematical model of a system or to an actual physical nonlinear system.

When the critical characteristics of nonlinear structural response are not preserved in a linearized system, then a nonlinear model is required for random vibration analysis. The techniques available for constructing a framework, identifying a nonlinear model, and analyzing its behavior depend on whether data are available for use in the system identification.

When data are available for the identification of a nonlinear model, then random vibration analyses can be performed by, first, identifying the parameters of the nonlinear model, second, generating realizations of excitation from the desired random source, then third, using the model to compute the response to arbitrary random excitation. This is a Monte Carlo approach. Several models have been used in this framework. For example, Hunter (1997, 1998) and Hunter and Theiler (1992) have utilized a local linear modeling approach to predict the response of a system known only through measured realizations of its excitation and response. They assume that finite length excitation vectors and their associated response vectors, grouped by a measure of excitation level, can be used to establish a local linear, state space model of the system. When an excitation vector is near the group of vectors used to create the linear model, then the model can be used to predict response. Of course, the modeling technique needs to be efficient because it is essential to identify the system with minimal data. They have chosen to use the canonical variate analysis technique (Larimore, 1983) for state space analysis. An implicit assumption is that the system under consideration is time invariant. Further, it is required that the known inputs and responses occupy regions of the hyperspace to be visited by the future, predicted excitations and responses of the system.

A similar approach identifies the parameters of an artificial neural network (ANN) to model system behavior. The ANN can be trained directly using measured input and response data, or it can be trained using transformed data. For example, data obtained in the process of training a local linear model, as the one mentioned above, can be used to train an ANN. Urbina, Hunter, and Paez (1998) and Paez and Hunter (1997) are examples of this type of analysis. The reason for using an ANN when the model that produces the training data is available is that the ANN is typically much more efficient than the former model, sometimes up to a few orders of magnitude more efficient. Chance, Worden, and Tomlinson (1998) and Bailer-Jones, MacKay, and Withers (1998) describe other ANNs that are suitable for random vibration analysis.

A very general model for nonlinear structural system behavior is the Volterra series model (see Volterra, 1959, Marmarelis and Marmarelis, 1978, Schetzen, 1980, Roy and Spanos, 1989). It is a generalization of the convolution model for linear structural response, and it is particularly good at representing harmonic

distortion phenomena. (Harmonic distortion phenomena are connected to nonlinear response behavior, and occur frequently in real structural systems under test.) The Volterra model has the form

$$\begin{aligned}
 x(t) = & h_0 + \int_0^\infty h_1(t-\tau)y(\tau)d\tau \\
 & + \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 h_2(t-\tau_1, t-\tau_2)y(\tau_1)y(\tau_2) \\
 & + \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \int_0^\infty d\tau_3 h_3(t-\tau_1, t-\tau_2, t-\tau_3)y(\tau_1)y(\tau_2)y(\tau_3) \\
 & + \dots
 \end{aligned}$$

$t \geq 0, \quad (5)$

where  $x(t)$  is the system response,  $y(t)$  is the system input excitation, and the  $h_0, h_1(t_1), h_2(t_1, t_2), h_3(t_1, t_2, t_3), \dots$ , are the system kernels. The quantity  $h_0$  is a constant;  $h_1(t)$  is analogous to the linear system impulse response function; and the  $h_i(t_1, \dots, t_i), i = 2, 3, \dots$ , are the higher-order kernels that encapsulate the nonlinear system behavior. Volterra argued that the input/output relation of a nonlinear, time-invariant, finite memory, analytic system can be expressed as in Eq. (5).

Identification of the kernels can, in principle, be done in the time or frequency domain. Identification is discussed in Marmarelis and Marmarelis (1978) and Schetzen (1980). When the objective is to identify the truncated Volterra series for a system known, for example, by its differential equation, then one set of identification procedures might be followed. For example, realizations of the input and response might be generated from the differential equation, then used to identify the system kernels. When data from an experimental system are available to identify the system kernels, then system identification can be done, especially directly in the frequency domain. However, the higher-order kernels require a particularly large amount of data to identify (Hunter and Paez, 1987).

The Wiener series model for nonlinear structural system behavior is a generalization of the Volterra series to the case where the system inputs are wide-band, Gaussian random processes (see Wiener, 1942, 1958; Marmarelis and Marmarelis, 1978; Schetzen, 1980). Specifically, Wiener created a functional series model for nonlinear system response to random excitation, where each functional in the series is orthogonal to all lower-order functionals in the series. The functionals appear like the ones in Eq. (5), except that each term has added terms involving the same order kernel whose purpose is to enforce the orthogonality.

In the same way that a general, nonlinear model for system behavior can be specified, as above, a specific, parametric nonlinear model can also be specified. In fact, when some characteristic or characteristics of a system are known through observation of the physical system or through knowledge of the form of its differential equation, that information can often be used to specify a parametric form. The problem then becomes one of identifying the parameters of the model, and this is almost always easier than identifying system kernels of a Volterra model. For example, Zhu and Lei (1997) discuss the identification of nonlinear models. Ghanbari and Dunne (1998) consider a parametric model for damping

A class of models that appears to be a special case of the Volterra series is that suggested by Bendat (1982, 1990, 1998) and Bendat and Piersol (1983). These comprise a set of robust and easy-to-use techniques, requiring response data. Bendat suggests that nonlinear structural models can be constructed by combining series of elements like those shown in Figure 1. In the diagram the first and final computational elements, denoted ZMNL, are zero memory, nonlinear operators. A ZMNL operator computes a function of its input at time  $t$ , to establish the output at time  $t$ . For example, some simple ZMNL operations on the signal  $y = y(t)$  are:

$$g_1(y) = y, \quad g_2(y) = y^2, \quad g_3(y) = y|y|, \quad g_4(y) = y^3. \quad (6)$$

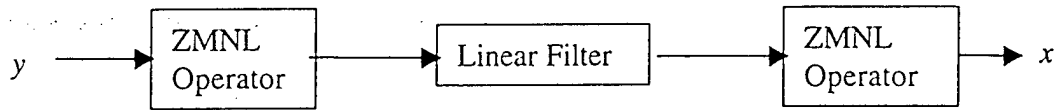


Figure 1. A series of operations on the signal  $y = y(t)$ , yielding  $x = x(t)$ .

The middle element in Figure 1 is linear filter operating on its input to yield a filtered output. For example, if we denote the Fourier transform of the filter input as  $\eta(f)$ ,  $-\infty < f < \infty$ , and the Fourier transform of the filter output as  $\xi(f)$ ,  $-\infty < f < \infty$ , then the filtering operation can be expressed as:

$$\xi(f) = H(f)\eta(f) \quad -\infty < f < \infty, \quad (7)$$

where  $H(f)$  is the filter transfer function. Bendat suggests that only one ZMNL operation normally be used in a series like the one shown in Figure 1 (though two such operations can be used).

Multiple series elements, as the ones shown in Figure 1, are used to model a structure input/output relation. For example, Figure 2 shows a three-element model that seeks to express the output as a function of linear, quadratic, and cubic terms.

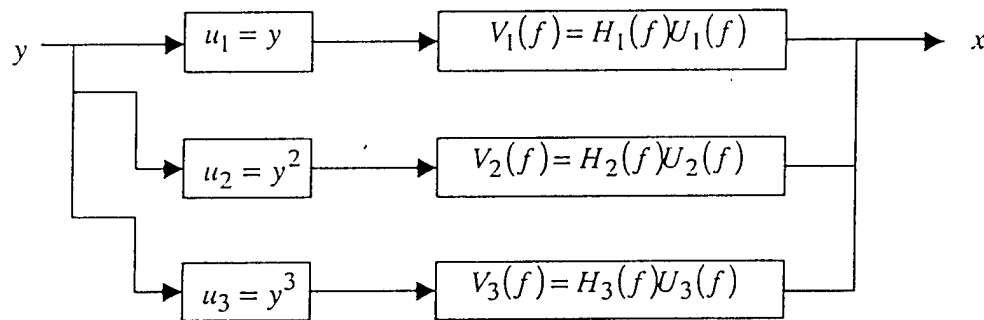


Figure 2. A nonlinear input/output model, expressing the response as a function of linear, quadratic, and cubic terms.

A model like the one shown in Figure 2 can be identified using a sequential application of the principles used in linear system identification.

The treatment developed above is simply a special use of the multiple input/single output system identification procedure first outlined in the mechanical system literature by Bendat and Piersol (1986) and based on a partial coherence estimation procedure developed by Dodds and Robson (1975). The nonlinear multiple input/single output problem can be treated simply by adding more inputs and nonlinear functions of them to the input set in Figure 2. The model shown in Figure 2 can also be augmented and modified via the inclusion of other nonlinear input forms.

In addition to models like the one shown in Figure 2, other types of models can be specified. For example, the ZMNL operation might follow the filtering operation. System identification follows the same general sequential procedure described above, in this case, except that the signal must be run from right to left



through the ZMNL operators to infer the filter outputs for  $H_2(f)$  and beyond. Richards and Singh (1998) and Zeldin and Spanos (1998a) consider the modeling of nonlinear systems with this type of model.

We showed in the previous section that there are techniques that can be used to bound the response of a linear system - the so-called least favorable response techniques - and, therefore, be used to find probability distributions that bound the peak response probability distribution. It so happens that Drenick (1977) published a paper showing that an analogous bound can be developed for nonlinear systems and that the bound on nonlinear system response is related to the behavior of a linearized version of the system. It appears that this result has never been used to establish a peak response probability distribution for nonlinear systems, though it could be.

A collection of methods known as reliability-based techniques have been used to solve problems not normally associated with random vibrations. However, recent investigations and the development of a finite element code called NESSUS have demonstrated that some problems connected with random vibrations, including nonlinear response problems, can be solved by these techniques. The subject of reliability-based techniques and the NESSUS code are discussed in the following section.

Researchers in the fields of random vibrations and chaotic vibrations appear, at least until recently, to have religiously avoided commenting on one another's fields in the frameworks of their own studies. This is an interesting omission in view of the fact that there appear to be many strong similarities between system behaviors in the two fields. Of course, the generation of pseudo-random numbers on digital computers during Monte Carlo analyses and random vibration tests uses carefully crafted procedures to generate numeric sequences that are simply chaotic realizations. Therefore, computed and experimental responses are, in these cases, simply functions of chaotic inputs. Still a few texts, including the ones by Ruelle (1991) and Schroeder (1991), have discussed these relations. The latter text discusses, among many other interesting subjects, the fractal character of Brownian motion. Among the many invariants used to characterize chaotic systems is the PDF. The PDF of a chaotic process can be estimated based on experimental data. Sums of chaotic processes obey the Central Limit Theorem. Further, many low-order chaotic processes transform into high-order chaotic, or random, processes with the variation of a fundamental parameter. These and many other related subjects are discussed by Eubank and Farmer (1990). Some other papers that discuss the relations between chaos and random vibrations are the ones by Lin and Yim (1996a, 1996b) and Feng and Pfeiffer (1998). Gregory and Paez (1990) and Paez and Gregory (1990) consider a chaotic system for its potential use in generating high frequency/high-level random environments.

Finally, we mention that a frequently used tool for the analysis of nonlinear random vibrations is the Fokker-Planck equation. The Fokker-Planck equation can be written for the transition probabilities of many structural systems. However, a closed form solution for system behavior, in the most general form, cannot always be practically obtained. Measures of system behavior, as the stationary state response probability distribution or moments of the response, can be obtained more easily. An early study in this area is that of Caughey (1963). Other examples of studies that involve the use of the Fokker-Planck equation are those by Parssinen (1998) and Jing and Young (1990).

## **Analysis of Large-Scale Systems in Numerical Analysis Codes/Stochastic System Analysis**

One of the areas of study in random vibrations that connects theoretical analysis techniques to the common application of random vibration in analysis, design, and diagnosis is the assessment of response character with computer codes meant to accommodate models of large systems. Many practical systems are analyzed using finite element codes today. Most commercial codes include the capability to perform the analysis symbolized in Eq. (2) for linear systems. Beyond this commercial finite element codes can be used to perform Monte Carlo analyses where the excitation, initial conditions, boundary conditions, and

material properties are varied randomly, but such analyses must usually be performed simply by using the code as a deterministic function analyzer, and this may be difficult. Engelhardt (1999) shows how one code is being automated to permit Monte Carlo analysis. Other tools could be directly incorporated into existing finite element codes. For example, capability to analyze response of structures excited by nonstationary random excitations (Masri, Smyth, Traina, 1998). In fact, though, this would require very substantial augmentation of existing codes.

The material and/or geometric properties of many systems must be treated as random when they affect structural response at the same level as inputs, initial conditions, and boundary conditions. The analysis of such systems is more complex than the analysis of systems whose random response is caused only by random excitation. As noted above, deterministic finite element tools can be used to perform probabilistic analysis of systems with both random excitations and random structural properties using a Monte Carlo approach. However, as the random character of excitation and system properties becomes more complex, the number of Monte Carlo simulations required for convergence of statistical measures of the response increases and may become prohibitive.

In view of this, there has been a continuing interest in the development of nonsampling techniques. An early approach to the analysis of complex structures was the perturbation method. The use of this method requires that coefficients in the system governing equations be separated into two parts: a mean, or deterministic part, and a random part. The random part of the governing equation is taken as a perturbation to the mean part; components of the response are matched to like-ordered components of the excitation, in the traditional way; and the equations of motion are solved. Some problems are that, first, only the first term in the perturbation can usually be maintained for reasons of computational complexity, thereby limiting potential accuracy of the series representation of the response, and second, convergence cannot typically be proven. Therefore, other techniques are required.

In the 1980's an entire class of techniques became available with the development of the reliability-based techniques, and these can be used for the analysis of large-scale probabilistic structural dynamics problems. Among these are first and second order reliability methods (FORM/SORM) (see Madsen, Krenk and Lind, 1986); fast probability integration (FPI) techniques (see Wu and Wirsching, 1987a); and the advanced mean value (AMV) technique (see Wu and Wirsching, 1987b).

The FORM is typical of other reliability-based techniques, and it seeks to approximately evaluate points on the cumulative probability distribution function (CDF) of a measure of the output of an arbitrary, deterministic function of a set of random variables. The function need not be explicitly defined; it may be a mathematical analysis computer code - e.g., a finite element code. The capability to approximately evaluate the CDF of a function of several random variables implies that we can characterize a measure of the response of a system excited by a dynamic excitation and, perhaps, with random system characteristics. The problem is posed in the following way. Let  $Y$  be an  $n$ -vector of random variables with known joint PDF,  $f_Y(y)$ ; and let  $X$  be a scalar, deterministic function of  $Y$ :

$$X = g(Y). \quad (8)$$

We seek the CDF of  $X$ ,  $F_X(x)$ , at a set of points  $x_j, j = 1, \dots, N$ . The CDF is defined

$$\begin{aligned} F_X(x) &= P(X \leq x) \\ &= \int_{g(y) \leq x} dy_1 \dots dy_n f_Y(y). \end{aligned} \quad (9)$$

The  $n$ -fold integral on the right of Eq. (9) is typically difficult to evaluate for arbitrary joint PDF,  $f_Y(y)$ . To simplify the approximation of the integral, we define a compound Rosenblatt transform (see

Rosenblatt, 1952) from the domain of the PDF of the  $n$ -vector  $Y$  to the domain of the PDF of  $n$  uncorrelated, standard normal random variables denoted by the vector  $Z$ . We denote the transformation

$$Z = T(Y), \quad (10)$$

and note that it is based on the marginal CDF of  $Y_1$ , and the conditional CDFs of the  $Y_j$  given  $Y_{j-1}, \dots, Y_1$ , for  $j = 2, \dots, n$ . The transformation is monotone increasing in each variable when the random variables in the vector  $Y$  are continuous valued; therefore, the transformation is invertible. Based on this, Eq. (9) can be rewritten

$$F_X(x) = \int_{g(T^{-1}(z)) \leq x} dz_1 \dots dz_n f_Z(z). \quad (11)$$

The integrand is much simpler in this expression than Eq. (9). To permit the approximate evaluation of the integral, we find the location,  $z^*$ , on the constraint

$$g(T^{-1}(z)) = x, \quad (12)$$

where the norm of  $z$ ,  $\|z\|$ , is minimum. This is the so-called design point, and we denote this distance to the origin as  $\|z^*\| = \beta$ . At this point we approximate the constraint,  $g(T^{-1}(z)) \leq x$ , with a version that is linearized in  $z$ ,  $\gamma_L(z) \leq x$ ; replace the integration limit in Eq. (11) with this approximate one; and evaluate the integral.

$$F_X(x) = \int_{\gamma_L(z) \leq x} dz_1 \dots dz_n f_Z(z) = \Phi(\pm \beta), \quad (13)$$

where  $\Phi(\cdot)$  is the CDF of a standard normal random variable. The plus sign is taken on  $\beta$  when  $x$  is greater than the mean of  $g(Y)$ , and the minus sign is taken otherwise. The integral can be evaluated because it is the CDF of a weighted sum of uncorrelated, standard normal random variables; and any weighted sum of uncorrelated, standard normal random variables is simply a normal random variable. By repeating the procedure described in Eqs. (8) through (13) for  $x = x_j$ ,  $j = 1, \dots, n$ , we approximately evaluate the CDF of the random variable,  $X$ , at these abscissa values.

Second order reliability methods work in precisely the same way as the FORM except that the linear approximation to the integration limit in Eq. (13) is replaced by a quadratic approximation. The advanced mean value technique works in the same way as the FORM except that we start the analysis with a linear approximation to  $X = g(Y)$ , and after completing the analysis make a correction that renders the result quite accurate. An iterative and more accurate, yet still very efficient, version of the AMV technique is available. (See Red-Horse and Paez, 1998.)

Normally we consider the solution of a random vibrations problem to be a probabilistic characterization of the temporal response. From this we normally derive the probabilistic character of critical response measures, like peak values, RMS values, first passage times, etc. The approaches described above yield results in a different form. Here we obtain directly the probabilistic character of response measures, and normally we cannot obtain the entire temporal response characterization. If the analyst requires the probabilistic characterization of the entire response, then one of the techniques would need to be adapted for that purpose. This appears not to have been done, though the Karhunen-Loeve approach for excitation

modeling, specified above, connected with nonstationary response analysis, may be useful in this connection:

One example of a reliability-based finite element code is the computer code called NESSUS (Numerical Evaluation of Stochastic Structures Under Stress). It was developed at Southwest Research Institute (see NESSUS, 1996). It is based on the AMV technique and has been implemented in connection with multiple finite element codes including Sandia National Laboratories' PRONTO code (See Taylor and Flanagan, 1987). The code is implemented as a "wrap-around" in the sense that the finite element code is only called to perform evaluations like the ones in Eq. (8). The code has been used to solve nonlinear structural response problems with up to one million elements and involving up to 36 random variable inputs. The AMV technique is elucidated with example applications in Wu (1994) and Wu, Millwater and Cruse (1990).

A class of stochastic finite element techniques that takes a different approach than the reliability-based techniques is the one introduced by Ghanem and Spanos (1991). [See also Ghanem and Brzakala (1996) and Ghanem (1999).] It is what might be termed a random field method. It casts the coefficient in the equation of equilibrium as separable into deterministic and random parts. In addition, the excitation may be stochastic. The parameters that underlie the coefficient are assumed to be random fields with known autocorrelation functions. The eigenproblem is solved for the autocorrelation function of the underlying parameter; then the stochastic coefficient is replaced with its truncated Karhunen-Loeve expansion. Next, the response variable (defined as a stochastic field over the system analyzed) is expressed as a truncated Karhunen-Loeve expansion with unknown coefficients and unknown eigenvectors. The coefficients in this expression are typically non-Gaussian distributed, and they are expanded in a homogeneous chaos - a multivariate Hermite polynomial. Finally, the eigenvectors in the response expression are expanded using a finite element basis. Both sides of the resulting expression are multiplied by an arbitrary term in the homogeneous chaos, and expected values are taken. The resulting approximate governing equations are collected into a block matrix equation. The dimension of one block in the set of block equations is the same as the dimension of the equivalent deterministic problem. The block dimension of the equilibrium equations is a function of the number of terms retained in the truncated Karhunen-Loeve expansion approximation to the coefficient random field and the order of the homogeneous chaos used in the representation of the response coefficients.

The objective in solution of the block equations is to find the amplitudes of the finite element shape functions associated with terms in the homogeneous chaos expansion for the response. Once these are obtained, the random field expression for the system response can be expressed, and this can be used to obtain arbitrary measures of the system response. For example, moments of any function of the response can be computed; further, marginal and joint probability distributions of the response can be obtained.

Other papers dealing with stochastic finite elements are those by Contreras (1980); Manohar and Adhikari (1998); Elishakoff, Ren, and Shinozuka (1996); Saigal and Kaljevic (1996).

An important problem that arises in connection with the stochastic finite element formulations is that of specifying and identifying the random fields that are used to characterize system properties. Some papers that deal with this issue are Zeldin and Spanos (1996, 1998b); Hoshiya and Yoshida (1996, 1998); and Noda and Hoshiya (1998).

The general problem described here is very practical in the sense that both excitation and material parameters often need to be considered as random. For this reason the field of stochastic finite elements and the implementation of stochastic capabilities into existing codes merits great attention.

## Experimental Probabilistic Structural Dynamics

Most of random vibration analysis is intended to realistically characterize the behavior of structural systems excited by random inputs; however, the responses of actual systems are only known when they are measured during the application of physical environments (and even then only approximately). The effects caused by the use of simplifying assumptions in the process of numerical simulation of physical systems are seldom evaluated. Most important, because of the depth and breadth of our capability to analyze linear systems subjected to stationary environments, we often idealize real systems as linear and inputs as stationary. Practically all real systems are nonlinear to a small or great extent; therefore, the response characterization can only be approximate.

Real physical system responses are measured, then characterized, in two frameworks. First, when real structural systems are designed, then built (or, perhaps, placed) in the field, their responses can be measured when subjected to natural or man-made environments. Second, when systems are of the appropriate size, they can be experimentally excited in the laboratory. Most laboratory tests of physical systems performed today are done on electrodynamic or electrohydraulic shakers and subject the system tested to a quasi-stationary random vibration environment.

Quasi-stationary random vibration tests to be performed on a shaker are controlled today using digital, closed loop control systems. The objective of a random vibration test is to maintain the autospectral density of a single measure of motion on or near a test system within some preestablished limits for a preestablished length of time. The algorithm used for control was derived in Tebbs and Hunter (1974). Prior work leading to this development is summarized in Hunter and Helmuth (1968) and Otts and Hunter (1970).

Before digital, closed loop control algorithms were available for the control of random vibration tests, analog, quasi-closed loop control systems that were operated manually were used. The control scheme employed during the 1950's and 1960's (and in some laboratories, the 1970's and even 1980's) is described by Metzgar (1958). The system used manual equalization to achieve the desired test spectral density. The purpose of the manual equalizer setting was to account for (a) the electro-mechanical frequency response function between the drive signal and the control point and (b) the shape of the desired spectral density at the control point. Because the electro-mechanical system, consisting of the signal generation electronics, the transmission system, the power amplifier and shaker, and the test item, is usually nonlinear, it was usually very difficult to establish and maintain the desired control. The control algorithm developed for digital control of random vibration tests is an attempt to mimic the analog control system.

With reference to Figure 3, the digital control algorithm consists of the following operations. Starting at top left a digital computer generates finite duration realizations of a stationary, Gaussian random process. Generated realizations are output to a digital-to-analog converter. The analog signal may then be filtered and amplified and transmitted to a shaker system. The signal is first amplified then used to drive the shaker. Motion is generated in the shaker armature. The test item is attached to the shaker armature, perhaps via a fixture or through a table that permits the testing of a much larger system. Control of the test is sought on or near the test item - control that is gauged in terms of the autospectral density of motion. Motion at the control point is measured, usually in terms of absolute acceleration. The motion is measured with a transducer, the output of which is amplified and filtered, then transmitted to the control digital computer. Before processing in the computer, the signal must be converted from analog to digital. The digitized signal is read by the control system. The continuously arriving signal is used to estimate the running, or real-time, spectral density (see Wirsching, Paez, and Ortiz, 1995).

A critical element permitting realization of the digital control system is the ability to combine finite duration realizations of stationary random process into a continuous realization of stationary random

process of arbitrary length. This is accomplished using a procedure called overlap processing, described in Gold and Rader (1969).

With the control system defined above, a random vibration test is performed following a two-step process. The first step is the system identification phase. In this phase a band-limited white noise drive signal is generated in the digital control computer and used to drive the shaker and the control point at a level that is low relative to the full level of the test to be performed. The drive signal and control point motion signals are stored and used to estimate the autospectral density function of the drive and the cross-spectral density function between the drive and the control. The latter is ratioed to the former to create an estimate of the electro-mechanical system frequency response function; denote this quantity  $H(f_k), k = 0, \dots, n/2$ .

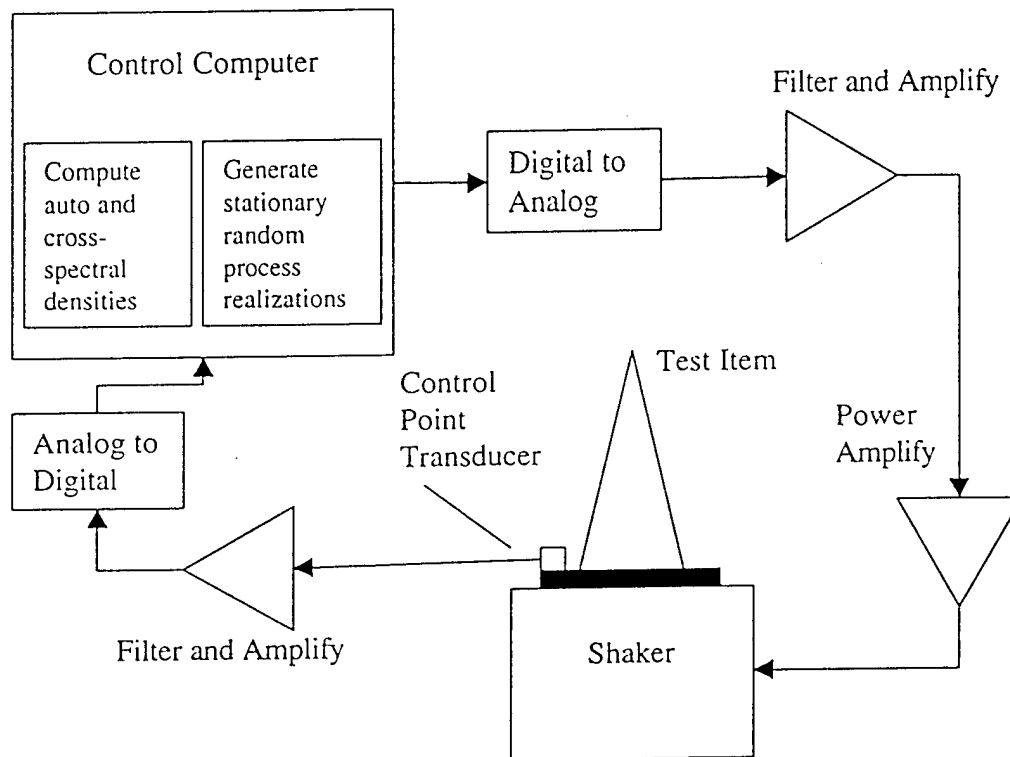


Figure 3. Elements and operations of the closed loop control system.

The second phase involves the performance of the actual test. This is done in a sequence of steps where the excitation level is gradually increased until the full level of the test is realized. Denote the autospectral density desired at the control point with  $G_{XX}(f_k), k = 0, \dots, n/2$ ; then the test is initiated by generating a drive signal from a Gaussian source with zero mean and spectral density

$$G_{DD}(f_k) = \alpha_0 \frac{G_{XX}(f_k)}{|H(f_k)|} \quad k = 0, \dots, n/2. \quad (14)$$

This signal is meant to drive the system so that the motion at the control point has the spectral density  $\alpha_0 G_{XX}(f_k), k = 0, \dots, n/2$ . The constant  $\alpha_0$  is normally chosen so that the RMS motion is some number of decibels (db) below the full level of the test. Data are gathered at this level, and the drive autospectral density and the drive to control cross-spectral density estimates are initiated and then updated using real-time spectral density estimation. These are used to update the estimate of the electro-mechanical system

frequency response function. When the test either equalizes at the current level - i.e., the estimated control autospectral density matches, within some limits, the target control autospectral density - or the test operator is satisfied that the test has equalized as well as it can, then the test level is increased, and the next step in the test is initiated. This is accomplished by changing the coefficient  $\alpha_0$  to  $\alpha_1$ , where  $\alpha_1$  is some multiple of  $\alpha_0$ . The running estimates of the drive autospectral density and the drive to control cross-spectral density are modified by the ratio  $\alpha_1 / \alpha_0$ , and the test continues. When the motion finally reaches its full level, the test is allowed to continue for a preestablished duration or until the test operator aborts the test. Though there are many other details to the operation of a random vibration test, and there are many potential pitfalls, the foregoing description characterizes the fundamental ideas behind random vibration test operation.

The digital computers used today for random vibration control are many orders of magnitude faster than the ones used when digital control was first introduced in the early 1970's. Nevertheless, the algorithm used in most commercial control systems is substantially the same as the one proposed decades ago and described above. Because of limitations in the capabilities of the standard control system and because of hardware limitations, the typical random vibration test performed in the laboratory is controlled in a single axis. Because of the physical limitations associated with testing hardware on a shaker - i.e., one axis of controlled-input motion, test item attached to a very stiff armature, etc. - there are many situations where it is thought preferable to control not a single measure of motion in one axis and at one point, but rather a single measure of many motions at multiple points and, perhaps, in multiple directions. In these situations the test operator uses a control scheme known as average control or one known as extremal control (see Smallwood and Gregory, 1977). As their names imply, the former seeks to control the average autospectral density at a number of locations, and the latter seeks to control the maximal spectral density at a set of locations. These control schemes are used when knowledge of the autospectral density in an actual system is inaccurate, or when, because of physical testing limitations, a known autospectral density at a particular critical location will be surpassed when motion is controlled to a particular level at another point.

More than 99 percent of "state-of-the-art" random vibration testing is performed on a single shaker, using single-point control; therefore, it is fair to state that the limitations to this type of testing are real limitations to the actual state of the art. The fundamental limitations are that testing is performed in a single axis, and sometimes equalization cannot be achieved - i.e., motion with the desired autospectral density cannot be excited at the control point. The limitation to single-axis testing arises from two sources - economics and the difficulty of performing more realistic multi-axis/multi-shaker tests. The rationale underlying the first source is obvious as is its effect on testing realism. Few real random vibration environments are limited to motion that occurs in a single axis. During a single-axis controlled random vibration test, only one measure of motion is sought to be controlled, though motions in all axes occur. Sometimes, depending on the system under test, the off-axis motions have greater RMS value than the controlled motions. The off-axis motions that occur during performance of a single-axis random vibration test have autospectral densities that practically never match the autospectral densities of the system motions in the field. Further, the cross-spectral densities between motions realized during a single-axis test practically never match the cross-spectral densities between motions realized in the field.

The inability to generate motion at the control point with the desired autospectral density during a single-axis controlled random vibrations test arises from the nonlinearity of the electromechanical test system and the system under test. This occurs in, perhaps, half of all random vibration tests of complex systems. A frequent side effect of nonlinear response is the occurrence of harmonic distortion - i.e., the generation of response harmonics associated with strong motion response at a particular frequency, especially system resonances. These harmonics occur because during strong motion, system response excited by a simple harmonic signal is often not a simple sine, cosine, or combination of these. Because the algorithm used to control motion assumes that the system under test is linear, the response at the harmonic frequencies is misinterpreted and cannot always be controlled. In particular when the response at a higher harmonic has an autospectral density greater than the desired control autospectral density, then the motion at the control

point will have an autospectral density that is out of tolerance on the high side, since the drive signal is not designed to diminish RMS motion at any frequency.

Clearly there are other limitations to random vibration testing in the laboratory. Among these are the force limits to any shaker. A large electrodynamic shaker might generate up to 50,000-lb force in the frequency range [5,5000] Hz; a large electrohydraulic actuator might produce the same force but in the frequency range [0,500] Hz. It would appear, of course, that if economics permit, many shakers can be used in parallel to overcome this limitation. Such appearance is not realized in real systems though. Multiple shakers tied together via a slip table or other fixture actually excite an elastic (or inelastic) system - the system that connects the shakers. The system has modes starting at a frequency that is a function of the size of the system and the acoustic velocity of the system materials (approximately the same for the materials used to construct armatures and fixtures). Large seismic simulations can have fundamental resonances below 100 Hz, and small component shakers can have fundamentals of 1000 to 2000 Hz. The control system must account for these in order to generate stable motion in the fixture/test article system. Separate control computers cannot, in general, be used to control the separate shakers; the system is sometimes unstable.

There are software plus hardware solutions to some of the problems mentioned above, although they are not widely applied, in practice, perhaps for economic reasons. One of these is the multi-axis/multi-shaker testing capability. This capability was developed by Smallwood (1982a, 1982b, 1999). It operates following the basic principles of Figure 3 except that multiple coherent drive signals are generated, these are separately conditioned, and separate power amplifiers are used to drive multiple shakers. The shakers connect to a test item or fixture at multiple points and perhaps in multiple directions. When  $N$  shakers are used to excite a system, then  $N^2$  measures of system motion can, within certain constraints, be controlled (see Paez, Smallwood and Buksa, 1987). For example, in a three-axis test the autospectral density of motion in each of three axes can be controlled, as well as the real and imaginary parts of the cross-spectral densities between the pairs of motions; these are nine quantities. Of course, motions at all the control points are required, and the control computer must be capable of estimating the auto and cross-spectral densities of the control point motions.

The fundamental capabilities required to make the control algorithm work is the ability to generate multiple signals with arbitrary auto and cross-spectral densities and the ability to make the algorithm stable. The former capability is achieved via Cholesky or eigenvalue decomposition of the cross-spectral density matrix of the multiple drives. See Smallwood and Paez (1993) for details on the signal generation algorithm.

There are many less obvious shortcomings of standard laboratory random vibration tests. Among the most important is the high - effectively infinite - impedance of the shaker system. This means a shaker will exert as much force as is required to match the control autospectral density. Overtesting problems can arise in this connection, and force limiting must be imposed to achieve realistic tests. Scharton (1995), Chang and Scharton (1998), and Smallwood and Coleman (1993) treat this issue.

There are many other activities in probabilistic testing that merit our attention including the generation of non-Gaussian environments (see Smallwood, 1996); nonstationary random vibration and random shock testing (see Smallwood, 1973); and the control of nonlinear systems in random vibration testing.

## Summary and Conclusions

Some recent and earlier activities in the area of random vibration analysis and testing have been summarized in this paper. A brief history was given, and linear random vibration was discussed. Some fundamental areas that merit continued investigation are: robust and convenient frameworks and algorithms for the analysis of nonstationary response of structures, general methods for the analysis of



structural response to non-Gaussian excitations, improved techniques for the analysis of first passage probabilities of complex systems.

Some nonlinear models and computation of response measures for nonlinear systems were discussed. This field is wide open in the sense that there is an extremely wide variety of types of nonlinear behavior in real systems, and the modeling of almost all could stand improvement. Among many other things general models and techniques for nonlinear analysis are required; general techniques for analysis of large systems are required.

Large system analysis is normally performed today using finite element codes. There are many commercial and proprietary codes, most of which are limited to spectral density analysis - i.e., the computation of response spectral density, given input spectral density. The capabilities of these codes need to be broadened. Further, reliability-based codes that yield a more traditional random vibration response characterization need to be developed, and stochastic finite element codes need to be made practical.

Random vibration testing is the most practical of the areas discussed in this paper. There are many investigations that could improve the state of the art in testing. Some areas requiring development work are: random vibration control algorithms for nonlinear systems, procedures for nonstationary excitation identification and nonstationary testing, means for making multi-shaker/multi-axis testing more robust and economical, improved hardware and standard procedures for force controlled testing

These are a few of the areas, among many others, that require the attention of investigators in the field of random vibrations.

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# SIGNAL PROCESSING IN VIBRATION ANALYSIS

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## 1. Introduction

Signal processing in vibration analysis can be divided into two main categories: single channel or “**signal analysis**”, and multiple channel, mainly used for “**system analysis**”. Vibration signals result from the action of forces on structures, and are a combination of the characteristics of the forcing function and the structural dynamic properties of the structure. Where one forcing function dominates, in so-called SIMO (single input, multiple output) situations, and where linearity applies as in a very large number of cases where stresses are below yield, the forcing function and structural response effect (transfer function) between the points of force application and response are convolved in the time domain, multiplied in the frequency domain and additive on taking logs of the frequency functions. This simplifying effect is one of the major reasons for the widespread use of **frequency analysis** (Fourier analysis) in processing vibration signals. A consequential property of linear systems is that when excited at one frequency, they respond only at that frequency, which is another reason for using frequency analysis. Even when the system is slightly nonlinear, this often just means that the response becomes distorted and higher harmonics (multiples) of the forcing frequency are generated, which are very easily related to the fundamental frequency. A further very important reason for the widespread use of frequency analysis is the ease with which it can now be carried out since the introduction of the so-called “**fast Fourier transform**” (FFT) algorithm in 1965 [1.1] which is a very efficient way of calculating the “**discrete Fourier transform**” (DFT).

Signal analysis is often carried out in cases where the forcing functions cannot be measured, but changes in the response signals are assumed to be dominated either by the forcing function or the structural response, based on other criteria. An example is the condition monitoring or diagnostics of operating machines. Even though frequency analysis using the FFT is the most commonly applied technique, other signal analysis procedures have advantages in certain circumstances as discussed below. For short signals, better frequency resolution can be obtained using parametric analysis techniques such as the **maximum entropy method** [1.2]. Since basic Fourier analysis assumes infinitely long time records, information is lost about time localisation of events. Analysis in both time and frequency can be achieved by moving a finite time window along a signal and calculating the frequency spectrum for each position (so-called **short time Fourier transform** - STFT). Because of the property of the DFT that the frequency resolution is the reciprocal of the length of record transformed, the better the time resolution the poorer the frequency resolution and vice versa. Techniques have been developed to improve the combined resolution in time-frequency analysis, based on the **Wigner-Ville distribution** [1.3, 1.4]. Another time-frequency analysis technique which has become increasingly used is “**wavelet analysis**” [1.5], where signals are decomposed in terms of a family of wavelets (of which there are many types) where the basic wavelet can be both translated and dilated in time. The dilation typically occurs in octave (2:1) steps so that the “scale”, which represents frequency, is naturally a logarithmic axis. This means that the time resolution of a wavelet analysis is much better at high scales or frequencies. The same applies to **octave-based analysis** with 1/n-octave filters such as the

1/3-octave filters which have long been used in the analysis of acoustic signals. This is because the human ear tends to interpret equal changes on a logarithmic scale (eg octaves) as equal intervals, and the same applies to the response of the human body to vibrations, where standards recommend analysis in 1/3-octave bands. Before the advent of the FFT algorithm, most frequency analysis was done with analogue filters, measuring the power of time signals transmitted by a series of band-pass filters each covering part of the frequency range to be analysed. Although most narrow band analysis is now done digitally using the FFT, **digital filters** working in the time domain in a similar manner to their analogue counterparts have advantages in certain situations, in particular for constant percentage bandwidth (1/n-octave) analysis over a wide frequency range on a logarithmic scale.

Some effects are more easily interpreted in the time domain rather than the frequency domain, and there are a number of techniques to enhance such effects. An example is **autocorrelation analysis** which gives a measure of how well a signal correlates with delayed versions of itself. This can be useful for enhancing periodicity and echoes, for example. A related time domain function is the **cepstrum**, which in general is better for detecting echoes, and which has two other main applications in vibration analysis

- Detecting families of equally spaced components in a spectrum such as harmonics and sidebands
- Helping to separate forcing and transfer functions in response signals as they are additive in the cepstrum.

A very important time domain technique is **synchronous averaging**, where signal sections are averaged together synchronously with a timing signal, for example a once-per-rev tachometer signal, to enhance events occurring at that repetition rate. The best known example is the application to gears, where the signals from a pair of gears in mesh can be separated from each other (and background noise) by averaging synchronously with each gear in turn. In signals from rotating machines synchronous averaging must often be combined with so-called "**order analysis**" to remove the effects of speed variation by sampling the signals synchronously with the speed of the machine. This is now most efficiently done by **digital interpolation** (resampling) techniques. Apart from eliminating the effects of small speed fluctuations, this process can also be used to study how the vibrations at the various harmonic orders vary over a wide speed range such as during a machine run-up or coast-down. Synchronous averaging is most useful when signals are exactly periodic (or synchronous); however if they are only approximately periodic but their autocorrelation function is periodic, then their **cyclostationarity** and **spectral correlation** properties may be of interest.

Other effects in vibration signals are often due to modulation, either amplitude or phase modulation, so that the signals obtained by **demodulation** are of more interest than the raw signals. An example where **amplitude demodulation** is of interest is in "envelope analysis" of signals from faulty rolling element bearings, where the series of impulse responses due to impacts with the fault contain the diagnostic information in the envelope signal, obtained by amplitude demodulation. As regards **phase demodulation**, this is useful in the measurement of torsional vibration of machines, which is a phase modulation if expressed in terms of shaft angular displacement, and a frequency modulation if expressed in terms of angular velocity. Demodulation is readily carried out using **Hilbert transform techniques**, which have a close relationship with Fourier analysis.

In multiple channel analysis, one or more forces are applied (and measured) by the user, and when combined with the corresponding response measurements from all over a structure can be used to determine the dynamic properties of the structure. Once again the most commonly

applied techniques are based on the FFT process to extract **frequency response functions** (FRFs) from which a complete scaled modal model can be determined. FRFs are only strictly valid for linear systems and a measure of the degree of linearity of the relationship between the forcing function and response is given by the **coherence**. For calculation of FRFs and coherence it is necessary to measure cross spectra between pairs of signals, the **cross spectrum** being the Fourier transform of the **cross correlation function**, similar to the autocorrelation function but relating one signal to a delayed version of the other. The inverse Fourier transform of an FRF is the corresponding **impulse response function**, which represents the system dynamic properties in the time domain. In some situations, such as when the vibration signals represent “free decay” after all forcing functions have been removed, then other techniques can be applied to determine the modal properties of a structure, such as the “**Ibrahim time domain**” (ITD) method [1.6] and **Prony** method. The **complex cepstrum** and **differential cepstrum** have also been found useful for determining modal properties of structures based on response measurements only, because of the previously mentioned property of the cepstrum that forcing and transfer function components in response signals are additive.

In what follows, the above highlighted techniques are explained in somewhat more detail, after a discussion of some basic theoretical concepts. However, the analysis of multiple channel signals for modal analysis is only treated sketchily as it is the subject of other specialist papers.

## **2. BASIC THEORY**

### **2.1 The Fourier Transform**

#### **2.1.1 Fourier Series**

The basic concept of Fourier analysis is to express signals as a summation of sinusoidal components, and with few exceptions virtually all signals can be decomposed in this way. Fourier's original analysis (now called Fourier Series) was applied to finite length signals (or periodic signals, as the resulting solution is periodic with the finite length as period). In vibration analysis it is used almost exclusively for periodic signals, as produced by a machine rotating at constant speed. Thus, for any periodic signal  $g(t)$  of period  $T$  for which  $g(t) = g(t + T)$  it can be shown that:

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t) \quad (2.1)$$

where  $\omega_0$  is the fundamental angular frequency in rad/s ( $= 2\pi / T$ ) and where the coefficients of the cosine and sine terms can be obtained by correlating them with  $g(t)$ , as follows:

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos(k\omega_0 t) dt \quad (2.2)$$

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \sin(k\omega_0 t) dt \quad (2.3)$$

Thus, the total component at frequency  $\omega_k (= k\omega_0)$  is given by:

$$a_k \cos(\omega_k t) + b_k \sin(\omega_k t) \quad \text{which can alternatively be written as:} \\ C_k \cos(\omega_k t + \phi_k) \quad (2.4)$$

where  $C_k = \sqrt{a_k^2 + b_k^2}$  and  $\phi_k = \tan^{-1}(b_k / a_k)$

Expression (2.4) can again be represented as:



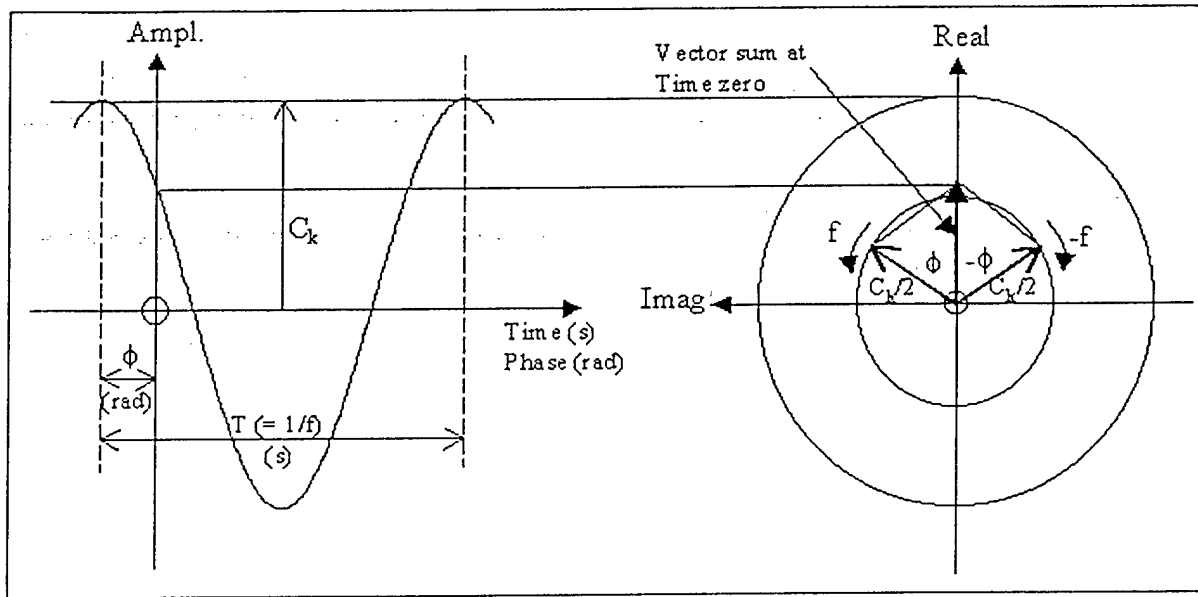


Figure 2.1. Representation of a sinusoid as a sum of two rotating vectors

$$\frac{C_k}{2} [\exp\{j(\omega_k t + \phi_k)\} + \exp\{-j(\omega_k t + \phi_k)\}] \quad (2.5)$$

which can be interpreted as two rotating vectors, each of length  $C_k/2$ , one rotating at angular frequency  $\omega_k$  with initial phase  $\phi_k$  and the other rotating at angular frequency  $-\omega_k$  with initial phase  $-\phi_k$ , as illustrated in Fig.2.1.

Using this interpretation of Fourier analysis as representing  $g(t)$  as a sum of rotating vectors leads to the alternative version of Equ.(2.1) as:

$$g(t) = \sum_{k=-\infty}^{\infty} A_k \exp(j\omega_k t) \quad (2.6)$$

where the coefficients  $A_k$  are now complex and incorporate the phase shift in the form

$$A_k = \frac{C_k}{2} \exp(j\phi_k) \quad (2.7)$$

The equation for calculating the coefficients  $A_k$  (equivalent to Equ.(2.2, 2.3)) now becomes:

$$A_k = \frac{1}{T} \int_{-T/2}^{T/2} g(t) \exp(-j\omega_k t) dt \quad (2.8)$$

This has the simple physical interpretation that multiplication by  $\exp(-j\omega_k t)$  subtracts angular frequency  $\omega_k$  from each component, meaning that the one originally rotating at  $\omega_k$  is stopped in the position it had at time zero (this then being extracted by the integral) while all other components still rotate at some other multiple of  $\omega_k$  (either positive or negative) and thus integrate to zero over the periodic time. Thus each frequency component  $A_k$  represents the position (and value) of the rotating vector at time zero, so that to obtain its position at any other time  $t$  it is necessary to cause it to rotate at angular frequency  $\omega_k$  by multiplying by  $\exp(j\omega_k t)$ . Summing then over all frequency components gives Equ.(2.6). Before leaving this interpretation of Fourier series analysis as a sum of rotating vectors each

of amplitude half that of the corresponding sinusoid (ie  $C_k/2$ ), but having a two-sided spectrum in that each positive frequency component is accompanied by its complex conjugate at negative frequency, it can be seen that the same result can be achieved by retaining the positive frequency components only, but doubling their length to  $C_k$  and then taking the projection of each vector on the real axis. This is illustrated in Fig. 2.2. A signal with a one-sided spectrum like this is complex (and known as an **analytic signal**) but it will later be shown that the projection on the imaginary axis is the Hilbert transform of the real part. This applies equally to the vector sum of a number of components as to each individual component.

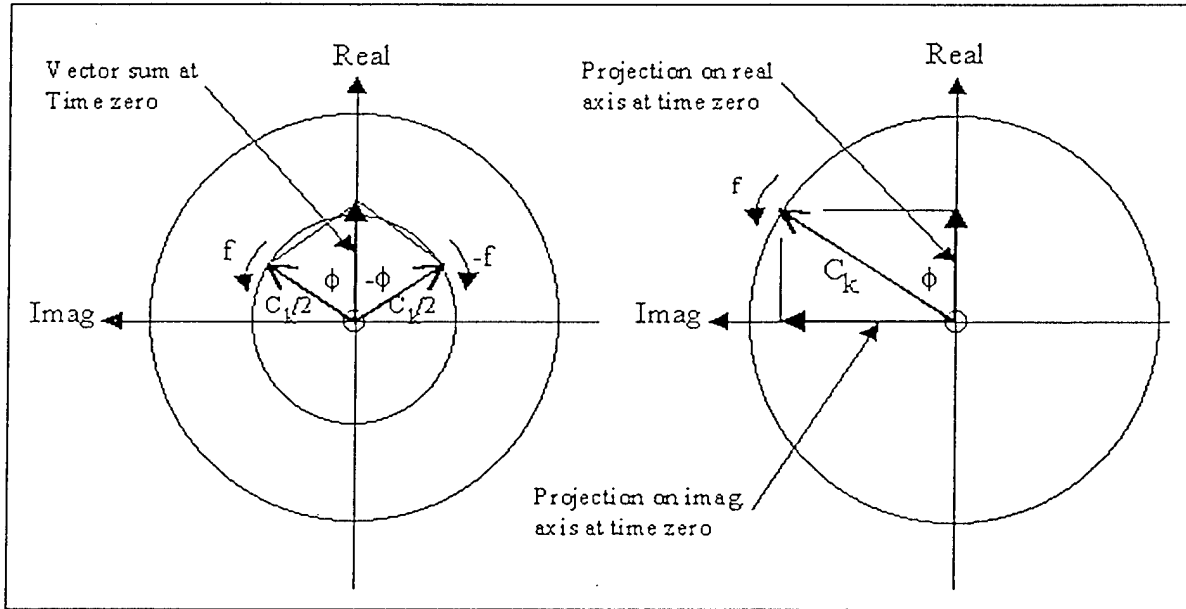


Figure 2.2. Equivalence of vector sum and projection on the real axis

### 2.1.2 Fourier integral transform

Signals other than periodic can also be expressed as a sum of complex exponentials, in particular transients to which the so-called Fourier transform applies. The Fourier transform can be derived from the Fourier series by allowing the periodic time to tend to infinity and at the same time removing the division by  $T$  because transients have finite energy rather than finite power. Equations (2.8) and (2.6) then become:

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-j2\pi ft) dt \quad (2.9)$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft) df \quad (2.10)$$

respectively, where angular frequency  $\omega_k$  in rad/s has been replaced by the continuous frequency function  $f$  expressed in Hz. Equations (2.9) and (2.10) are known as the forward and inverse Fourier (integral) transforms, respectively.

At this point it is convenient to highlight the connection between the Fourier transform of Equ.(2.9) and the Laplace transform, defined by:

$$G(s) = \int_0^{\infty} g(t) \exp(-st) dt \quad (2.11)$$

where  $s$  is a complex variable which can be represented as  $\sigma + j\omega$  in terms of its real and imaginary parts. Thus, for impulse response functions, which are necessarily causal (ie they do not exist for negative time) their Fourier transform, known as the frequency response

function (FRF) is equal to their Laplace transform (transfer function) for the special case that  $s$  is restricted to the imaginary axis ( $s = j\omega = j2\pi f$ ).

Note the symmetry between the forward and inverse transforms of Equ.(2.9, 2.10), the only difference being the sign of the exponent. This means that most often results which apply to the forward transform also apply to the inverse transform, for example the convolution theorem to be discussed below. This is particularly the case for real, even functions, for which it makes no difference if time or frequency run forwards or backwards.

### 2.1.3 Sampled time signals

All signals which are to be processed digitally have to be digitised or discretely sampled. As will be seen in Fig.2.3(c) [2.1], this is the inverse case of the Fourier series (Fig.2.3(b)), where the spectrum is discretely sampled, and the symmetry of the Fourier transform means that the spectrum of a sampled time signal is periodic. The corresponding versions of the forward and inverse transforms are:

$$G(f) = \sum_{n=-\infty}^{\infty} g(t_n) \exp(-j2\pi f t_n) \quad (2.12)$$

$$g(t_n) = \frac{1}{f_s} \int_{-f_s/2}^{f_s/2} G(f) \exp(j2\pi f t_n) df \quad (2.13)$$

where  $t_n = n \Delta t = n / f_s$

### 2.1.4 The discrete Fourier transform (DFT)

The sampled time signals in 2.1.3 are in principle of infinite length, but when the record length is finite, this leads to the same situation as with the Fourier series in that the spectrum is discrete and the time record implicitly periodic. As seen in Fig.2.3(d), this leads to a combination of the cases of Fig.2.3(b, c) so that both the time record and frequency spectrum are discretely sampled and periodic. The continuous infinite integrals of the Fourier transform become finite sums, usually expressed as:

$$G(k) = 1/N \sum_{n=0}^{N-1} g(n) \exp(-j2\pi kn/N) \quad (2.14)$$

$$g(n) = \sum_{k=0}^{N-1} G(k) \exp(j2\pi kn/N) \quad (2.15)$$

This version corresponds most closely to the Fourier series in that the forward transform is divided by the length of record  $N$  to give correctly scaled Fourier series components. If the DFT is used with other types of signals, eg transients or stationary random signals, the scaling must be adjusted accordingly as discussed below. Note that with the very popular signal processing package Matlab®, the division by  $N$  is done in the inverse transform, which requires scaling in every case, as even though the forward transform is then closer to the Fourier integral, it still must be multiplied by the discrete equivalent of  $dt$ .

Note also that for convenience, the time and frequency zero positions have been shifted from the centre of the record to the beginning, but because of the implicit periodicity this just means that the second half of each record represents the negative axes (see Fig.2.3(d)).

The forward DFT operation can be understood as the matrix multiplication:

$$\mathbf{G}_k = \frac{1}{N} \mathbf{W}_{kn} \mathbf{g}_n \quad (2.16)$$

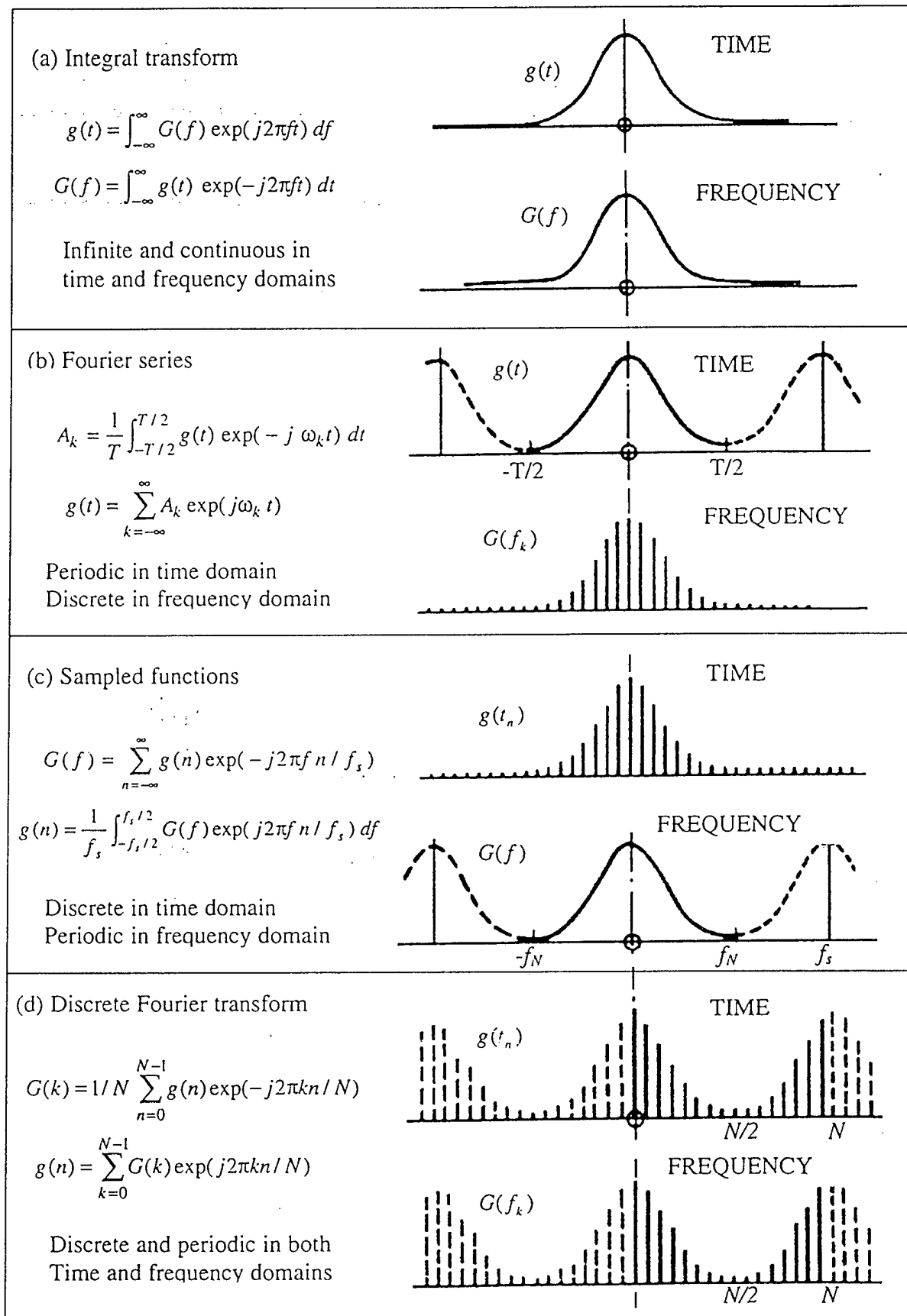


Figure 2.3. Various forms of the Fourier transform [2.1](a) Fourier integral transform (b) Fourier series (c) Sampled functions (d) Discrete Fourier transform

where  $G_k$  represents the vector of  $N$  frequency components, the  $G(k)$  of Equ.(2.14, 2.15), while  $g_n$  represents the  $N$  time samples  $g(n)$ .  $W_{kn}$  represents a square matrix of unit vectors  $\exp(-j2\pi kn/N)$  with angular orientation depending on the frequency index  $k$  (the rows) and time sample index  $n$  (the columns). This is illustrated graphically in Fig.2.4.

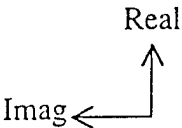
$$\begin{bmatrix} G_0 \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_6 \\ G_7 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \uparrow & \nearrow & \rightarrow & \searrow & \downarrow & \swarrow & \leftarrow & \nwarrow \\ \uparrow & \rightarrow & \downarrow & \leftarrow & \uparrow & \rightarrow & \downarrow & \leftarrow \\ \uparrow & \searrow & \leftarrow & \nearrow & \downarrow & \nwarrow & \rightarrow & \swarrow \\ \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \\ \uparrow & \swarrow & \rightarrow & \nwarrow & \downarrow & \swarrow & \leftarrow & \searrow \\ \uparrow & \leftarrow & \downarrow & \rightarrow & \uparrow & \leftarrow & \downarrow & \rightarrow \\ \uparrow & \nwarrow & \leftarrow & \swarrow & \downarrow & \nwarrow & \rightarrow & \swarrow \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \end{bmatrix}$$


Figure 2.4. Matrix representation of the DFT

For  $k=0$  the zero frequency value  $G(0)$  is simply the mean value of the time samples  $g(n)$  as would be expected. For  $k=1$  the unit vector rotates  $-1/N$ th of a revolution for each time sample increment, resulting in one complete (negative) revolution after  $N$  samples. For higher values of  $k$  the rotation speed is proportionally higher. For  $k=N/2$  (half the sampling frequency, the so-called Nyquist frequency) the vector turns through  $-\pi$  for each time sample, but it is not possible to see in which direction it has turned. For  $k > N/2$  the vector turns through more than  $\pi$  (in the negative direction) but is more easily interpreted as having turned through less than  $\pi$  (in the opposite direction) and thus if the time signal has been lowpass filtered at half the sampling frequency (as should always be the case) the second half of  $G_k$  will contain the negative frequency components from the negative Nyquist frequency to just below zero.

### 2.1.5 The fast Fourier transform (FFT)

The fast Fourier transform is simply a very efficient algorithm for calculating the DFT equations (2.14, 2.15). Starting with the matrix version (2.16), in the simplest form (the so-called radix 2 algorithm) the FFT is based on  $N$  being a power of 2, and factorises the matrix  $W_{kn}$  into  $\log_2 N$  matrices each with the property that multiplication by them only requires  $N$  complex operations as compared with the  $N^2$  operations required for direct multiplication by  $W_{kn}$ . Thus the total number of complex operations is reduced from  $N^2$  to  $N \log_2 N$ , a saving by a factor of more than 100 for the typical case where  $N = 1024 (= 2^{10})$ . Other savings can be made in special cases, and a similar but not so effective gain can be made for factorisation other than in powers of 2, but the main point is that the properties of the FFT are those of the DFT.

### 2.2 The Convolution Theorem

This very important theorem states that a Fourier transform in either direction converts a multiplication into a convolution and vice versa. One example already mentioned is that the

convolution between an input force and the impulse response of a structure results in the frequency domain in a product of their respective Fourier transforms to form the Fourier transform of the response vibration. An example of the inverse is that multiplication of a time signal by a windowing function results in the frequency domain in a convolution of their respective Fourier transforms.

The convolution operation represents a moving average or swept filtration of one function by the other reversed, as expressed by the Duhamel integral:

$$x(t) = \int_{-\infty}^{\infty} g(\tau)h(t-\tau) d\tau \quad \text{represented as} \quad x(t) = g(t) * h(t) \quad (2.17)$$

When one of the functions is a Dirac delta function (or impulse function), with or without scaling and physical dimensions, the convolution operation is very simple and involves shifting the origin of the convolving function to the position of the delta function and multiplication by the scaling factor. Note that the scaling factor can be complex and thus involve a phase shift as well as a gain factor. In what follows, considerable use will be made graphically of these convolution relationships.

### 2.2.1 Fourier Series from the Fourier transform

As a first example it can be seen that any periodic signal can be generated by convolving the transient representing one period with a train of delta functions with spacing  $T$ , the periodic time. It can be shown that the Fourier (Series) transform of the latter is likewise a train of delta functions with spacing  $1/T$  but also value  $1/T$ , so that the Fourier series is obtained by multiplying the Fourier transform of the transient by the train of delta functions (thus sampling it at the harmonics of the fundamental frequency) and scaling the samples by the factor  $1/T$ . Note that at the same time this corrects the physical dimensions of the result so that the Fourier series components have the same units as the original signal.

Figure 2.5 shows an example of how the Fourier series of a full-wave rectified cosine can be obtained from the Fourier transform of a single half-cosine pulse. To obtain the Fourier series of a half-wave rectified cosine it is simply necessary to sample the spectrum at intervals of  $1/2T$  and multiply by  $1/2T$ .

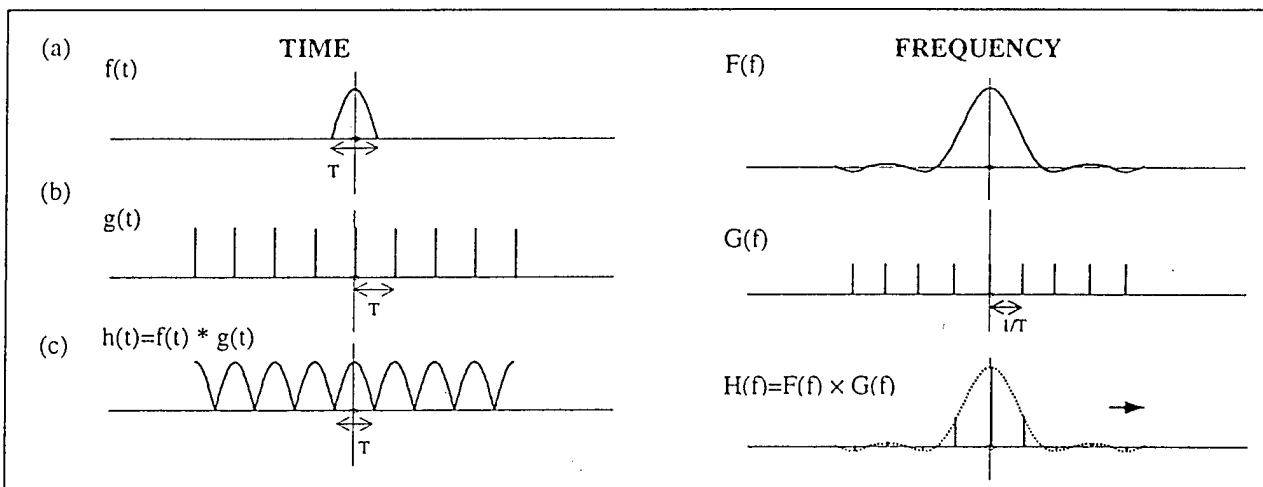


Figure 2.5. Determining a Fourier series from a Fourier transform

(a) half-cosine pulse and its Fourier transform

(b) train of delta functions and its spectrum

(c) full-wave rectified cosine and its Fourier series spectrum

### 2.2.2 Multiplication in the time domain

The case taken for illustration is that of the response of a single-degree-of-freedom (SDOF) system, which in the time domain is an exponentially damped sinewave represented by

$$\exp(-\sigma t)\sin(2\pi f t) \quad (2.18)$$

The spectrum of the sinewave can be represented by two delta functions, each of amplitude  $1/2$ , the one at  $+f_0$  having a phase angle of  $-\pi/2$  and the one at  $-f_0$  having a phase angle of  $+\pi/2$ . The Fourier transform of the exponential function is:

$$\frac{1}{\sigma + j2\pi f} \quad (2.19)$$

which changes in phase from  $+\pi/2$  at  $-\infty$  through zero at zero frequency to  $-\pi/2$  at  $+\infty$ . Figure 2.6 shows how the multiplication of these two functions in the time domain results in a convolution of their respective Fourier transforms, and giving the well-known frequency response function with a peak at the resonance frequency (where the phase is turned through  $-\pi/2$ ), zero phase at zero frequency and  $-\pi$  at  $+\infty$ .

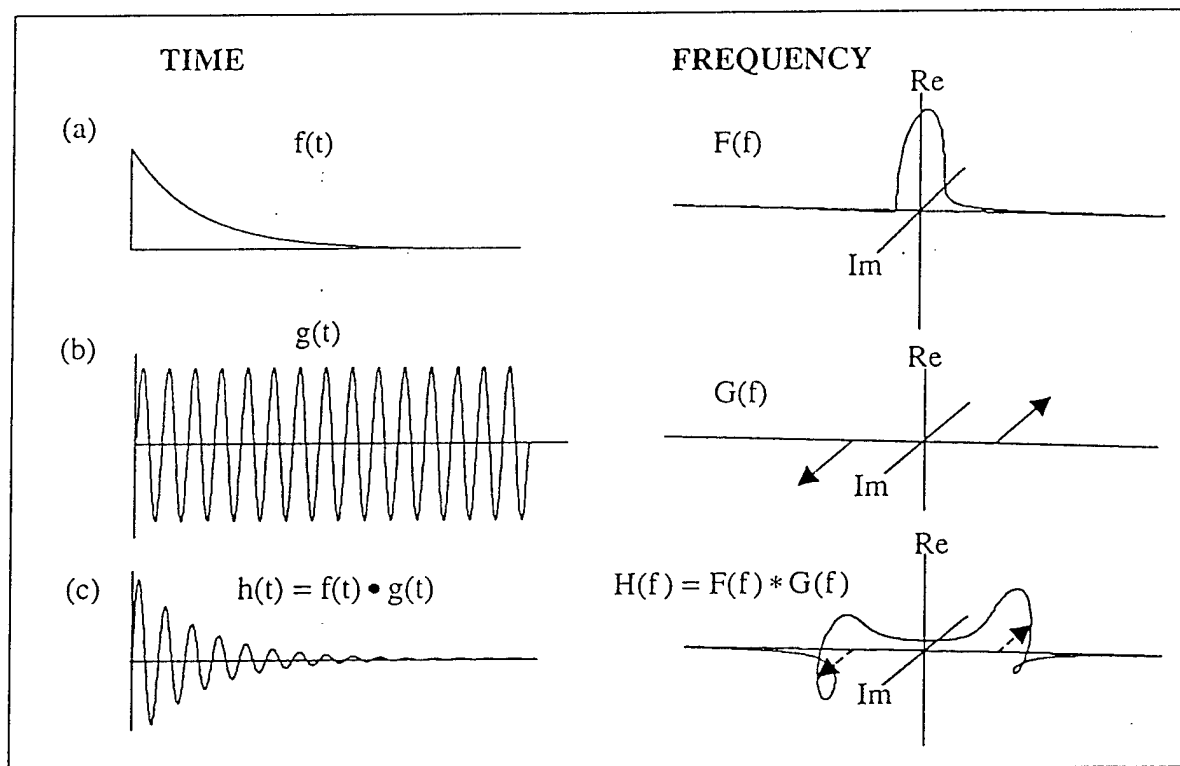


Figure 2.6. Spectrum of an exponentially damped sinewave.  
In the spectrum of (c) the arrows depict the resonance peak

### 2.3 Hilbert transform relationships

The Hilbert transform can be said to be the relationship between the real and imaginary parts of the Fourier transform of a one-sided function. For example, any impulse response function is causal and thus one-sided in the time domain, and this means that the real and imaginary parts of the corresponding frequency function (eg that shown in Fig.2.6) are related by a

Hilbert transform. That there should be a relationship becomes evident when it is considered that a causal function is made up of even and odd components which are identical for positive time, and thus cancel for negative time. Thus:

$$x(t) = x_e(t) + x_o(t) \quad (2.20)$$

and

$$x_e(t) = x_o(t) \times \text{sgn}(t) \quad (2.21)$$

where  $\text{sgn}(t)$  is the sign function. Since the even part of a time function transforms to the real part of its Fourier transform, and the odd part to the imaginary part, by applying the convolution theorem to Equ.(2.21) it can be seen that:

$$X_e(f) = X_o(f) * \mathfrak{F}\{\text{sgn}(t)\} \quad (2.22)$$

The Fourier transform of the sign function is the imaginary hyperbolic function  $\frac{1}{j\pi f}$  so that the final expression relating the real part of the Fourier transform ( $X_R(f) = X_e(f)$ ) to the imaginary part ( $X_I(f) = X_o(f)/j$ ), and writing out the convolution in full, is given by:

$$X_R(f) = \frac{1}{\pi} \int_{-\infty}^{\infty} X_I(\phi) \cdot \frac{1}{(f - \phi)} d\phi \quad (2.23)$$

The equivalent equation for the Hilbert transformation of a time function  $x(t)$  is:

$$\tilde{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} x(\tau) \frac{1}{(t - \tau)} d\tau \quad (2.24)$$

Taking the Fourier transform of Equ.(24) gives:

$$\tilde{X}(f) = X(f) \cdot (-j \text{sgn}(f)) \quad (2.25)$$

which shows that a Hilbert transform can be achieved more simply by transforming into the frequency domain, shifting the phase of positive frequency components by  $-\pi/2$  and of negative frequency components by  $+\pi/2$ , and then transforming back to the time domain.

A one-sided frequency spectrum can similarly be divided into conjugate even and conjugate odd components which transform by the inverse Fourier transform to real and imaginary time signals, respectively, which are related by a Hilbert transform. The sum of these two components is known as an "analytic signal", which can be formed from a given real time signal by adding  $j$  times its Hilbert transform. Alternatively, it can be obtained more simply by transforming the real time signal into the frequency domain, obtaining the equivalent one-sided spectrum by multiplying by  $2H(f)$ , where  $H(f)$  is the Heaviside or unit step function, and transforming back to the time domain. This is also a very efficient way of performing a Hilbert transform. Note from Fig.2.2 that the Hilbert transform of a cosine is obviously a sine function.

As a corollary, it is worth pointing out that when working with a signal processing package such as Matlab®, modifying frequency spectra and then transforming back to the time domain, it is not necessary to adjust all negative frequency components in the same way (but complex conjugate) as the positive frequency components; it is much simpler to multiply the positive frequency components by 2 (but not the zero frequency component), set the second half of the spectrum (the negative frequency components) to zero, perform an inverse transform to an analytic signal and simply take the real part (this last operation is usually necessary even when working with 2-sided spectra, as the program does not know that the answer is supposed to be real, and will usually calculate a very small imaginary part). This process is illustrated in Fig.2.7.



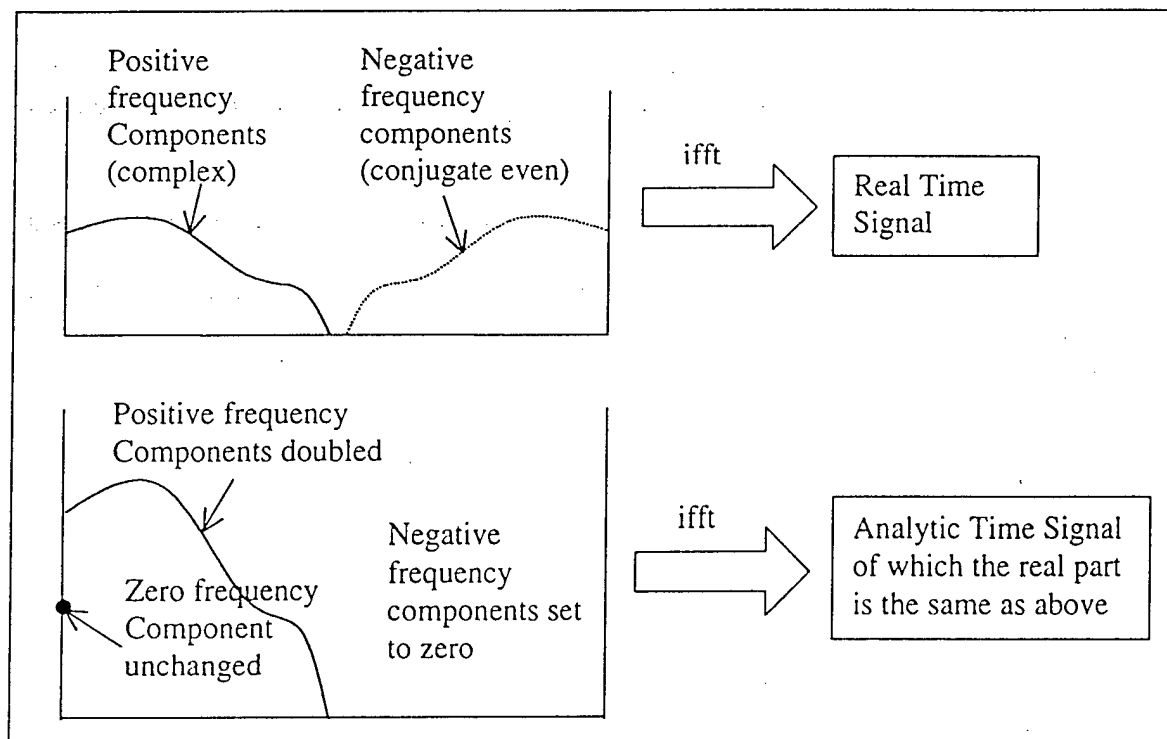


Figure 2.7. Manipulation of the positive frequency spectrum to obtain a real time signal

### 3. PRACTICAL FFT ANALYSIS

#### 3.1 Pitfalls

The so-called pitfalls of the FFT are all properties of the DFT and result from the three stages in passing from the Fourier integral transform to the DFT. The first step is digitisation of the time signal which can give rise to **aliasing**; the second step is truncation of the record to a finite length, which can give rise to **leakage** or **window effects**, while the third results from discretely sampling the spectrum, which can give rise to the **picket fence effect**. Figure 3.1 shows these three steps graphically, using the convolution theorem [2.1].

In Fig.3.1(a-c) the infinite continuous time signal is sampled as in Fig.2.3(c) producing a periodic spectrum with a period equal to the sampling frequency  $f_s$ . It can be seen that if the original signal contains any components outside the range  $\pm f_N$ , where  $f_N$  is the "Nyquist frequency" or half the sampling frequency, then these will overlap with the true components giving "aliasing" (higher frequencies represented as lower ones). Once aliasing is introduced it cannot be removed, so it is important to use appropriate analogue lowpass filters before digitising any time signal for processing. After initial correct digitisation, digital lowpass filters can be used to permit resampling at a lower sampling rate (the Matlab® function "decimation" achieves this, but not the function "resampling"). In Fig.3.1(d-e) the signal is truncated to length  $T$  by multiplying it by a finite (rectangular) window. The spectrum is thus convolved with the Fourier transform of the window, which acts as a filter characteristic. Energy at a single frequency is spread into adjacent frequencies in the form of this characteristic, hence the term "leakage". Finally, in Fig.3.1(f-g) the continuous spectrum is discretely sampled in the frequency domain, which corresponds in the time domain to convolution with a train of delta functions of spacing  $T$ , making the time signal periodic. The spectrum is not necessarily sampled at peaks; hence the term "picket fence effect"; it is as though the spectrum is viewed through the slits in a picket fence.

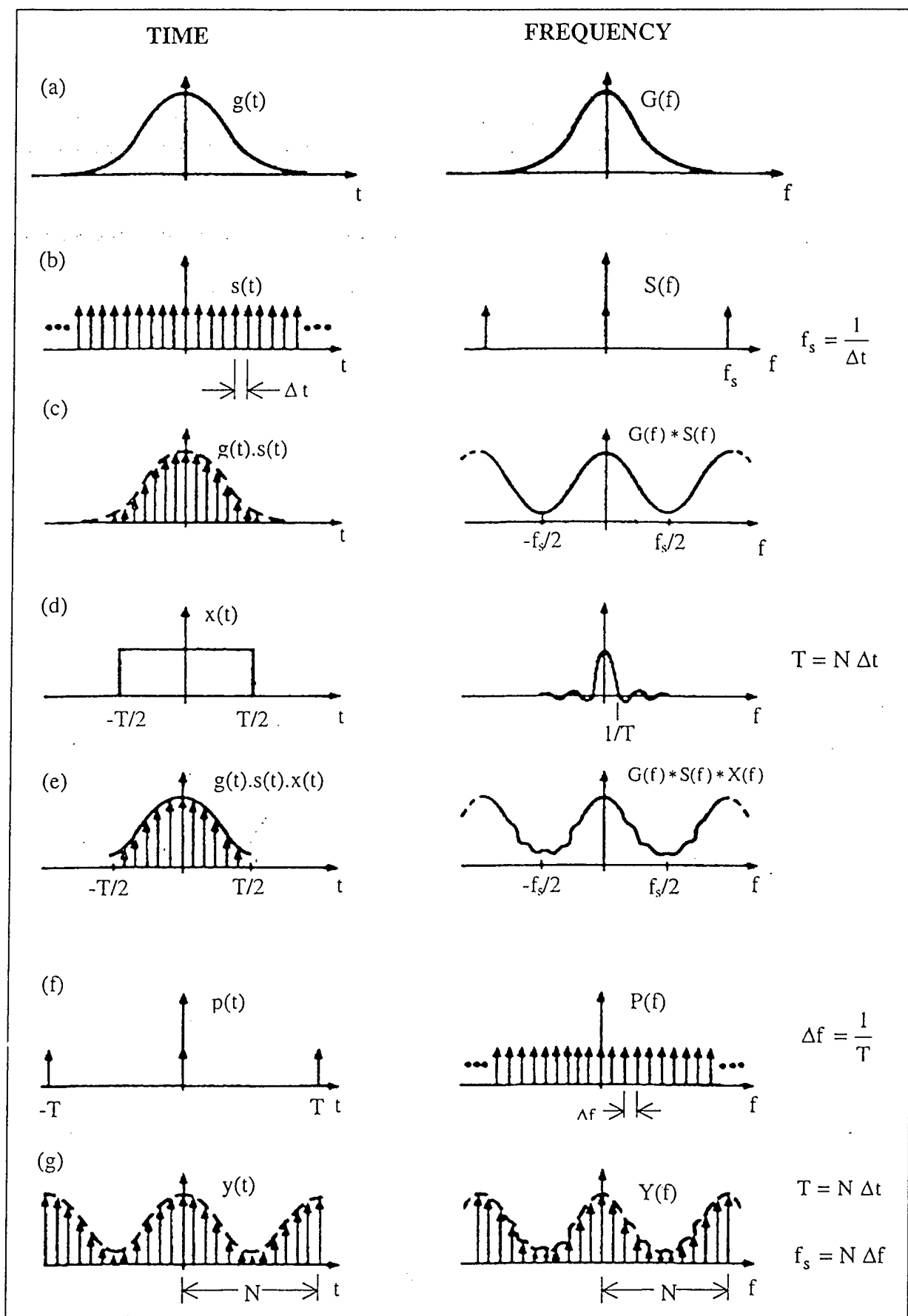


Figure 3.1. Three steps in passing from the FT to the DFT [2.1]  
 (a-c) Time sampling (d-e) Truncation (f-g) Frequency sampling

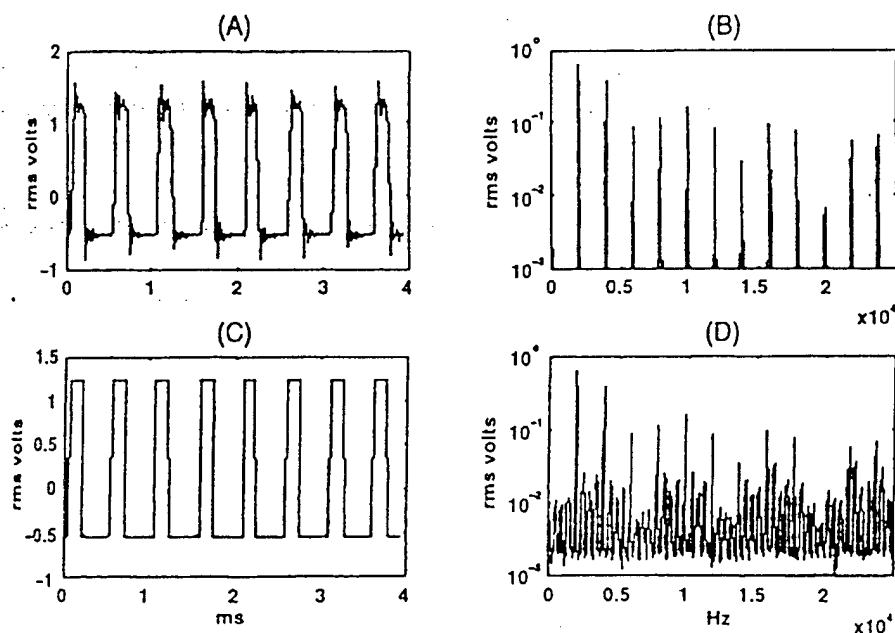


Figure 3.2. (A) Time signal correctly lowpass filtered (B) Spectrum of (A) (C) Time signal without lowpass filtration (D) Spectrum of (C) (note aliasing)

To avoid aliasing it is virtually always necessary to use an antialiasing filter with a very steep roll-off. It has become fairly standard to use filters with a roll-off of 120 dB/octave, allowing approximately 80% of the calculated spectrum to be used. Thus with a 1K (1024 point) transform, spectrum line number 512 is at the Nyquist frequency, and higher frequencies fold back towards the measurement range. Line number 624 folds back into the top of the desired measurement range (line number 400), and is only 64% of an octave above it, and so is attenuated by 77 dB, taking it below the typical dynamic range. The antialiasing filters typically result in considerable distortion of the time signal, and are thus usually not included in digital oscilloscopes (which thus should not be used for digitisation of signals for further processing). Figure 3.2 (from [3.1]) illustrates how antialiasing filtering corrects the spectrum while distorting the time signal, and vice versa.

The effects of leakage are influenced by the final spectrum sampling, and Fig.3.3 illustrates that for a rectangular window (whose FT is a  $\frac{\sin(x)}{x}$  or sinc(x) function) if the window contains an integer number of periods of a sinusoid, even though each spectral line is associated with a sinc function, these are sampled at the zeros and are thus not apparent. On the other hand, in the worst case of a residual half period, the effective filter characteristic is very poor. Use is made of this phenomenon in "order tracking" of machines, where the signal sampling is synchronised with machine speed and it can be arranged that there is an integer number of periods of (all harmonics of) the rotational frequency within the record length, in which case a rectangular window can be used. Otherwise, for continuous signals it is usually necessary to choose a data window other than rectangular to achieve a better filter characteristic. This is discussed below.

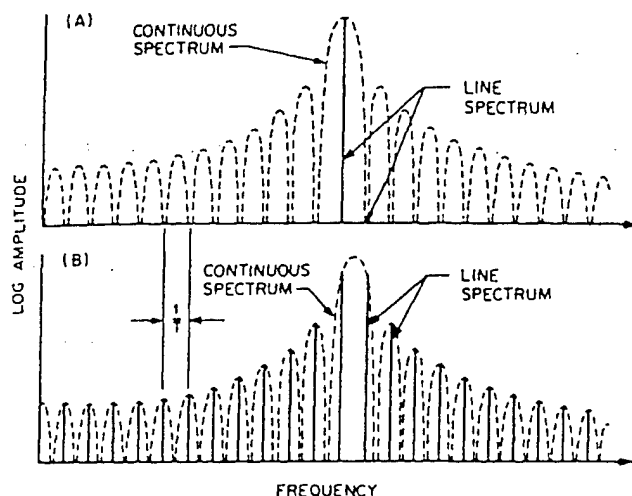


Figure 3.3. How frequency sampling affects the apparent filter characteristic of a window  
(A) Integer number of periods in record (B) Extra half period in record

Because of the picket fence effect, spectral functions are not necessarily sampled at their peaks and the "picket fence error" is the difference between the true value and the value of the maximum spectral line. For rectangular weighting this can be as much as 3.9 dB, and most other windows have a reduced value.

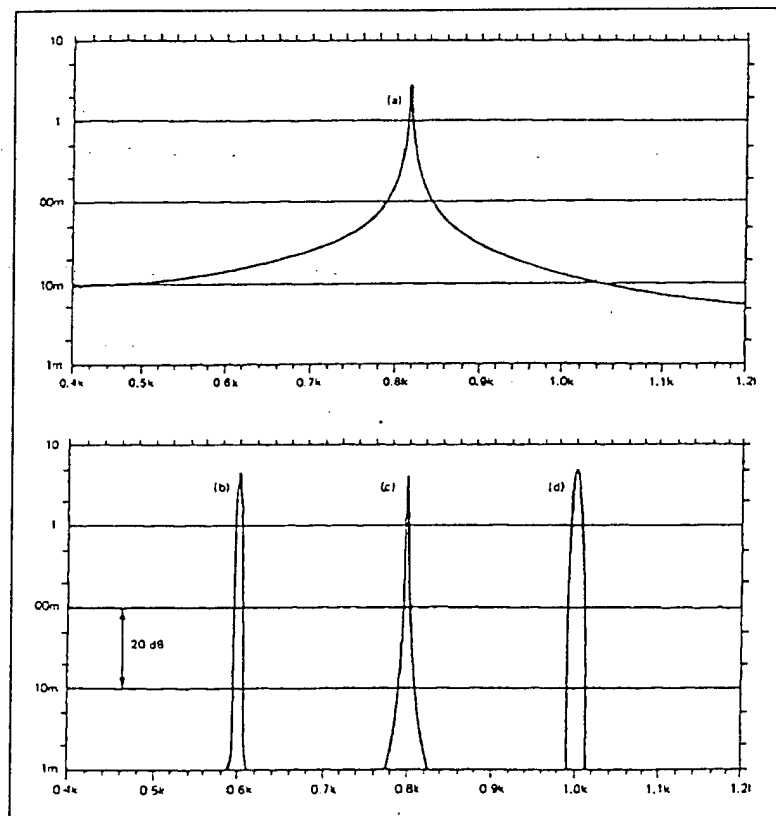
### 3.2 Data windows

#### 3.2.1 Continuous signals

For continuous signals, a major function of the window is to reduce the effect of the discontinuity which usually arises when a random section of signal is made periodic. Practically, it means minimising the sidelobes in the filter characteristic, both the highest and the remaining ones (by maximising their rate of falloff). To improve enhancement of discrete frequency components with respect to broadband noise it is desirable to minimise the noise bandwidth of the characteristic, but on the other hand, attention must also be paid to minimising the picket fence effect. Table 3.1 gives a comparison of the properties of the most common windows applied to stationary signals, and Figure 3.4 (from [3.1]) compares their worst case filter characteristics. The Hanning window, which can be considered as one period of a sine squared function, is a good general purpose window, with picket fence effect limited to 1.4 dB, noise bandwidth 1.5 (times  $\Delta f$  the line spacing) and desirable characteristics with respect to overlap averaging, to be discussed below. The best window with respect to separating adjacent components of widely differing levels is probably the Kaiser-Bessel, but

Table 3.1. Comparison of window properties

Window type	Highest sidelobe (dB)	Sidelobe roll-off (dB/decade)	Noise bandwidth ( $\times \Delta f$ )	Max. picket fence error, (dB)
Rectangular	-13.4	-20	1.00	3.9
Hanning	-32	-60	1.50	1.4
Hamming	-43	-20	1.36	1.8
Kaiser-Bessel	-69	-20	1.80	1.0
Truncated Gaussian	-69	-20	1.90	0.9
Flattop	-93	0	3.70	<0.1



*Figure 3.4. Worst case filter characteristics for some common window functions  
(a) Rectangular (b) Kaiser-Bessel (c) Hanning (d) Flattop*

the same can usually be achieved by simply analysing with more resolution (zoom analysis or a larger transform). The flattop window is specifically designed to minimise the picket fence effect, and is thus usually the best choice when calibrating measurements with a calibration signal whose frequency can fall anywhere between two analysis lines. It can also be useful when analysing a signal dominated by one or more families of harmonics, since as long as they are resolved (keeping in mind that the noise bandwidth is  $3.7 \Delta f$ ) there is no need to compensate the indicated values of the various harmonics. Figure 3.5 shows how compensation can be made for both picket fence error and frequency error when using a Hanning window. Provided frequencies are stable along the record length, the difference in dB between the two highest samples around a frequency peak ( $\Delta \text{dB}$ ) determines the errors. As mentioned above, the Hanning window has the desirable property that the effective weighting in overlap averaging (see later) can be made completely uniform with an overlap of  $2/3$ ,  $3/4$ , etc. With 50% overlap, the weighting varies by 2:1, but this is not serious with stationary signals. When finding the averaged spectrum of a long transient signal (longer than the transform size), a uniform weighting is preferable.

### 3.2.2 Transient signals

Analysis of transient signals is common in impact measurements for modal analysis. The force signal is always short, and a rectangular window is suitable, although this may be tailored to just more than the length of the force pulse in order to exclude noise. For the response signal it must be ensured that it has died away (ie by 50 to 60 dB) by the end of the record. This can be achieved by extending the record length (ie by zoom or a larger transform size), but where this is restricted by other constraints it is common to apply an exponential

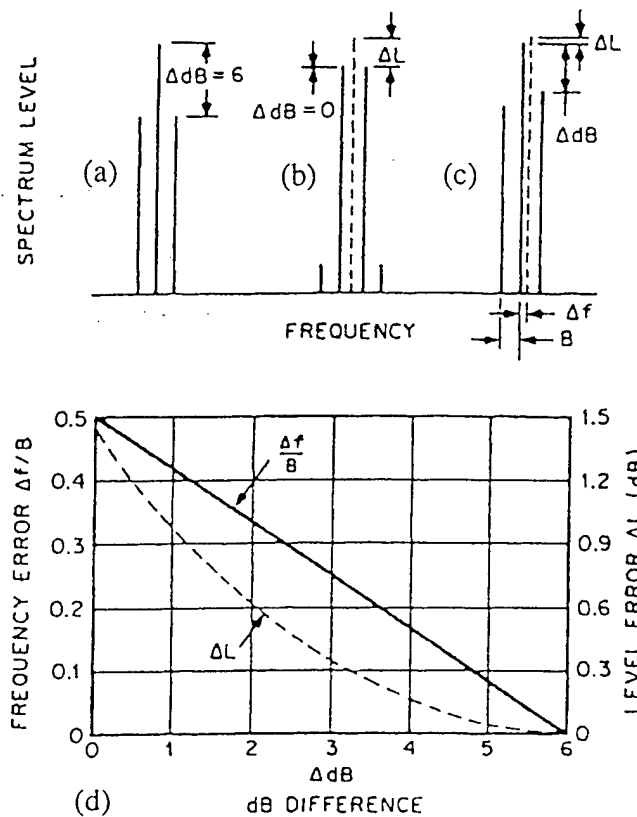


Figure 3.5 Compensation for the picket fence effect with a Hanning window.

$\Delta dB$  = difference between two highest spectrum samples.

$\Delta L$  = picket fence error.  $\Delta f$  = frequency error

(a), (b), (c) Minimum, maximum and intermediate error cases (d) Error nomogram

window, starting just before the response signal, which attenuates the signal sufficiently. This is equivalent to applying additional damping, which is known very precisely, and so can be subtracted from the resulting measurements.

### 3.2.3 Application in the frequency domain

Since multiplication by a window function corresponds in the frequency domain to a convolution with its FT, this is sometimes the most efficient way to apply them. Examples of where this is advantageous are firstly where the FT of the window is very simple, such as with the Hanning function, secondly where only a part of the spectrum is required as with zoom spectra, and thirdly where several different windows can be applied to exactly the same FT. The basic principle can be explained using the Hanning window, which when repeated periodically (as happens implicitly with the DFT) can be represented as  $\sin^2 \theta$  or

$\frac{1}{2} - \frac{1}{2} \cos 2\theta$  which has the convolution coefficients  $\left[ -\frac{1}{4}, \frac{1}{2}, -\frac{1}{4} \right]$  (keeping in mind

that the frequency corresponds to one period along the record length, and thus to one line spacing. Convolution with such a simple function is often more efficient than direct multiplication in the time domain. Note that the coefficients as stated are for a window with maximum value one, and are usually modified to scale the result (see below under "Scaling").

### 3.3 Zoom Analysis

The basic DFT transform of Equ.(2.14) extends in frequency from zero to the Nyquist frequency and has a resolution equal to the sampling frequency  $f_s$  divided by the number of samples  $N$ . Sometimes it is desired to analyse in more detail in a limited part of the

frequency range, in which case use can be made of so-called "zoom analysis". Since resolution  $\Delta f = f_s / N$ , the two ways to improve it are:

- 1) Increase the length of record  $N$ . Some analysers include this option in the form of "non-destructive zoom", which make use of an algorithm to perform a transform of size  $m \times N$  by combining the results of  $m$  undersampled transforms of size  $N$ . This was useful when hardware restrictions limited the size of transform which could be performed, but in modern analysers, and in signal processing packages such as Matlab® there is virtually no restriction on transform size, and so zoom can be achieved by performing a large transform and then viewing only part of the result.
- 2) Reduce the sampling frequency  $f_s$ . This can be done if the centre of the desired zoom band is shifted to zero frequency so that the zoom band around the centre frequency can be isolated by a lowpass filtration. The highest frequency is then half the zoom band and the sampling frequency can be reduced accordingly without aliasing problems. This process is illustrated in Fig.3.6. The lowpass filtering and resampling process is usually done in octave (2:1) steps, as a digital filter will always remove the highest octave, relative to the sampling frequency, and halving the sampling frequency simply means discarding every second sample. This type of zoom is normally done in real-time by a specialised hardware processor, the advantage being that the sampling rate is reduced before signals have to be stored. It is discussed below that the zoom process is a useful precursor to demodulation, even where the further processing is to be done in a computer. Note that the time signal output from the zoom processor is complex, as the corresponding spectrum is not conjugate even.

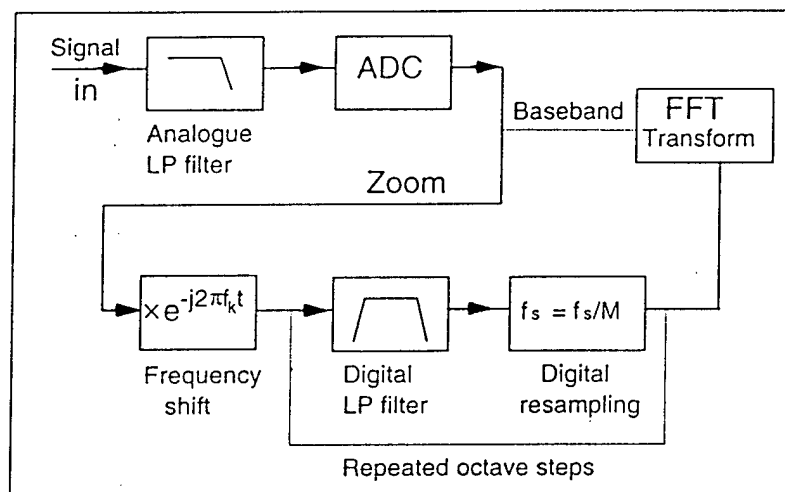


Figure 3.6. Schematic diagram of FFT zoom process

### 3.4 Spectrum Averaging

The need for averaging of FFT spectra is determined by whether the signal contains random components or not. Averaging should always be done in terms of signal power (ie amplitude squared) as it is this which is conserved independent of phase. The DFT spectra of discrete frequency components always have the same amplitude, and therefore little is achieved by averaging the squared amplitudes, although this can be useful for clarifying which components are discrete frequency and which are random. Where analysis is being done for diagnostic purposes (eg a zoom spectrum to measure a single frequency or family of sidebands very accurately) it is preferable not to average, as in practice machine speeds vary slightly with time and averaging results in a smearing of frequencies and in particular disguises frequency spacings.

When meaningful spectra are to be obtained from random signals, which in mechanical signals are typically caused by fluid flow (turbulence, cavitation) road roughness etc, it is necessary to average a number of power spectrum estimates. The number of averages required is determined by the desired accuracy, as the standard deviation of the result (for gaussian signals) is given by [2.1]:

$$\epsilon = \frac{1}{2\sqrt{n}} \quad (3.1)$$

where  $n$  is the number of independent averages. Thus, for  $n = 16$ ,  $\epsilon = 12.5\%$  or 1 dB, meaning that there is a 68% probability that the result will be within  $\pm 1$  dB, 95% probability that it will be within  $\pm 2$  dB and 99.7% probability that it will be within  $\pm 3$  dB. To halve the error it is necessary to make four times as many averages etc.

With a rectangular window, "independent" means non-overlapping, but with other windows such as Hanning, advantage can be gained by overlapping, as information is lost near the two ends where the weighting is near zero. In fact, very little is lost statistically by overlapping 50%, and so this is recommended for stationary random signals, as twice as many effective averages can be obtained from a given length of signal. As mentioned above, the overall weighting is not uniform in that case, as at the point where the successive Hanning windows overlap, their amplitude weighting is  $\frac{1}{2}$  and thus their power weighting  $\frac{1}{4}$ , meaning that the final weighting of that part of the signal in the overlap average is  $\frac{1}{2}$  and the total power weighting along the signal varies between  $\frac{1}{2}$  and unity. This gives no problem for stationary signals, but to extract all information from a given length of record, in particular if it is non-stationary, it is advisable to overlap by a factor of at least  $\frac{2}{3}$  although with typical FFT record lengths in powers of 2 it is often simpler to overlap by  $\frac{3}{4}$ . In the latter case the effective number of averages to insert in Equ.(3.1) is half the actual number. The Matlab® function PSD performs overlap averaging (as does CSD for multiple signals).

### 3.5 Scaling

#### 3.5.1 Discrete frequency signals

As stated above, the DFT operations of Equ.(2.14, 2.15) result in a correctly scaled Fourier series spectrum for the forward transform and exact reconstitution of a (periodic) time signal for the inverse transform. Note that this means that the resulting value  $A_k$  at the positive frequency  $\omega_k$  has an amplitude half that of the corresponding sinewave ( $C_k$ ). To obtain the equivalent RMS (root mean square) value,  $|A_k|$  must be multiplied by  $\sqrt{2}$  to take account of the power in the negative frequency component. The RMS value is also equal to  $C_k/\sqrt{2}$ . If

the signal contains discrete frequency components which do not have an integer number of periods along the record length, then a window function such as Hanning will generally be used to reduce leakage. However, if the window is scaled to a maximum value of unity, the average "power" (ie mean square value) of the signal will obviously be reduced, and it is necessary to compensate for this. From section 3.2.3 it will be seen that the convolution

coefficients for a Hanning window scaled to a maximum value of 2 are  $\left[ -\frac{1}{2}, 1, -\frac{1}{2} \right]$

meaning that a single component in one line would be replaced by three components of which the central one would have the same power (ie the peak value would be scaled correctly). This



is usually the best scaling to use for discrete frequency components as it means that the maximum value around a spectral peak can be read off directly after correction for picket fence error (eg using Fig. 3.5). For windows other than Hanning, the same effect can be achieved by scaling the window such that its central convolution coefficient is unity. Note that because of the extraneous sidebands introduced, the total "power" in the spectrum has been increased as discussed below.

### 3.5.2 Parseval's theorem

At this stage it is convenient to introduce Parseval's theorem which in broad terms states that the total power (or energy) in a signal can be obtained by integrating over all time or all frequency, and in both domains is related to amplitude squared. For a stationary signal with finite power, the frequency spectrum will either contain discrete frequency components whose amplitude squared directly represents the power at each frequency, or for random signals the squared amplitude spectrum is continuously distributed over frequency and represents "power spectral density" (PSD) which has to be integrated over a finite bandwidth to give finite power. In both cases the equivalent "power" in the time domain is the mean square value, obtained by integrating the instantaneous squared value (instantaneous power) over a sufficiently long time and dividing by that time. For transient signals with finite "energy" (integral of "power" over time) the squared amplitude of its Fourier transform represents "energy spectral density" (ESD) which when integrated over all frequency gives the same total energy as integrating the instantaneous power of the signal over all time.

Henceforth, the terms power and energy will be used without inverted commas to represent signal amplitude squared and its time integral, respectively, as these are generally related to physical power and energy by an impedance or admittance function (eg electrical power  $= I^2 R = V^2/R$  in terms of current  $I$ , voltage  $V$  and resistance  $R$ ). Thus for a signal with units  $U$  (where  $U$  represents  $m$ ,  $ms^{-1}$ ,  $ms^{-2}$ ,  $g$ ,  $N$  etc) power has units  $U^2$ , energy has units  $U^2s$ , PSD has units  $U^2/Hz (= U^2s)$ , ESD has units  $U^2s/Hz (= U^2s^2)$ .

### 3.5.3 Stationary random signals

Each signal record transformed will be treated by the DFT algorithm as a periodic signal, but the power in each spectral line can be assumed to represent the integral of the PSD over the frequency band of width  $\Delta f (= 1/T)$ , and thus the average PSD is obtained by multiplying the squared amplitude by  $T$ . The required averaging over a number of records does not change this scaling. How well the average PSD represents the actual PSD depends on the width of peaks (and valleys) in the spectrum. The width of such peaks is typically determined by the damping associated with a structural resonance excited by the broadband random signal, and the 3dB bandwidth is given by twice the value of  $\sigma$  (of Equ.(2.19)) expressed in Hz. The PSD will be sufficiently accurate if the 3dB bandwidth is a minimum of five analysis lines.

If a window such as Hanning has been used to reduce leakage, and if it is scaled so as to read the peak value of discrete frequency components (as recommended in 3.5.1) the calculated PSD value will have to be divided by the "noise bandwidth" indicated in Table 1 to compensate for the extra power given by the spectral sidebands. This noise bandwidth is the sum of the squares of the convolution coefficients (for Hanning it is  $0.5^2 + 1^2 + 0.5^2 = 1.5$ ). When integrating over several frequency lines (eg to convert a constant bandwidth spectrum to constant percentage bandwidth) the total power in each integrated band must be divided by the noise bandwidth of the window because of the extra power associated with each line; this is the same as integrating the PSD over the required bandwidth.

Note that discrete frequency components cannot be represented on a PSD scale as they are concentrated in an infinitely narrow bandwidth and thus have infinite PSD. Note also that their power is independent of the analysis bandwidth  $\Delta f$  used to analyse them, whereas the power of spectral lines of random signals varies directly with the analysis bandwidth. Zoom analysis is sometimes used to make discrete frequency components stand out from random background noise.

#### 3.5.4 Transient signals

Transient signals are also treated as being one period of a periodic signal, so not only does the power in a spectral line have to be converted to an average spectral density by dividing by  $\Delta f$ , but also the average power must be converted to energy per period by a further

multiplication by  $T$ , altogether a multiplication by  $T^2$  to obtain a result scaled as ESD. Generally, transient signals will be shorter than the transform length and thus a rectangular window will be used, and if the signal has decayed to near zero at the end of the record (eg following the recommendations of 3.2.2) the signal bandwidth will be sufficiently greater than the analysis bandwidth for the average ESD to represent the true ESD. The extra damping given by an exponential window will genuinely give a reduction in signal energy.

### 4. OTHER SPECTRAL ANALYSIS TECHNIQUES

#### 4.1 Parametric Spectral Analysis

With Fourier analysis, as is evident from Fig.3.1(d), the spectral resolution is determined by the Fourier transform of the time window which limits temporal resolution. For a window of length  $T$  the spectral resolution is of the order of  $1/T$ , and thus the better the time localisation the poorer the frequency localisation, and vice versa. This is one expression of Heisenberg's uncertainty principle, and is because no assumption is made about the behaviour of the time function outside the window (effectively it is set to zero, which is extremely improbable).

With parametric spectral analysis [4.1, 4.2], better spectral resolution can be obtained for short records, basically because it assumes that the behaviour of the function outside the window is most similar to its behaviour inside the window. This is valid for sinusoidal or near sinusoidal signals. With parametric analysis, the signal is modelled as the output of a physical system described by a limited number of parameters when excited by a unit white noise input. Thus the frequency response of the system represents the signal spectrum. Generally, the improvement in spectral resolution is accompanied by a deterioration in amplitude accuracy.

##### 4.1.1 MA models

Perhaps the easiest case to understand is where the system is modelled as an FIR (finite impulse response) filter, in which case the output is the (digital) convolution of the input signal with the finite length impulse response of the filter, as expressed by the equation:

$$y_i = \sum_{k=0}^M b_k x_{i-k} \quad (4.1)$$

where  $x_i$  represents the input signal,  $y_i$  represents the output signal, and the  $b_k$  represent the convolution weights or samples of the impulse response. Equ.(4.1) is a digitised, finite length version of the convolution equation (2.17) and is effectively a "moving average", giving rise to the term MA model. Applying a Z-transform to Equ.(4.1), which is the equivalent of a Laplace transform for discrete time signals, the convolution becomes the product:

$$Y(z) = \sum_{k=0}^M b_k z^{-k} X(z) = B(z) X(z) \quad (4.2)$$

from which the transfer function can be seen to be:

$$B(z) = \sum_{k=0}^M b_k z^{-k} = b_0 \prod_{k=1}^M (1 - z^{-1} z_k) \quad (4.3)$$

which has no poles and is thus an "all-zero" model.

This type of model is obviously most efficient when the effective length of the impulse response is short, meaning that it is highly damped and thus without sharp spectral peaks.

#### 4.1.2 AR models

AR or "autoregressive" models are more efficient where there are sharp spectral peaks, and thus the required transfer function has poles. This is the case with IIR (infinite impulse response) filters where outputs are generated recursively from the previous outputs and the current input.

The relationship between input and output signals can be expressed as:

$$y_i = -\sum_{k=1}^N a_k y_{i-k} + x_i \quad (4.4)$$

where in principle  $N \rightarrow \infty$  but can be truncated when the terms become sufficiently small. After Z-transformation this gives:

$$Y(z)A(z) = X(z) \quad (4.5)$$

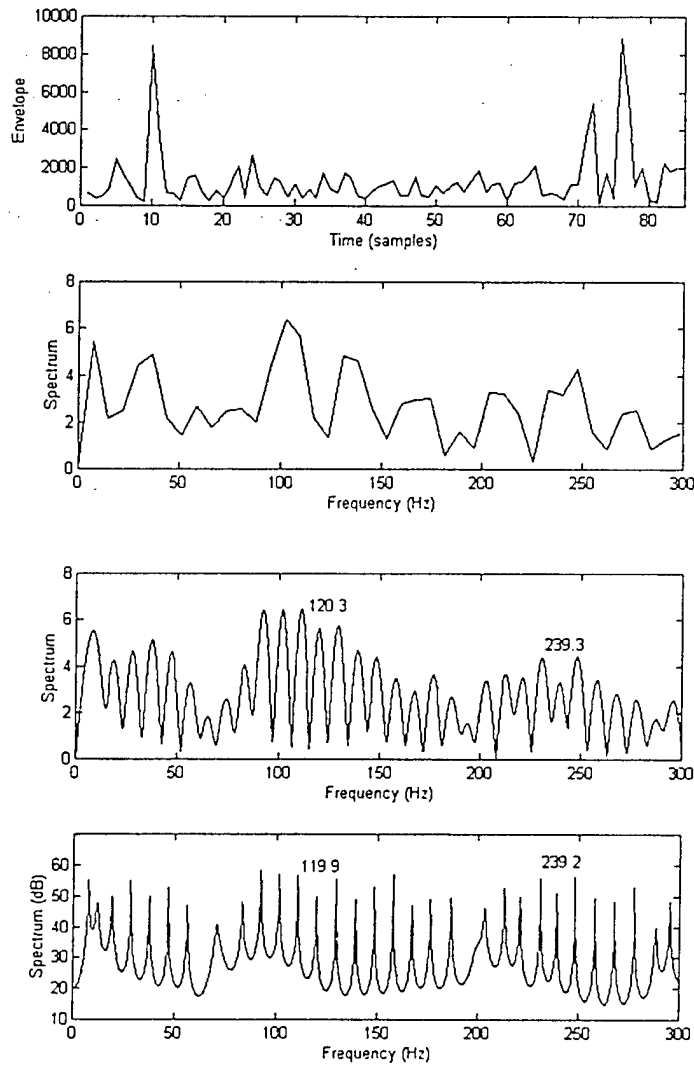
from which the transfer function can be seen to be:

$$1/A(z) = 1/\sum_{k=0}^N a_k z^{-k} = 1/\prod_{k=1}^N (1 - z^{-1} p_k) \quad (4.6)$$

which has no zeros and is an all-pole model.

There are a number of techniques which result in such an AR model, one of which is the "maximum entropy" method mentioned above. In this, the coefficients are found by maximising the entropy (disorder) of the signal, while ensuring that the autocorrelation function is determined by the signal within the window. This really means that the signal outside the window will be most similar to the signal within the window because of the elimination of any biasing effect. Other AR techniques include "linear prediction" used in speech analysis, and statistical "autoregression" from which it takes its name.

Figure 4.1 shows the results of applying maximum entropy analysis to a very short record of envelope signal from a bearing with an inner race fault [4.3]. The record length comprised only 1.29 revolutions of the shaft speed which determined the spacing of modulation sidebands in the envelope spectrum. The maximum entropy spectrum of Fig.4.1(d) appears to give very good resolution of the sidebands, but Fig.4.1(c) shows that Fourier analysis can give almost as much information provided a sufficient degree of spectrum interpolation is used. The spectrum interpolation was achieved by padding the data record with zeros to 7 times its original length. Note that the maximum entropy spectrum had to be represented on a logarithmic amplitude scale because of the much wider range of amplitude values than for the Fourier analysis cases.



*Figure 4.1 Envelope spectra of short time recording (1.29 revolutions) just spanning the first two sections where the inner race fault passes the loading zone. [4.3]*  
 (a) Envelope signal after bandpass filtration in frequency range 2.7 ~ 3.3 kHz,  
 (b) Envelope spectrum without interpolation, (c) Envelope spectrum with interpolation, (d) Maximum entropy envelope spectrum

AR models have been found useful for characterising a signal with a minimum number of parameters, eg to use as a feature vector for input to a neural network to distinguish different cases. Where signals are nonstationary, the concept of “evolutionary” AR models [4.4] may be useful. In this case the AR parameters are updated by moving a finite length window along the record. In [4.4] an evolutionary AR model gave a better separation of faults in reciprocating machine vibration signals.

#### 4.1.3 ARMA models

Where the spectrum contains sharp peaks and valleys, it may be preferable to model it with an ARMA model which contains both poles and zeros. The relationship between input and output signals is then:

$$y_i = \sum_{k=0}^M b_k x_{i-k} - \sum_{k=1}^N a_k y_{i-k} \quad (4.7)$$

for which the transfer function is:

$$B(z)/A(z) \quad (4.8)$$

with both poles and zeros. In general it is much more onerous in terms of computational effort to fit an ARMA model than either an AR or MA model, although this may be compensated by a reduction in the number of parameters required to obtain a good fit. Note that software to fit these models is available in signal processing packages such as the Matlab® System Identification Toolbox.

#### 4.1.4. Other models

For vibration measurements made when the structure is in "free decay" after being excited, it is natural to fit a number of complex exponentials corresponding to the impulse responses of the various modes. One method which achieves this is the ITD method [1.6] referred to in the Introduction while another is the Prony method described in [4.1]. These should give a result similar to an ARMA model, in that the transform of a sum of complex exponentials results in a ratio of polynomials with poles and zeros.

### 4.2 Digital Filters

Section 4.1 discussed one application of digital filters, with both infinite and finite impulse responses, to model a system which would produce a measured signal when excited by a white noise source. Digital filters can also be used to frequency analyse a signal in the same way as an analogue filter, ie by measuring the power transmitted by a series of filters covering the required frequency band. Where analysis in constant bandwidths is required, this is most efficiently done using the FFT algorithm, but when constant percentage bandwidth is required it is most efficiently done using a series of 1/n-octave digital filters, the different octaves being covered by repetitively halving the sample rate. There are two ways in which the properties of a digital filter are varied:

- 1) By varying the filter coefficients (either FIR or IIR)
- 2) By varying the sample rate, as for given coefficients the frequency characteristic is proportional to the sampling frequency.

Thus for 1/3-octave analysis, for example, three bandpass filters are required (ie three sets of coefficients) covering the octave from 33% to 66% of the Nyquist frequency. The signal is sampled at a rate corresponding to the highest octave (ie the upper cutoff frequency of the upper 1/3-octave at 66% of the Nyquist frequency) and is passed through the three 1/3-octave filters and a digital lowpass filter cutting off at 40% of the current Nyquist frequency. The sample rate of the lowpass filtered signal can then be halved (by discarding every second sample) after which passage through the same digital filters will produce the filter outputs corresponding to the next lower octave in actual frequency. By repeating this process, as many octaves can be covered as desired. Note that by being able to perform the filtering calculations twice as fast as required for the upper octave, it is possible to process any number of octaves in real-time, as each time the sample rate is halved only half as many samples have to be processed in a given time. If the time taken to process the upper octave is taken to be unity the time taken to process all octaves is  $\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right] = 2$ .

This process is best done in real-time by specialised hardware, but can be done as post-processing using a signal processing package. However, in general the record to be post-processed would have to be extremely long if the analysis is to cover, say, three decades or more, with a reasonable amount of averaging. For example, for a 1/12-octave (6% bandwidth) analysis over three decades (eg 20 Hz - 20kHz) of a random signal for which a BT product (equivalent to  $n$  in Equ.(3.1)) of 16 is required, the filter output would have to be averaged over  $16 \times 80 = 1280$  samples of the lowest filter sample rate, corresponding to 640,000

samples of the original record.

The power of each filter output is measured by squaring it, and either calculating the average over a defined time period, or smoothing it with an "exponential averager", a first order lowpass filter (which can also be applied as a digital filter in real-time to give a running average). The equivalent linearly weighted averaging time is twice the time constant of the first order filter.

#### **4.3 Order Tracking**

In analysing rotating machine vibrations it is often desired to have an x-axis based on harmonics or "orders" of shaft speed. This can be to avoid smearing due to speed fluctuations or can be to see how the strength of the various harmonics changes over a greater speed range, for example as they pass through various resonances. If a constant amplitude signal which is synchronous with the rotation of a shaft, for example, is sampled a fixed number of times per revolution, the digital samples are indistinguishable from those of a sinusoid, and thus give a line spectrum, whereas if normal temporal sampling is used the spectrum spreads over a range corresponding to the variation in shaft speed. Thus, for order analysis it is necessary to generate a sampling signal from a tacho signal synchronous with shaft speed. It is sometimes possible to use a shaft encoder mounted on the shaft in question to provide a sampling signal, but more often the latter has to be generated electronically. Formerly, this was done using a phase-locked loop to track the tacho signal and then generate a specified number of sampling pulses per period of the tracked frequency. However, an analogue phase-locked loop has a finite response time and cannot necessarily keep up with random speed fluctuations such as occur with an internal combustion engine from cycle to cycle. The best method is to digitally resample each record based on the corresponding period of the tacho signal. This can be done in a number of ways, based on digital interpolation. One way is simply to increase the sample rate of each section by say a factor of 10, and then select the nearest sample to the theoretical interpolated position. Increasing the sample rate by an integer factor can be achieved by inserting the appropriate number of zeros in between each actual sample, and then applying a digital lowpass filter to limit the frequency range to the original maximum, thus smoothing the curve (it will also require rescaling proportional to the resampling factor). Resampling by a factor of 4 is illustrated in Fig.4.2. The same result can be achieved using the FFT by padding the spectrum with zeros in the centre (ie around the Nyquist frequency) and then inverse transforming the increased (2-sided) spectrum to the same increased number of time samples. Note that the record length in seconds is the reciprocal of the frequency line spacing in Hz which is not affected by the zero padding. This latter procedure can also be used to resample a record consisting of an integer number of samples to another (though greater) integer number, and is the basis of the Matlab® function INTERPFT. In general, more accurate interpolation, not limited to a ratio of integer numbers, can be achieved by fitting a curve to a group of samples (eg two for a linear curve, three for a quadratic etc) and then calculating the value of the polynomial at the interpolated positions. The accuracy of the interpolation can be judged by considering that the interpolation in the time domain corresponds to a multiplication in the frequency domain by a filter characteristic which aliases back into the measurement range. For example, choosing the nearest sample value is the same as convolving the original samples with a rectangular function of width equal to the sample spacing, while a linear interpolation corresponds to a convolution with a triangular function of base width twice the sample spacing (the convolution of the rectangular function with itself). In the former case the filter characteristic is a sinc(x) function with zeros at multiples of the sampling frequency (so that all sidelobes fold back into the measurement

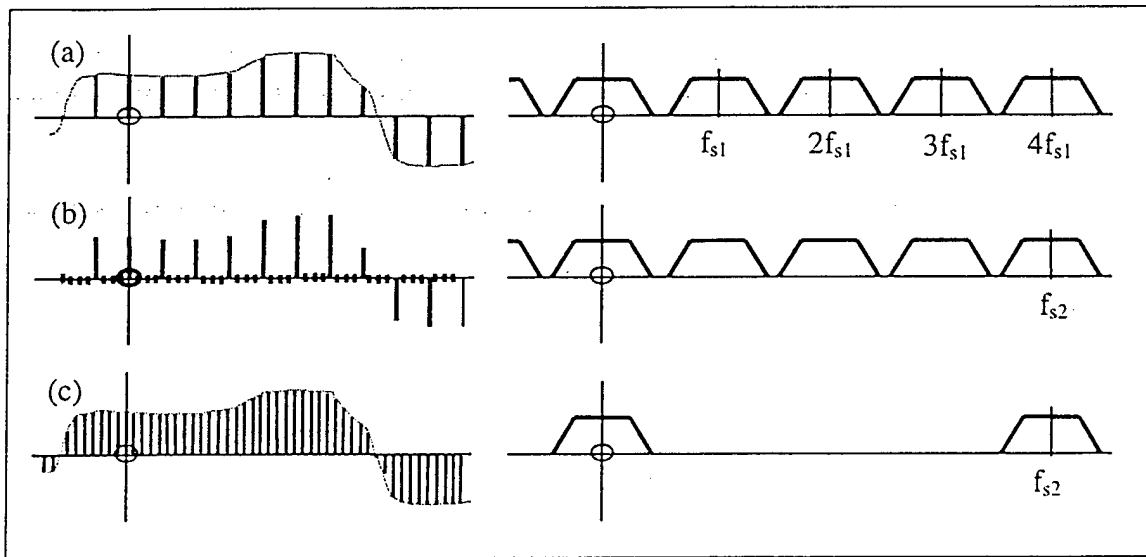


Figure 4.2. Digital resampling with four times higher sampling frequency  
 (a) Signal sampled at  $f_{s1}$  and its spectrum (b) Addition of zeros which changes sampling frequency to  $f_{s2}$  (c) Lowpass filtration and rescaling

range), while in the latter case it is the square of the  $\text{sinc}(x)$  function, which has much smaller sidelobes. In practice, cubic interpolation involving two samples on each side of a central one gives good results without excessive computational effort [4.5].

Quite apart from errors introduced by the interpolation, when resampling at a lower frequency (for example as a machine speed reduces), it is necessary to ensure that the signal is adequately lowpass filtered to prevent aliasing. Digital filtering can be useful here as the cutoff frequency varies directly with the sampling frequency, but the initial analogue lowpass filtration must be such that aliasing components do not enter the measurement range. Digital oversampling can solve this problem, as from Fig.4.2(c) it can be seen that the sampling frequency can be reduced by a large factor before overlap occurs. In fact, the factor is 5.9 in this case because a tracking digital filter cutting off at a particular shaft order proportionally reduces the useful band in terms of frequency.

Figure 4.3 illustrates the use of tracking to avoid smearing in the spectrum of the vibration signal from a gearbox in a variable speed mining shovel. The discrete frequency components in the spectrum after tracking come mainly from gear-related components which were removed using synchronous averaging as described in the next section.

## 5. TIME DOMAIN ANALYSIS

### 5.1 Time Synchronous Averaging

Synchronous averaging is useful to extract that part of a signal which is periodic with the same period as a trigger signal, eg a once-per-rev tacho signal from a shaft in a rotating machine. In practice it is done by averaging together a series of signal segments each corresponding to one period of the synchronising signal. Thus:

$$y_a(t) = 1/N \sum_{n=0}^{N-1} y(t + nT) \quad (35)$$

This can be modelled as the convolution of  $y(t)$  with a train of  $N$  delta functions displaced by integer multiples of the periodic time  $T$ , which corresponds in the frequency domain to a

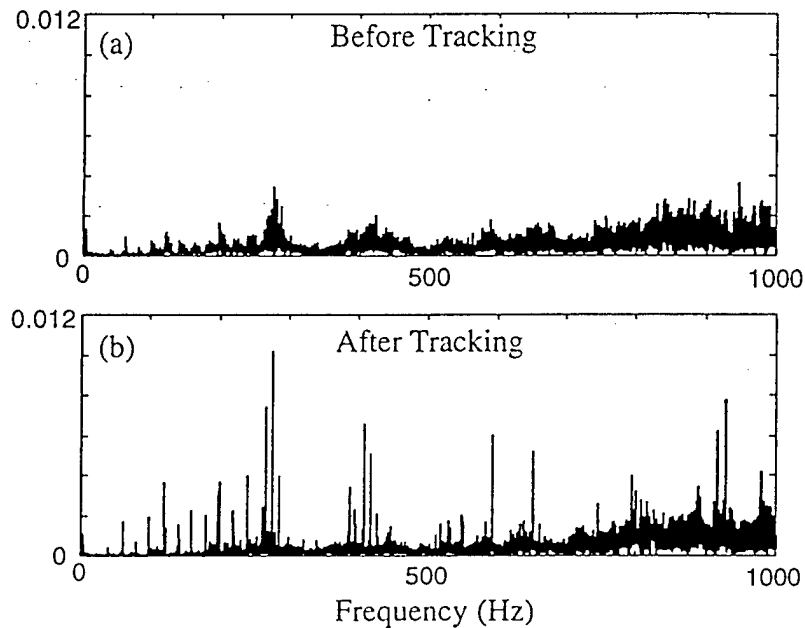


Figure 4.3. Use of tracking to avoid smearing of shaft speed related components

multiplication by the Fourier transform of this signal, which can be shown to be given by the expression [4.1, 5.1]:

$$C(f) = 1/N \sin(N\pi Tf) / \sin(\pi Tf) \quad (36)$$

The filter characteristic corresponding to this expression is shown in Fig.(5.1) for the case where  $N = 8$ , and is seen to be a comb filter selecting the harmonics of the periodic frequency. The greater the value of  $N$  the more selective the filter, and the greater the rejection of non harmonic components. The noise bandwidth of the filter is  $1/N$ , meaning that the improvement in signal/noise ratio is  $10 \log_{10} N$  dB for additive random noise. For masking by discrete frequency signals, it should be noted that the characteristic has zeros which move with the number of averages, so it is often possible to choose a number of averages which completely eliminates a particular masking frequency. The above characteristic is for an infinitely long time signal  $y(t)$ , and in Ref.[5.1] it is shown that for the practical situation of a finite length of signal with finite sampling frequency, it is possible to calculate an optimum number of averages to completely remove a discrete masking signal, in particular when the frequency is related by a rational fraction to the synchronous frequency. This is always the case for different shafts in gearboxes.

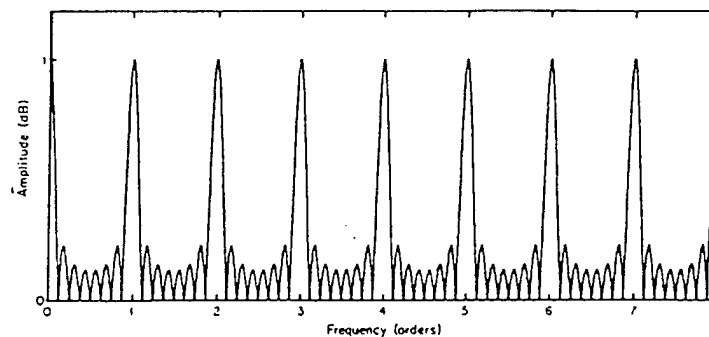
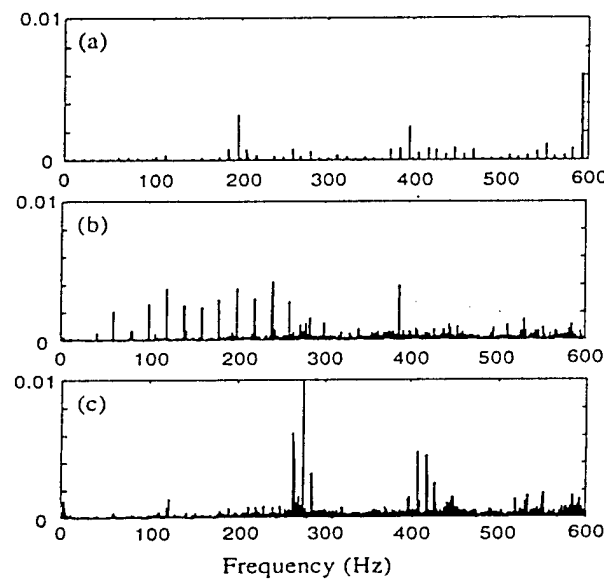


Figure 5.1. Filter characteristic corresponding to 8 synchronous averages (from [5.1])



For good results the synchronising signals should correspond exactly with samples of the signal to be averaged, as one sample spacing corresponds to  $360^\circ$  of phase of the sampling frequency, and thus to  $144^\circ$  of phase at 40% of it which is a typical maximum signal frequency. Moreover, even a 0.1% speed fluctuation would cause a jitter of the same order of the last sample in a (typical) 1K record, with respect to the first, and thus an even greater loss of information at the end of the record, after averaging.

Sampling the signal using a sampling frequency derived from the synchronising (tacho) signal, as described in Section 4.3, solves both these problems and is always to be recommended. Figure 5.2 shows the results of using synchronous averaging on the data of Fig.4.3. The order tracked data was arranged to have an integer number of samples per period of the low speed gear, which allowed determination of the harmonics of this gear speed by synchronous averaging. The spectrum of this signal is shown in Fig.5.2(a). After a periodic repetition of this signal was subtracted from the overall tracked signal (Fig.4.3(b)) the data was resampled to have an integer number of samples per period of the high speed gear, after which its harmonics could be determined in the same way (Fig.5.2(b)). Finally, after subtraction of this periodic signal from the data, the remaining signal was dominated by the effects of an inner race bearing fault (Fig.5.2(c)).



*Figure 5.2. Application of synchronous averaging to data of Fig.4.3.  
 (a) Spectrum synchronous with low speed gear (b) Spectrum synchronous with high speed gear (c) Spectrum dominated by bearing fault after effects of two gears removed*

## 5.2 Autocorrelation Analysis

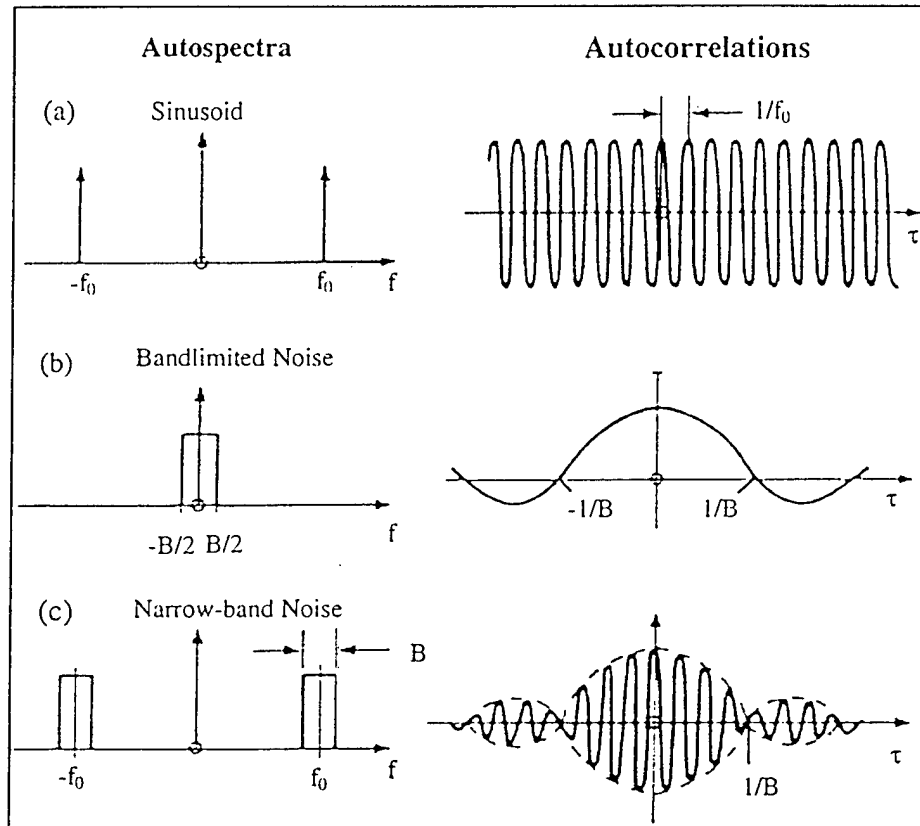
The autocorrelation function measures how well a signal correlates with delayed versions of itself. The equation for calculating it for a transient is:

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt \quad (5.3)$$

while for a stationary function (for which the above integral would be infinite) it is:

$$R_{xx}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)dt = E[x(t)x(t+\tau)] \quad (5.4)$$

For  $\tau = 0$ , Equ.(5.3) gives the total energy, and Equ.(5.4) the mean square value or power, and it is common to normalise the autocorrelation function to a maximum value of unity by dividing through by this value (the correlation cannot be better than the perfect correlation with itself). The Wiener-Khinchin relationship states that the autocorrelation function is the inverse Fourier transform of the power (amplitude squared) spectrum, even for a stationary random function. This can easily be understood for a transient (with power spectrum replaced by energy spectrum) by comparing Equ.(5.3) with (2.17) where it can be seen that the autocorrelation represents a convolution with the same function reversed in time, so that its spectrum is the product of the original spectrum with its complex conjugate, and thus is the squared amplitude or energy spectrum.



**Figure 5.3.** Autocorrelation vs autospectrum for three signals.  
Note that spectrum of (c) is the convolution of (a) and (b)

Figure 5.3 uses this relationship to derive the autocorrelation for a sinusoid and a bandlimited noise, and then a narrow-band noise as its spectrum can be generated as the convolution of the first two. This shows that the narrower the bandwidth of a noise signal, whether or not it is shifted from zero frequency, the more its autocorrelation is spread out in time. On the other hand, for a white noise, whose spectrum extends uniformly to infinity, its autocorrelation is concentrated at zero time lag.

The autocorrelation function can be used to enhance periodicity, as it separates this from broadband noise, and converts all sinusoids to cosines. This means that periodic functions become more pulse-like and thus apparent, since the phase of all harmonics is aligned once per period. The autocorrelation function can also be used to detect echos, but this is limited by the abovementioned spreading out of narrow-band functions, since even a perfect echo will

just reproduce the autocorrelation at a lag corresponding to the echo delay time. As seen below, the cepstrum is a better echo detector, as in principle the echo delay is given by a delta function, independent of the bandwidth.

Figure 5.4 shows a case where masking noise was reduced considerably in an envelope spectrum for a bearing fault, by clipping the autocovariance function near zero time lag. The autocovariance is the autocorrelation performed after removing the mean value, which gives considerable distortion in an all-positive envelope signal. A further squaring of the autospectrum, corresponding to a further autocorrelation operation on the autocorrelation function, gives even better rejection of the noise but at the same time enhances large discrete peaks at the expense of small ones.

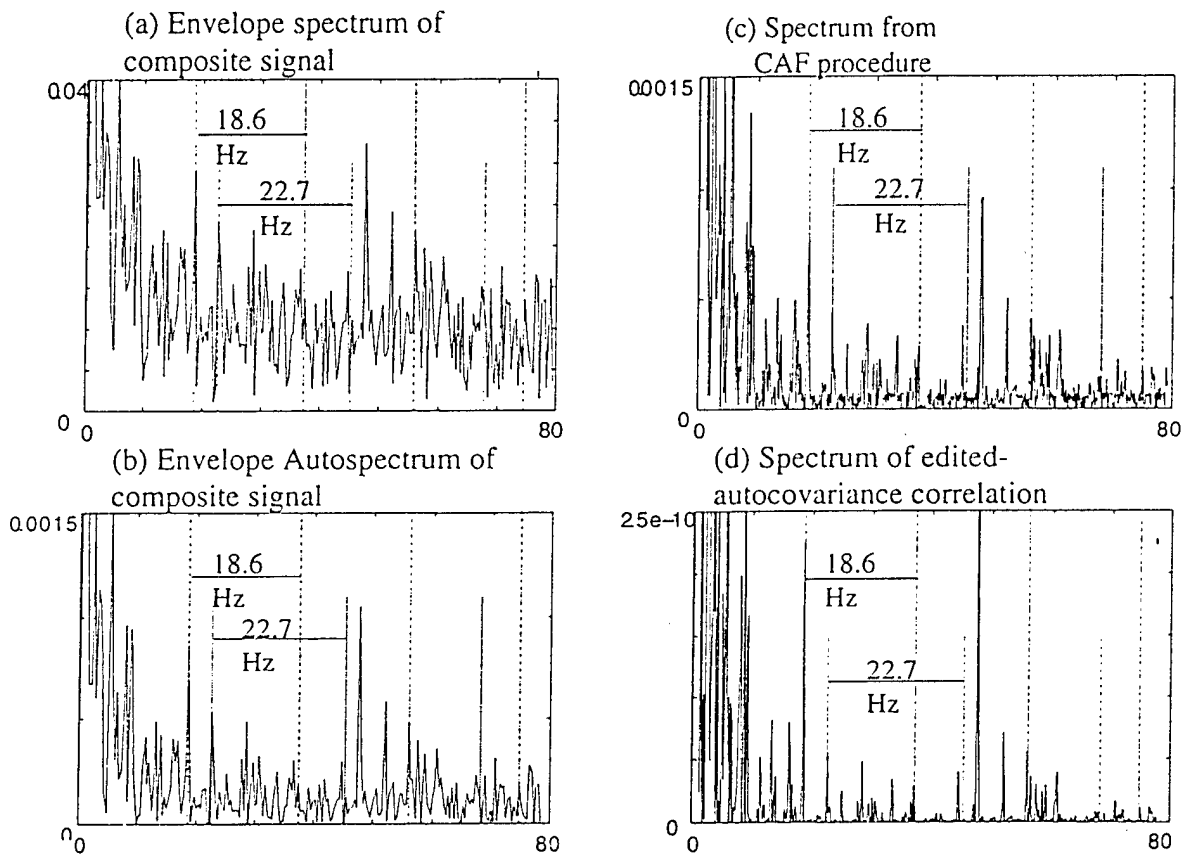


Figure 5.4. Use of CAF (clipped autocovariance function) (c) and its squared spectrum (d) to reduce noise effects

### 5.3 Cepstrum Analysis

The cepstrum is usually calculated from the spectrum, and often used to extract information about a spectrum, but will be treated here as a time domain technique as its x-axis is time, and it is closely related to the autocorrelation function just discussed. The most general definition of the cepstrum is as the inverse Fourier transform of a log spectrum, or:

$$C(\tau) = \mathcal{F}^{-1}[\log(X(f))] \quad (5.5)$$

where

$$X(f) = \mathcal{F}[x(t)] = A(f)\exp(j\phi(f)) \quad (5.6)$$

in terms of its amplitude and phase, so that:

$$\log(X(f)) = \ln(A(f)) + j\phi(f) \quad (5.7)$$

When  $X(f)$  is complex as in this case, the cepstrum of Equation (5.5) is known as the “complex cepstrum”, although since  $\ln(A(f))$  is even and  $\phi(f)$  is odd, the complex cepstrum is real-valued. When the power spectrum is used to replace the spectrum  $X(f)$  in Equation (1), the resulting cepstrum, known as the “power cepstrum” or “real cepstrum”, is given by:

$$C_{xx}(\tau) = \mathfrak{F}^{-1}[2\ln(A(f))] \quad (5.8)$$

and is thus a scaled version of the complex cepstrum where the phase of the spectrum is set to zero. It can also be seen from Equ.(5.5) and (5.8) that the difference between the cepstrum and the autocorrelation function is given by taking the log of the amplitude squared spectrum. Since forcing function and transfer function effects are multiplied in both the complex and power spectra, the log converts this to an additive relationship, which remains in the cepstrum. Moreover, the two additive components are often better separated in the cepstrum than in the spectrum.

There is yet another type of cepstrum known as the “differential cepstrum”, which is defined as minus the inverse Z-transform of the derivative of the log spectrum, or:

$$C_d(n) = -Z^{-1}\left[z \frac{d}{dz}(\log\{X(z)\})\right] \quad (5.9)$$

and can be directly calculated from the time signal as:

$$C_d(n) = \mathfrak{F}^{-1}\left[\frac{\mathfrak{F}\{n x(n)\}}{\mathfrak{F}\{x(n)\}}\right] \quad (5.10)$$

where the Fourier transforms are to be interpreted as evaluating the Z-transform on the unit circle. One advantage is that it is not then necessary to unwrap the phase of the spectrum to a continuous function as is the case with the complex cepstrum.

Where the frequency spectrum  $X(f)$  in Equ.(5.5) is a frequency response function (FRF) which can be represented in the Z-plane by a gain factor  $K$  and the zeros and poles inside the unit circle,  $a_i$  and  $c_i$ , respectively, and the zeros and poles outside the unit circle,  $1/b_i$  and  $1/d_i$ , respectively, (where  $|a_i|, |b_i|, |c_i|, |d_i| < 1$ ) then it can be shown that the complex cepstrum is given by the analytical formulae:

$$\begin{aligned} C(n) &= \ln(K) & , n = 0 \\ C(n) &= -\sum_i \frac{a_i^n}{n} + \sum_i \frac{c_i^n}{n} & , n > 0 \\ C(n) &= \sum_i \frac{b_i^{-n}}{n} - \sum_i \frac{d_i^{-n}}{n} & , n < 0 \end{aligned} \quad (5.11)$$

in terms of the time index  $n$  (known as quefrency). When grouped in complex conjugate pairs, these terms represent exponentially damped cosines, further weighted by the hyperbolic function  $1/n$ . For the differential cepstrum, the differentiation in the frequency domain results in a multiplication by  $n$  in the quefrency domain, giving exponentially damped cosines which are mathematically similar to modal impulse responses. For minimum phase FRFs, there are no poles or zeros outside the unit circle, and the cepstrum becomes causal (positive quefrency only), so that the real and imaginary parts of its Fourier transform (the log amplitude and phase of the spectrum, respectively) are related by a Hilbert transform. This

means that the phase does not have to be measured or unwrapped, as the complex cepstrum can be obtained from the real cepstrum by multiplying by 2 times the Heaviside function  $H(n)$ .

The cepstrum was originally proposed by Bogert et al [5.2] to detect echos, as it can be shown that an echo manifests itself in both the log amplitude and phase spectra as a periodic function and thus transforms to a series of delta functions (known as "rahmonics"), with a spacing equal to the echo delay time, in the cepstrum. Figure 5.5 illustrates this for a signal with two evenly spaced echos, and shows how the latter may be removed, even when they overlap the original function. This leads to one set of applications for the cepstrum.

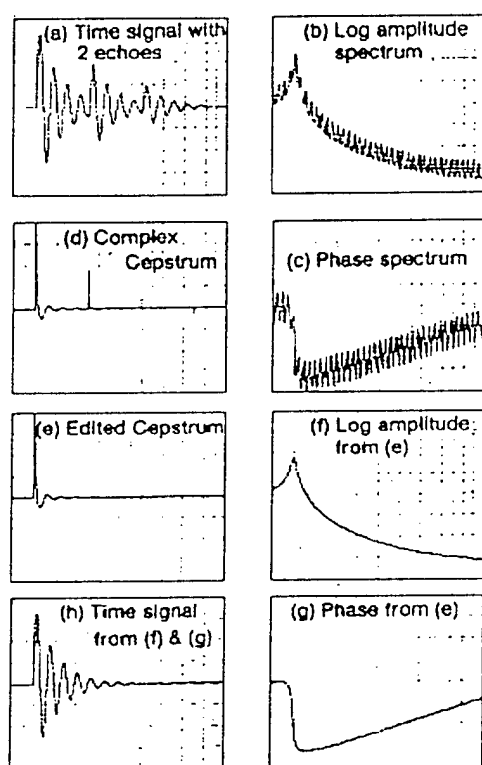
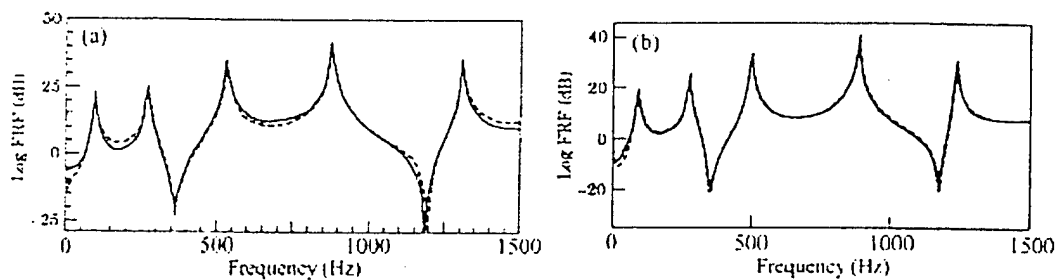


Figure 5.5. Use of complex cepstrum to remove echos

Taking the logarithm of the spectrum often enhances the pattern of equally spaced spectral components, such as harmonics and sidebands, in for example signals from gearboxes, where on a linear scale only the very largest would be visible. The cepstrum condenses such patterns into a small number of rahmonics and makes for easier comparison [5.3]. Recently it has been shown that the two sets of rahmonics produced by two gears in mesh are complementary in that the sum of the values of the first rahmonic in each series is a constant equal to 0.5 in the absence of noise, or somewhat less with noise [5.4]. If the condition of one gear deteriorates its component will increase, but is accompanied by a corresponding decrease in the other. This leads to a very general and sensitive indicator of change which is only dependent on the signal/noise ratio. This research has also led to a technique to detect and locate spalled gear teeth [5.5].

Because the forcing function and transfer function are additive in the response cepstrum, at least in SIMO situations, it is possible to separate them in response measurements if the log spectrum of the excitation is smooth and flat, so that the corresponding cepstrum is short. In [5.6] it was shown how the poles and zeros of the FRF could be extracted from the response

cepstrum, in a windowed range of frequency with negligible force effect. To use this information to reproduce the FRF requires an overall scaling factor, and an equalisation curve to compensate for the effects of unmeasured out-of-band modes. In [5.6] it was shown that these could be determined from an initial measurement, or a reasonably accurate finite element model, even when actual natural frequencies were different by up to 10%. Figure 5.6 illustrates this for two cases, one where a number of FRFs for a beam were determined by combining the poles and zeros from the response cepstrum with a scaling and equalisation function from an FEM model, and another where this was further extended by tracking the changes due to milling a slot in the middle of the beam. In [5.6] the measurements were made using impulsive excitation with very little background noise, but the method has recently been extended [5.7] to a case where broadband random excitation was used in the presence of other excitations. The response to the dominant excitation could be extracted by singular value decomposition provided it was larger by a factor of four than the next largest force.



*Figure 5.6. FRFs regenerated from response cepstra (full) compared with direct measurement (dashed) for impact measurements on a beam.  
 (a) Using an FEM model for scaling and equalisation  
 (b) After milling a slot in the middle of the beam  
 (note reduction in frequency of odd-numbered modes)*

## 6. OTHER SIGNAL ANALYSIS TECHNIQUES

### 6.1 Demodulation

Modulation occurs when an otherwise sinusoidal signal, a so-called carrier signal, has its amplitude or frequency made to vary with time. In the first case it is known as amplitude modulation, and in the second it can be considered as a frequency or phase modulation. Phase modulation is the deviation in phase (angular displacement) from the linearly increasing phase of the carrier, while frequency modulation is the difference in instantaneous frequency (angular velocity) from the constant carrier frequency. Thus, frequency modulation is the derivative of phase modulation. A direct mechanical example of phase/frequency modulation is shaft torsional vibration, which when expressed in terms of shaft angle is a phase modulation, and when in terms of shaft speed is a frequency modulation. There is no modulation term for the angular acceleration obtained by further differentiation. A mechanical example of amplitude modulation is the variation in vibration amplitude at the meshing frequency in a gearbox, as the increase in tooth deflection with load gives an increasing departure from ideal involute profiles, and often tooth load varies periodically with the rotation of the gears.

The signals produced by faults in rolling element bearings are a series of high frequency bursts as resonance frequencies are excited by near periodic impacts. The diagnostic information is contained in the repetition frequency, not in the resonance frequencies excited, but spectra obtained by direct Fourier analysis are dominated by the latter, and the important

information is disguised by smearing of the high order harmonics. Such signals can be modelled as an amplitude modulation of a carrier signal at the resonance frequency by a near periodic series of exponential pulses (though in general there will also be a jump in phase at the start of each new pulse). In so-called "envelope analysis" the signal envelope is extracted by amplitude demodulation, and frequency analysed to reveal the repetition frequencies even when these have a small random fluctuation.

Thus, a generally modulated signal can be represented by:

$$A_m(t) \cos(2\pi f_c t + \phi_m(t)) \quad (6.1)$$

where  $A_m(t)$  represents the amplitude modulation function and  $\phi_m(t)$  represents the phase modulation function in radians. The corresponding frequency modulating function (in Hz) is

$\frac{1}{2\pi} \frac{d\phi_m(t)}{dt}$ . Expression (6.1) will be seen to be the real part of the rotating vector:

$$A_m(t) \exp\{j(2\pi f_c t + \phi_m(t))\} \quad (6.2)$$

whose modulus is the amplitude modulating function and whose phase is the phase modulating function plus the linear carrier component. Thus, if it is desired to demodulate a real signal such as (6.1), it is desirable to find the corresponding imaginary part so as to form the complex expression (6.2). Provided the fluctuating part of (6.2),

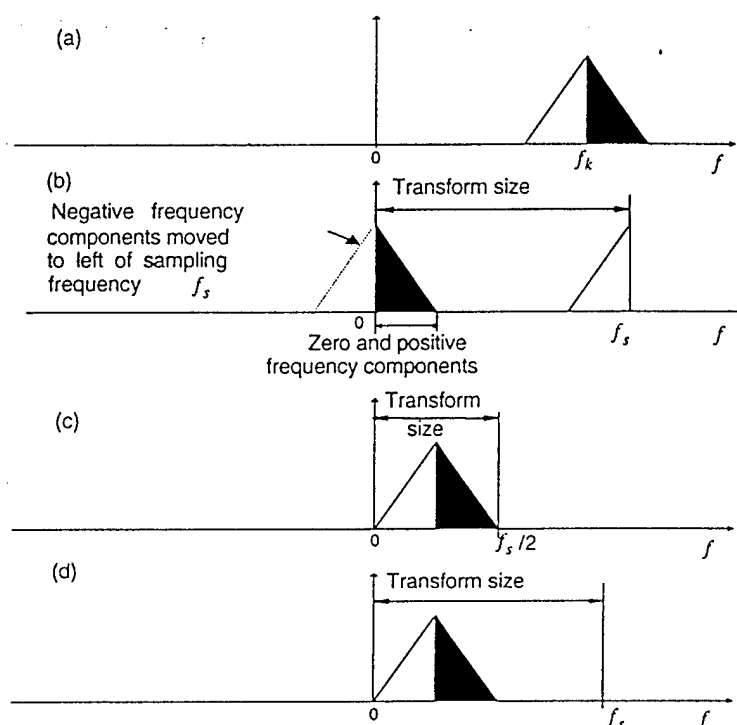
$$A_m(t) \exp\{j\phi_m(t)\} \quad (6.3)$$

has a half bandwidth less than the carrier frequency  $f_c$ , the spectrum of (6.2) will be one-sided, and (6.2) will be an analytic function. In this case the required imaginary part is the Hilbert transform of the real part, and the methods of Section 2.3 can be used. As the spectra of the two parts are convolved, the total bandwidth is less than the sum of the individual bandwidths. The bandwidth of the amplitude part is directly that of  $A_m(t)$ , and even though that of  $\exp\{j\phi_m(t)\}$  is theoretically infinite, if the maximum phase deviation is less than 1 radian, the effective bandwidth (within the dynamic range) is less than twice that of  $\phi_m(t)$ .

Note that a zoom processor can be used directly both to extract that part of the spectrum to be demodulated, and to remove the carrier component by zooming at the carrier frequency. Generally, the zoom process results at the same time in a considerable reduction in the sampling rate to be more compatible with the bandwidth of the modulating functions. The modulus of the complex output from the zoom processor is the amplitude modulating function, while the argument is the phase modulating function. This may have to be unwrapped to a continuous phase function (ie eliminating jumps over  $2\pi$ ), but in general this is not a problem for well-behaved functions. Demodulating a larger bandwidth decreases the time step, and thus phase jump, between samples and may facilitate unwrapping.

As illustrated in Figure 6.1, the same thing can be achieved using FFT transforms, although the first one will have to be large to accommodate the high carrier frequency while being long enough to contain sufficient periods of the lower modulating frequencies. Where phase demodulation is required, the centre of the demodulation band will have to be shifted to zero frequency (and negative frequency components shifted to the other end of the frequency record). However, for amplitude demodulation the result is unaffected by the frequency shift, and it is more convenient to shift the left hand end of the band to zero frequency, and pad the negative frequency side with zeros, thus maintaining an analytic signal. In either case there should be at least as many contiguous zeros in the spectrum as components, since the modulus is the square root of the amplitude squared, and the latter corresponds to the convolution of the spectrum with its complex conjugate reversed end-for-end. The zeros prevent extraneous

wrap-around errors.



*Figure 6.1 Block shift procedure for selecting frequency band for demodulation (a) Original spectrum of one-sided bandpass section (b) Frequency shift by  $f_k$  (centre of passband) (c) Frequency shift by amount corresponding to lower passband limit (half size transform) (d) Frequency shift by amount corresponding to lower passband limit (full size transform)*

## 6.2 Cyclostationarity and Spectral Correlation

Many machine generated signals are almost but not completely periodic, even after compensation for small speed changes by order tracking. One example are the vibration signals from rolling element bearing faults, where even though theoretical fault impact frequencies can be calculated on the basis of pure rolling contact, the actual frequencies fluctuate a small amount because of random slip. Another is the vibration signal from an internal combustion engine, as even though some events such as piston motion and valve operation are completely periodic, the combustion pressure signal varies randomly from cycle to cycle. In such cases synchronous averaging cannot be used, at least without losing the variable part of the signal. However, such signals are often cyclostationary [6.1] meaning that their second order statistics, such as autocorrelation functions, are periodic.

In the definition of the autocorrelation function of Equ.(5.4), it was assumed to be a function of delay time  $\tau$  only, as the function was assumed stationary with statistical properties independent of  $t$ . However, in the more general case of non-stationary signals, the second order statistical moment, the autocorrelation function, varies with both  $t$  and  $\tau$ , and it is possible to make two-dimensional Fourier transforms with respect to each. By the Wiener-Khinchin relationship that with respect to  $t$  gives autospectra against normal frequency, while that with respect to  $\tau$  gives the spectral correlation against "cyclic frequency" normally represented by  $\alpha$ . For stationary signals, there is a peak only for  $\alpha = 0$ , but for cyclostationary signals there will be peaks also for frequency shifts where spectrum bands



correlate with each other. As an example, where two different carrier frequencies are amplitude modulated by the same narrow-band noise signal, there will be a perfect correlation for a frequency shift corresponding to the separation of the two carrier frequencies, giving a very narrow peak in the cyclic frequency direction, even though the width of peaks in the normal frequency direction is determined by the bandwidth of the modulating noise function. Ref.[6.1] is an excellent discussion of the various applications of cyclostationarity, while [6.2] and [6.3] describe the application to the detection of gear faults.

### 6.3 Time-Frequency Analysis

In theory the Fourier transform requires integration over all time, but we are all aware that the ear can detect changes in frequency with time (what is music?), and so an analysis technique has been sought which matches the ear's ability to follow changing frequency patterns. A simple approach is to move a short time window along the record and obtain the Fourier spectrum as a function of time shift. However, the uncertainty principle means that the frequency resolution is the reciprocal of the effective time window length, and this does not seem to accord with the ear's appreciation of a tonal quality of a note even if it lasts for a short time. However, the so-called STFT (short time Fourier transform) is sometimes useful for tracking changes in frequency with time, even with the restriction of resolution. It is described by the formula:

$$S(f, \tau) = \int_{-\infty}^{\infty} x(t) w(t - \tau) \exp(-j2\pi ft) dt \quad (6.4)$$

where  $w(t)$  is a window which is moved along the record. Normally, the amplitude squared  $|S(f; \tau)|^2$  is displayed on a time vs frequency diagram, in which case it is sometimes known as a spectrogram. The window could be of finite length such as a Hanning window, or theoretically infinite such as a gaussian window, but in practice of course it must be truncated.

#### 6.3.1 The Wigner-Ville distribution

The Wigner-Ville distribution (WVD) appears to violate the uncertainty principle in appearing to give better resolution than the STFT, but suffers from interference components between the actual components. The original Wigner distribution [1.3] was modified by Ville [1.4] who proposed the analysis of the corresponding analytic signal so as to eliminate interference between positive and negative frequency components. The WVD is one of the so-called "Cohen's class" of time-frequency distributions [6.4], most of which have been proposed to improve on the WVD in some way. Even the STFT falls into this class. Cohen's class may be represented by the formula:

$$C_x(t, f, \phi) = \mathfrak{I}\{R(t; \tau)\} \quad (6.5)$$

where  $R(t; \tau)$  is a weighted autocorrelation-like function defined by:

$$R(t; \tau) = \int_{-\infty}^{\infty} x\left(u + \frac{\tau}{2}\right) x^*\left(u - \frac{\tau}{2}\right) \phi\left((t - u), \tau\right) du \quad (6.6)$$

and  $\phi(u, \tau)$  is a kernel function used to smooth the WVD (with  $\phi = 1$  the WVD is obtained). The "pseudo Wigner-Ville distribution" is a finite windowed version of the WVD and the "smoothed pseudo Wigner-Ville distribution" suppresses interference in both the time and frequency directions. Figure 6.2 compares the WVD and the smoothed pseudo-WVD against the STFT for a vibration signal from a portion of a diesel engine cycle, and shows that at least in this case the smoothing gives a simultaneous resolution in both directions that is better than

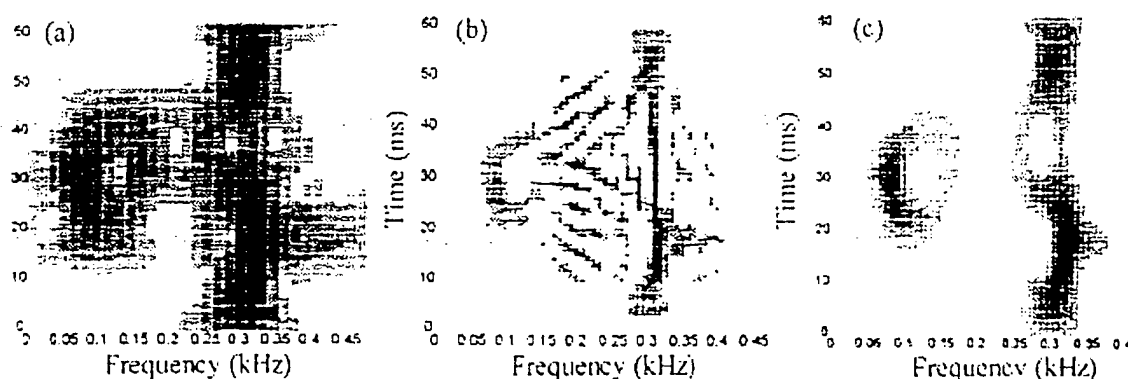


Figure 6.2. Comparison of time-frequency distributions for a diesel engine vibration signal  
(a) STFT (b) Wigner-Ville distribution (c) Smoothed pseudo Wigner-Ville

the STFT while still suppressing the major interference components. In Ref.[6.5] the proposal is made to use various smoothing techniques to locate the interference components, and then remove them from the unsmoothed WVD to retain optimum resolution.

### 6.3.2 Wavelet analysis

Another approach to time-frequency analysis is to decompose the signal in terms of a family of “wavelets” which have a fixed shape, but can be shifted and dilated in time. The formula for the wavelet transform is:

$$W(a;b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{(t-b)}{a} \right) dt \quad (6.7)$$

where  $\psi(t)$  is the mother wavelet, translated by  $b$  and dilated by factor  $a$ . Since this is a convolution, the wavelets can be considered as a set of impulse responses of filters, which because of the dilation factor have constant percentage bandwidth properties. In principle, they are not very different from  $1/n$ -octave filters, but with zero phase shift because the mother wavelet is normally centred on zero time. The dilation factor  $a$  is known as scale, but represents log frequency, as for constant percentage bandwidth filters. Wavelets give a better time localisation at high frequencies, and for that reason can be useful for detecting local events in a signal. Many authors have described their use for detecting local faults in gears and bearings (eg [6.6, 6.7]).

### 6.3.3 Other time-frequency techniques

A number of other time-frequency techniques have been proposed, including the evolutive AR methods mentioned in Section 4.1.2, and time-frequency “worms” [6.8]. It is still an area where considerable development is going on. One problem with time-frequency representations is the problem with interpreting the results. Even if a trained eye can see the desired effect, quite advanced image processing is sometimes required to extract the information.

## 7. MULTIPLE CHANNEL ANALYSIS

For system analysis, it is normally necessary to measure both input and output signals at the same time, and process them in pairs to obtain transfer function information, such as FRFs. With multiple inputs and/or outputs, it can be necessary to process more than two signals at a time, by matrix methods, but that will not be considered in detail here, as it is covered in other

specialist papers.

### 7.1 The Cross Spectrum

The cross spectrum between two signals is obtained by multiplying the spectrum of one by the complex conjugate of the spectrum of the other (the latter being considered as the input, since the phase of the result has its phase as datum). As with the autospectrum, to which it reduces if the two signals are identical, for random signals the individual estimates must be averaged to obtain meaningful results. Thus:

$$G_{AB}(f) = E[G_A^*(f) \cdot G_B(f)] \quad (7.1)$$

where  $E[.]$  is the statistical expectation, or averaging, operation.

By the Wiener-Khinchin relationship, the cross spectrum inverse transforms to the cross correlation function:

$$g_{AB}(t) = E[a(t) b(t + \tau)] \quad (7.2)$$

which reverts to the autocorrelation function (Equ.(5.4)) when the signals are identical.

### 7.2 The Frequency Response Function (FRF)

In the frequency domain the FRF is basically the ratio of output over input, or:

$$H_{AB}(f) = \frac{G_B(f)}{G_A(f)} \quad (7.3)$$

but in the presence of noise a better estimate can be obtained as described in other specialist papers. If noise is primarily located in the output signal (over which the experimentalist has less control), the procedure is modified by multiplying numerator and denominator by the complex conjugate of the input spectrum, and averaging, to give:

$$H_1(f) = \frac{E[G_B(f) G_A^*(f)]}{E[G_A(f) G_A^*(f)]} = \frac{G_{AB}(f)}{G_{AA}(f)} \quad (7.4)$$

since averaging reduces the effect of noise on the cross spectrum. If noise is primarily located in the input signal, a better estimate is given by:

$$H_2(f) = \frac{E[G_B(f) G_B^*(f)]}{E[G_A(f) G_B^*(f)]} = \frac{G_{BB}(f)}{G_{BA}(f)} \quad (7.5)$$

The system impulse response function can be obtained by inverse transformation of the FRF.

### 7.3 Coherence

The coherence gives a measure of the degree of linear relationship between two signals as a function of frequency. It is calculated by the formula:

$$\gamma^2(f) = \frac{|G_{AB}(f)|^2}{G_{AA} G_{BB}} \quad (7.6)$$

This has values between zero and one depending on the degree of linearity. For a single estimate, it will always be one, since  $|G_{AB}| = |G_A| \cdot |G_B|$  and in the absence of noise and nonlinearity each estimate of  $G_{AB}$  will have the same amplitude and orientation so that the average value will not change. However, in the presence of noise and/or nonlinearity the various estimates will change in length and particularly phase, so that the average obtained by vector summation will be less than if they were aligned. If there is no linear relationship, the various estimates will have random orientation and the average will tend to zero.

It is possible for signals to have partial coherence [7.1], but to determine this requires multiple channel processing and is beyond the scope of this paper.

#### 7.4 Separation of Sources

This is an area of signal processing which is receiving considerable attention and will presumably result in far-reaching consequences in the next millennium. So far there has been little application to mechanical problems, but if the type of result which has already been achieved in for example communications can be translated to structural dynamics, there is a strong likelihood that it will be possible to determine modal properties of structures from response measurements only. Such results obtained in service would give the values which actually apply in each situation. In Section 5.3 a case was mentioned where it was possible using singular value decomposition to separate out the response to the largest forcing function, provided it was at least four times larger than the next largest. This is a limitation given by "second order statistics". However, in Refs.[7.2-7.4] it is demonstrated that it should be possible to separate responses to different sources, even of comparable strength, if their higher order statistics are different.

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# Structural Dynamics Measurements

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## Introduction

In this paper, the term "structural dynamics measurements" will more specifically mean the *measurement of the vibration of mechanical structures and machinery*. Because this topic is so broad in scope, modal analysis and signal processing are also discussed here, but other papers at this conference are specifically devoted to those topics and cover them in more detail.

## Why Vibration Measurements?

Why are vibration measurements important? Because vibration contributes to a variety of undesirable behavior in machinery and structures. A machine or structure,

- may be *uncomfortable* to ride in,
- is *too difficult to control*,
- makes *too much noise*,
- doesn't *maintain tolerances*,
- *wears out* too fast,
- *fatigues* prematurely,
- or *breaks* unexpectedly.

## Types of Vibration

All structural vibration can be characterized as a combination of *forced* and *resonant* vibration. No vibration can occur at all unless forces are applied to the structure. However, resonant vibration can still occur after the forces have been removed. Resonant vibration is also conveniently characterized in terms of the *modes of vibration* of a structure.

## Resonant Vibration

A structure's modal parameters (resonant *frequency*, *damping*, and *mode shape*) can be estimated from certain kinds of structural dynamics measurements. If excited, modes (or resonances), can act like "*mechanical amplifiers*". Modes can cause excessive vibration responses that are *orders of magnitude* greater than responses due to static loading.

## Key Issues in Structural Dynamics Testing

Since dynamic behavior can be unpredictable due to the excitation of structural resonances, the most important question to be answered from structural dynamics testing is,

- Is a *structural mode* being excited?"

In addition, several other questions need to be answered,

- What are the *excitation forces*, and where are they coming from?
- Is the system *non-stationary*?
- Is the system *non-linear*?

Modes are only defined for *linear, stationary* mechanical systems. Most real structures can exhibit non-stationary (*non-steady state*) and non-linear dynamic behavior. When testing for the modes of a structure, these issues and others must be taken into account.

## Spectral Analysis

Probably the most convenient way to analyze a vibration signal is to obtain its frequency content, or frequency spectrum. There are at least two good reasons for this,

1. Excitation forces (especially in rotating equipment), often provide sinusoidal excitation at specific frequencies. These forces are manifested as peaks in a frequency spectrum.
2. Resonances are also manifested as peaks in a frequency spectrum.

Prior to the late 1960's, all structural dynamic testing was done with analog instrumentation. Sine wave generators were used to artificially excite structures, one frequency at a time. Oscilloscopes were used to look at the signals. Analog filters were used to limit the frequency content (band limit) the signals. Special analog filters that changed with the frequency of excitation, known as swept filters, were used to obtain the structural response, one frequency at a time. This response to each sinusoidal excitation frequency is a *frequency spectrum*.

In the 1960's, commercial spectrum analyzers were marketed that utilized swept analog filters, and constructed the frequency spectrum of structural vibrations, one frequency at a time.

## The FFT Analyzer

The Fourier transform is a mathematical procedure that was invented by a Frenchman named Jean-Baptiste-Joseph Fourier in the early 1800's. The Fourier transform yields the frequency spectrum of a time domain function. It is defined for continuous (or analog) functions, and is usually applied in situations where the functions are assumed to be continuous.

The discovery of the Fast Fourier Transform (FFT) algorithm in the late 1960's opened up a whole new area of signal processing using a digital computer [1]. The FFT computes a discretized (sampled) version of the frequency spectrum of a sampled time signal. This discretized, finite length spectrum is called a Discrete Fourier Transform (DFT). Following its discovery, the FFT was implemented in a new kind of spectrum analyzer called an FFT, or Fourier analyzer.

Present day FFT analyzers can compute a DFT in milliseconds, whereas it used to take hours using standard computational procedures. From a DFT, FFT analyzers can calculate a variety of

other frequency domain functions, including Auto Power Spectra (APSs), Cross Power Spectra (XPSs), Frequency Response Functions (FRFs), Coherences, etc.

### Rules of Digital Measurement

There are three key equations that govern the use of the DFT. The first one describes the sampled signal in the time domain, the second describes the sampled spectrum in the frequency domain, and the third is Shannon's Sampling Theorem, also called the Nyquist sampling rate.

**Time Waveform:** The DFT assumes that the sampled time waveform contains  $N$  uniformly spaced waveform samples, with an increment of  $(\Delta t)$  seconds between samples. (The most common FFT algorithms restrict  $N$  to being a power of 2, although this is not necessary.) The total time period of sampling (also called the *sampling window*), starts at  $(t = 0)$  and ends at  $(t = T)$ . Therefore,

$$T = (\Delta t) N \text{ (seconds)}$$

**Frequency Waveform:** The DFT assumes that the digital frequency spectrum contains  $N/2$  uniformly spaced samples of complex valued data, with frequency resolution  $(\Delta f)$  between samples. The frequency spectrum is defined for the *frequency range*  $(f = 0)$  to  $(f = F_{\max})$ . Therefore,

$$F_{\max} = (\Delta f) (N/2) \text{ (Hertz)}$$

**Nyquist Sampling:** Shannon's Sampling Theorem says that a frequency spectrum can only contain unique frequencies in a range from  $(f=0)$  up a maximum frequency  $(f = F_{\max})$  equal to *one half the sampling rate* of the time domain signal. Therefore,

$$F_{\max} = (1/2) (1/\Delta t) \text{ (Hertz)}$$

### Fundamental Rule: To Improve Frequency Resolution, You Have to Wait

The three equations above can be used to derive the most fundamental rule of digital spectrum based testing,

$$\Delta f = (1/T)$$

This equation says that the frequency resolution obtainable in a digital spectrum *depends on the time domain sampling window length*  $(T)$ , not the sampling rate. Stated differently, to get better frequency resolution, you have to sample over a longer time period.

### Zoom Measurements

A popular digital signal processing technique that is implemented in most FFT analyzers is the Zoom transform, or Zoom measurement. A Zoom transform is essentially a digital filtering operation that takes place after the time waveform has been sampled. It involves re-sampling, frequency shifting, and low pass filtering of the sampled data to yield a DFT with increased frequency resolution, but over a smaller frequency band.

The Zoom transform is very useful for obtaining better frequency resolution without having to perform an FFT on a very large number of samples. From a practical standpoint, the Zoom transform is much faster than using a base band

FFT (starting at zero frequency) with more samples to obtain more frequency resolution.

As an example, in order to obtain 1 milli-Hz of resolution in the vicinity of 100 Hz, a base band FFT would have to transform at least 262,144 samples. This would yield a base band spectrum between 0 and 132 Hz.

$$132 \text{ Hz} = (0.001 \text{ Hz}) (262,144 / 2)$$

Even though the Zoom transform starts with the same 262,144 time samples, the Zoom band can be centered around 100 Hz, and, assuming that a 1 Hz bandwidth is sufficient, the FFT only needs to transform 2048 samples,

$$1 \text{ Hz} = (0.001 \text{ Hz}) (2048 / 2)$$

### Digital Measurement Difficulties

The rules above are basically all that is required to make digital measurements. However, there are two remaining difficulties associated with the use of the FFT. They are called aliasing and leakage.

#### Aliasing

Aliasing of a signal occurs when it is *sampled at less than twice the highest frequency* in the spectrum of the signal. When aliasing occurs, the parts of the signal at frequencies above the sampling frequency add to the part at lower frequencies, thus giving an incorrect spectrum.

All modern FFT analyzers guarantee that aliasing will not occur by passing the analog signals through *anti-aliasing filters* before they are sampled. An anti-aliasing filter band limits (low pass filters) the signal so that it contains no frequencies higher than the sampling frequency. Since all filters have a roll off frequency band, the cutoff frequency of the anti-aliasing filters is typically set to *40% of the sampling frequency*. Therefore, *80% of a DFT frequency band* is considered to be alias-free.

#### Leakage

The FFT assumes that the signal to be transforming is *periodic in the transform window*. (The transform window is the samples used by the FFT). To be periodic in the transform window, the waveform must have no discontinuities at its beginning or end, if it were repeated outside the window.

Signals that are always periodic in the transform window are,

1. Signals that are completely contained within the transform window.
2. Cyclic signals that complete an integer number of cycles within the transform window.

However, many other types of signals (such as random signals), may not be periodic in the transform window. If a time signal is *not periodic* in the transform window, when it is transformed to the frequency domain, a *smearing* of its spectrum will occur. This is called leakage. *Leakage distorts the spectrum and makes it inaccurate.*

### Minimizing the Effects of Leakage

If a signal is non-periodic in its sampling window, it will have leakage in its spectrum. In this case, leakage can never be eliminated but it can be minimized. To minimize the effects of leakage, specially shaped windows are applied to the time waveforms *after* they are sampled, but *before* they are transformed using the FFT.

- **Hanning Window:** The Hanning window is effective for minimizing the effects of leakage in the spectra of *broad band signals*, such as random signals.
- **Flat Top Window:** The Flat Top (Potter P301) window is effective for minimizing the effects of leakage in the spectra of *narrow band signals*, such as sinusoidal signals.
- **Exponential Window:** This window is effective for minimizing the effects of leakage in impulse responses that don't damp out within the sampling window.

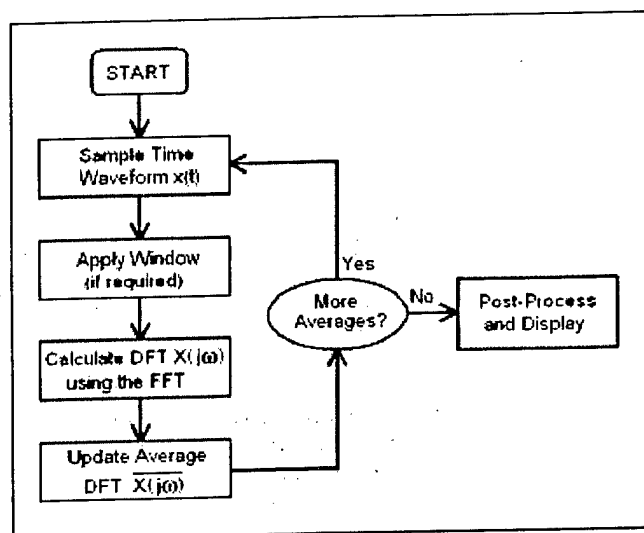


Figure 1. Spectrum Averaging Loop

### Spectrum Averaging

Spectrum averaging is an option in most modern FFT analyzers. It is done with a spectrum averaging loop, as shown in Figure 1.

Spectrum averaging is used to remove the effects of,

1. *extraneous random noise.*
2. *randomly excited non-linearities.*

In a spectrum averaging loop, multiple spectral estimates of the same signal are averaged together to yield a final estimate of the spectrum. Different types of averaging can be used, but the most common type (called *stable averaging*), involves summing all of the estimates together and dividing by the number of estimates.

The FFT is a linear, one-to-one and onto transformation. That means that it *uniquely transforms* the vibration signal from a linear dynamic system into its correct digital spectrum, and vice versa. If a signal contains any *additive Gaussian random noise or randomly excited non-linear behavior*, these portions of the signal are transformed into spectral components that *appear randomly in the spectrum*.

#### Removing Random Noise & Non-Linearities

By summing together (averaging) multiple spectral estimates of the same signal, the linear spectral components will add up (re-enforce one another), while the random noise and non-linear components will sum toward zero, thus removing them from the resultant average spectrum.

In order to remove random noise and non-linearities while retaining the spectral components of the linear dynamics, we must guarantee that the *magnitudes & phases* of the linear portion of all spectral estimates are the same. This depends on how the data is sampled in each sampling window.

#### Single Channel Versus Multi-Channel Measurements

FFT Analyzers can be classified into two categories, *single channel* and *multi-channel*. Each channel can process a unique signal. Single channel analyzers are the most popular because they cost less, but they also have limited measurement capability. The distinguishing feature of a multi-channel analyzer is that all channels are *simultaneously sampled*. (It is also assumed that filtering and other signal conditioning match within acceptable tolerances among all channels).

If an analyzer has multiple channels, but they are *multi-plexed* instead of simultaneously sampled, then each channel must be treated like a single channel analyzer channel.

Simultaneously sampled signals contain the correct *magnitudes & phases relative to one another*, since they are all sampled at the same moments in time. Therefore, any two simultaneously sampled signals can be used to form a Cross Power Spectrum (XPS), a fundamental cross channel measurement function.



Using spectrum averaging, a single channel analyzer can remove noise and non-linearities from a spectrum if the measurement process is a *repeatable process*. A multi-channel analyzer requires a less restrictive *steady state (stationary)* process.

### Repeatable Process

In a repeatable measurement process, data acquisition must occur so that *exactly the same time waveform* is obtained in the sampling window, every time one is acquired. Figure 2 depicts a repeatable process. For a repeatable process, the *magnitude & phase* of each sampled signal are assumed to be unique and repeatable.

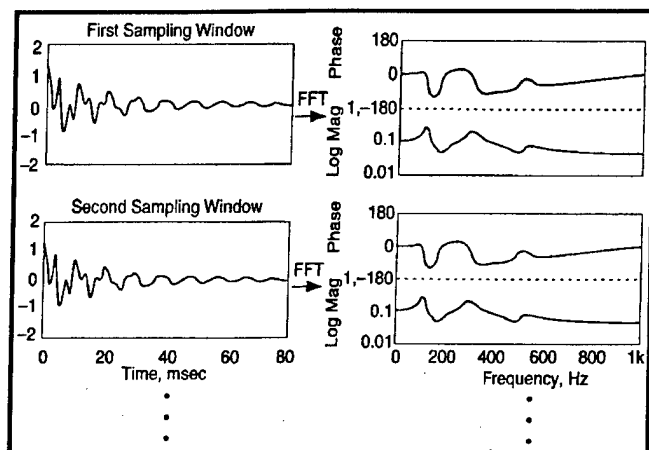


Figure 2. Repeatable Measurement Process.

A repeatable process guarantees the same results as simultaneous sampling. That is, it guarantees that multiple signals will have the *correct magnitudes & phases relative to one another*, whether they are acquired one at a time or simultaneously. Therefore, if a repeatable measurement process can be achieved, multiple channels of data can be acquired one at a time if necessary.

To insure a repeatable process, an *external trigger* is usually required to capture the repeatable event in the sampling window. In machinery applications, the trigger is usually obtained as a tachometer signal from a rotating shaft.

With a repeatable process, time domain averaging can also be done to remove random noise and random non-linearities. This is also called *synchronous averaging*.

Unfortunately, a repeatable measurement process cannot be achieved in many test situations.

### Steady State Process

A steady state measurement process can be achieved in situations where a repeatable process is not achievable. A steady state process is achieved when the *Auto Power Spectrum (APS) of a signal does not change* from measurement to measurement. (An APS is merely the magnitude squared of an FFT, or linear spectrum.) Figure 3 shows a steady state process. Notice that the time waveform can be different in

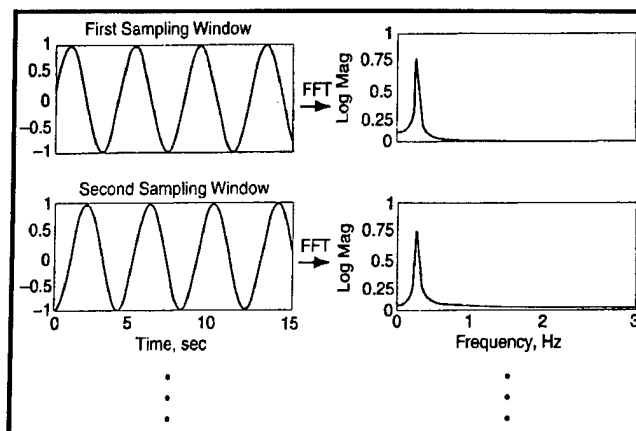


Figure 3. Steady State Measurement Process.

each sampling window, but its APS does not change. No special triggering is required for steady state measurement.

### Tri-Spectrum Averaging

The measurement capability of a multi-channel FFT analyzer is built around a tri-spectrum averaging loop, as shown in Figure 4. This loop assumes that two or more time domain signals are simultaneously sampled. Three spectral estimates, an Auto Power Spectrum (APS) for each channel, and the Cross Power Spectrum (XPS) between the two channels, are calculated in the tri-spectrum averaging loop. After the loop has completed, a variety of other cross channel measurements (including the *FRF*), are calculated from these three basic spectral estimates.

In a multi-channel analyzer, tri-spectrum averaging can be applied to as many signal pairs as desired. Tri-spectrum averaging will remove random noise and randomly excited

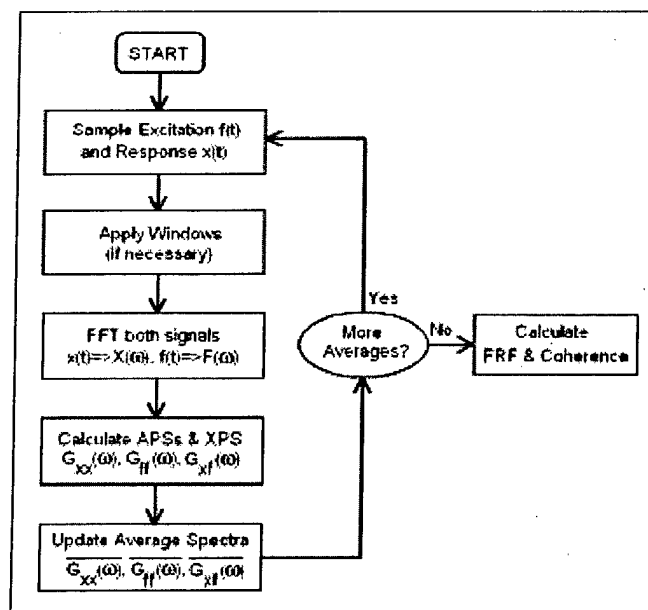


Figure 4. Tri-Spectrum Averaging Loop

non-linearities from signals taken during a steady state measurement process. This is particularly useful for measuring FRFs.

## The FRF

The Frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of mechanical structures. Experimental modal parameters (resonant frequency, damping, and mode shape) are obtained from a set of FRF measurements.

The FRF describes the input-output relationship between two points on a structure as a function of frequency, as shown in Figure 5. That is, the FRF is a measure of how much displacement, velocity, or acceleration response a structure has at an *output* point, per unit of excitation force at an *input* point.

The FRF is defined as the ratio of the Fourier transform of a motion output (or response) divided by the Fourier transform of the force input that caused the output. This is represented by the diagram in Figure 5.

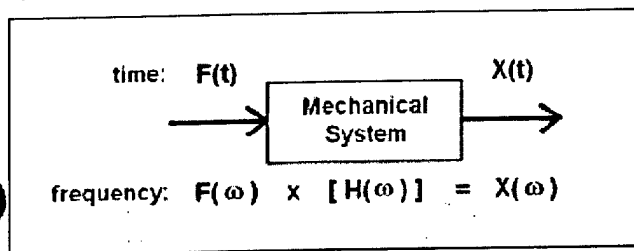


Figure 5. Block Diagram of an FRF.

Since both force and motion are vector quantities (they have directions associated with them), each FRF is actually defined between an input DOF (point and direction), and an output DOF.

An FRF is a complex valued function of frequency, that can be displayed in various forms, as shown in Figure 6.

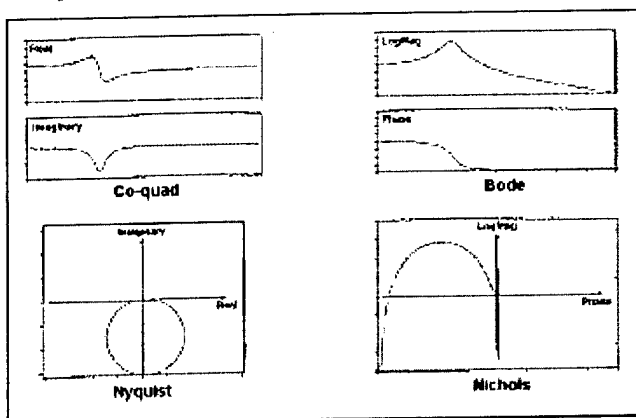


Figure 6. Alternate Forms of the FRF.

Depending on whether motion is measured as displacement, velocity, or acceleration, the FRF and its inverse have a variety of names,

- Compliance  $\Leftrightarrow$  displacement / force
- Mobility  $\Leftrightarrow$  velocity / force
- Inertance  $\Leftrightarrow$  acceleration / force
- Dynamic Stiffness  $\Leftrightarrow$  1 / Compliance
- Impedance  $\Leftrightarrow$  1 / Mobility
- Dynamic Mass  $\Leftrightarrow$  1 / Inertance

On a real structure, an *unlimited number* of FRFs can be measured between pairs of input and output DOFs, as shown in Figure 7.

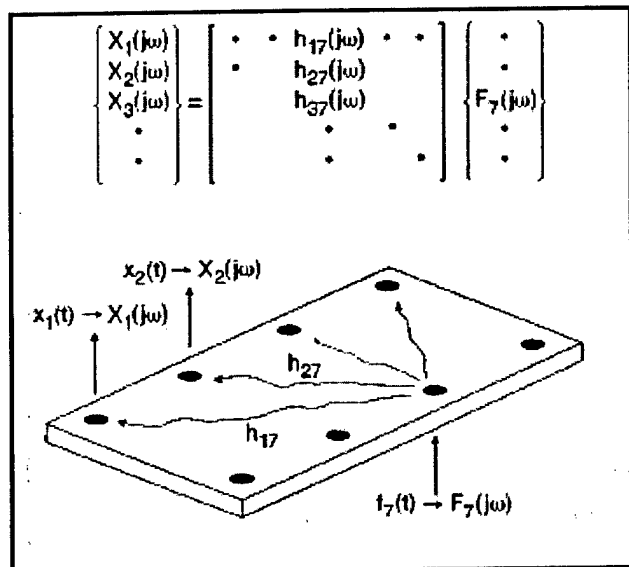


Figure 7. Measuring FRFs on a Structure

Although the FRF is defined as a ratio of Fourier transforms, it is actually computed differently using APS and XPS estimates. This is done to remove random noise and non-linearities (distortion) from the FRF, by using spectrum averaging as described earlier. There are several different ways to calculate the FRF. These are called FRF estimators.

### Noise on the Output ( $H_1$ )

This FRF estimator assumes that random noise and distortion are summing into the output, but not the input of the system. For this model, the FRF is calculated as,

$$H_1 = \frac{\text{XPS}}{\text{Input APS}}$$

It can be shown that  $H_1$  is a *least squared error estimate* for the FRF when extraneous noise and randomly excited non-linearities are modeled as Gaussian noise *added to the output*.

### Noise on the Input ( $H_2$ )

This FRF estimator assumes that random noise and distortion are summing into the input, but not the output of the system. For this model, the FRF is calculated as,

$$H_2 = \frac{\text{Output APS}}{\text{XPS}}$$

It can be shown that  $H_2$  is a least squared error estimate for the FRF when extraneous noise and randomly excited nonlinearities are modeled as Gaussian noise *added to the input*.

### Noise on the Input & Output ( $H_V$ )

This FRF estimator assumes that random noise and distortion are summing into both the input but and output of the system. The calculation of  $H_V$  requires more steps, and is detailed in [2].

## Measuring Rows & Columns of the FRF Matrix

Structural dynamics measurement involves measuring elements from a FRF matrix model for the structure, as shown in Figure 7. This model represents the dynamics of the structure between all pairs of input and output DOFs.

### FRF Matrix Model

The FRF matrix model is a frequency domain representation of a structure's linear dynamics, where linear spectra (FFTs) of multiple inputs are multiplied by elements of the FRF matrix to yield linear spectra of multiple outputs.

The FRF matrix model is written as,

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\}$$

where:

$\{X(\omega)\}$  = Linear spectra of output motions ... (n vector)

$[H(\omega)]$  = FRF matrix ... (n by m)

$\{F(\omega)\}$  = Linear spectra of input forces ... (m vector)

m = number of inputs

n = number of outputs

$\omega$  = frequency variable

**Columns** of the FRF matrix correspond to inputs, and **rows** correspond to outputs. Each input and output corresponds to a measurement Point or DOF of the test structure.

### Modal Testing

In modal testing, FRF measurements are usually made under controlled conditions, where the test structure is artificially excited by one or more shakers driven by broad band signals, or is excited by an impactor. A multi-channel FFT analyzer is then used to make FRF measurements between input and output DOF pairs on the test structure.

### Measuring FRF Matrix Rows or Columns

Modal testing requires that FRFs be measured from *at least one row or column* of the FRF matrix. Modal frequency & damping can be obtained from any FRF measurement. A row or column of FRF measurements is required to obtain mode shapes.

When the input is fixed and FRFs are measured for multiple outputs, this corresponds to measuring elements from a *single column* of the FRF matrix. This is typical of a shaker test.

On the other hand, when the output is fixed and FRFs are measured for multiple inputs, this corresponds to measuring elements from a *single row* of the FRF matrix. This is typical of a roving hammer impact test.

### Single Reference (or SIMO) Testing

The most common type of modal testing is done with either a *single fixed input* or a *single fixed output*. A roving hammer impact test using a single fixed motion transducer is a common example of single reference testing. The single fixed output is called the reference in this case.

When a single fixed input (such as a shaker) is used, this is called SIMO (Single Input Multiple Output) testing. In this case, the single fixed input is called the reference.

### Multiple Reference (or MIMO) Testing

When two or more fixed inputs are used, and FRFs are calculated between each of the inputs and multiple outputs, then FRFs from *multiple columns* of the FRF matrix are obtained. This is called Multiple Reference or MIMO (Multiple Input Multiple Output) testing. In this case, the inputs are the references.

Likewise, when two or more fixed outputs are used, and FRFs are calculated between each output and multiple inputs, this is also multiple reference testing, and the outputs are the references.

## Impact Measurements

Impact testing is the most commonly used method for finding the resonances of structures and machines. A typical impact test is depicted in Figure 8.

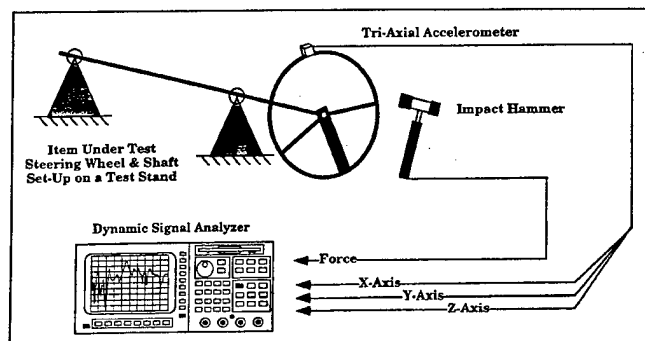


Figure 8. Impact Test Setup.

Impact testing requires a minimum of equipment,

1. A hammer with a load cell attached to its head to measure the impact force,
2. An accelerometer fixed to the structure to measure response motion,
3. A 2-channel FFT analyzer.

A wide variety of structures and machines can be impact tested. Of course, different sized hammers are necessary to provide the appropriate impact force to the structure. Not all structures can be impact tested, however. A structure or machine with delicate surfaces probably should not be impact tested. Typical signals from an impact test are shown in Figure 9.

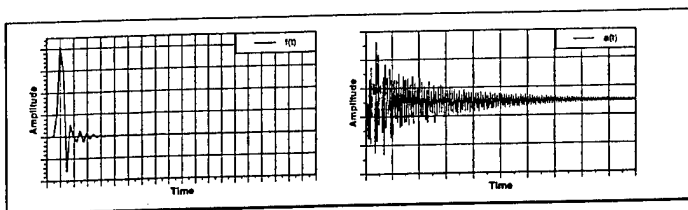


Figure 9A. Impact Force and Response Signals

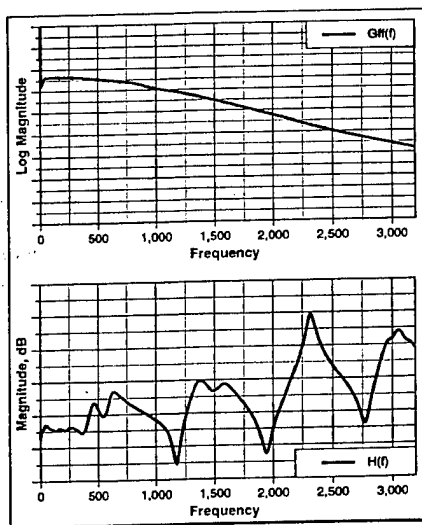


Figure 9B. Impact APS and FRF.

### Roving Hammer Test

A roving hammer test is the most common type of impact test. In this test, the accelerometer is fixed at a single DOF (point and direction), and the structure is impacted at as many DOFs as desired to define the mode shapes of the structure.

### Tri-axial Measurements

The only drawback to the roving hammer approach is that many points on a structure cannot be impacted in three directions, so tri-axial (3D) motion cannot be obtained for all points. When 3D motion is desired at each test point, a roving tri-axial accelerometer can be used, and the structure impacted at a fixed DOF. However, in order to process the

tri-axial and force data together, however, a 4-channel FFT analyzer is required instead of a 2-channel analyzer.

### Pre-Trigger Delay

Because the impulse signal exists for such a short period of time, *it is important to capture all of it* in the sampling window. To insure that the entire signal is captured, the analyzer must be able to capture the impulse and impulse response signals *prior to the occurrence of the impulse*. This is called a pre-trigger delay. In other words, the analyzer must begin sampling data before the trigger point occurs, which is usually set to a small percentage of the peak value of the impulse.

### Force & Exponential Windows

Two common time domain windows that are used in impact testing are the force and exponential windows. These windows are applied to the signals after they are sampled, but before the FFT is applied to them.

*The force window is used to remove noise from the impulse (force) signal.* Ideally, an impulse signal is non-zero for a small portion of the sampling window, and zero for the remainder of the window time period. Any non-zero data following the impulse signal in the sampling window is assumed to be measurement noise. The force window *preserves* the samples in the vicinity of the impulse, and *zeros* all of the other samples in the sampling window.

The exponential window is applied to the impulse response signal. *The exponential window is used to reduce leakage in the spectrum of the response.* If the response *decays to zero* (or near zero) before the end of the sampling window, then there will be no leakage, and the exponential window need not be used.

In the response *does not decay* to zero before the end of the window, then the exponential window *must be used* to reduce the leakage effects on the response spectrum. The exponential window *adds artificial damping* to all of the modes of the structure in a known manner. This artificial damping can be subtracted from the modal damping estimates. But more importantly, if the exponential window causes the impulse response to be completely contained within the sampling window, leakage is removed from its spectrum.

### Accept/Reject

Because impact testing relies, to some degree, on the skill of the one doing the impacting, it should be done with spectrum averaging, using 3 to 5 impacts per measurement. Since one or two of the impacts during the measurement process may be *bad hits*, an FFT analyzer designed for impact testing should have the ability to accept or reject each impact. An accept/reject capability saves a lot of time during impact testing since you don't have to restart the measurement after each bad hit.

### Advantages of Impact Testing

The advantages of impact testing are,

- Low equipment cost.
- Ease test setup.
- Fast measurement time.
- Signals are periodic (or near periodic) in the sampling window.

### Disadvantages of Impact Testing

The disadvantages of impact testing are,

- Special analyzer capabilities are required.
- Some skill required to impact correctly.
- Low energy density in impact signal.
- Doesn't remove non-linear behavior.
- Can't be used on some structures.

## Shaker Measurements

When impact testing cannot be used, then structural dynamic measurements are made by providing excitation with one or more shakers attached to the structure. Common types of shakers are electro-dynamic and hydraulic shakers. A typical shaker test is depicted in Figure 10.

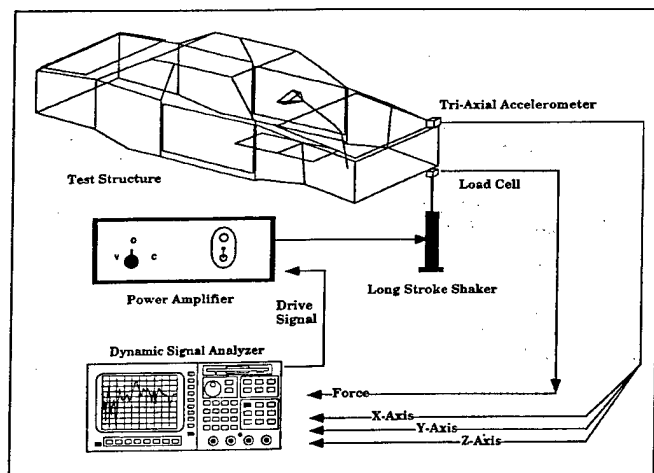


Figure 10. Shaker Test Setup.

A shaker is usually attached to the structure using a stinger (long slender rod), so that the shaker will only impart force to the structure along the axis of the stinger, the axis of force measurement.

A load cell is attached *between the structure and the stinger* to measure the excitation force. At least a 2-channel FFT analyzer and a uni-axial accelerometer are required to make FRF measurements using a shaker. If an analyzer with 4 or more channels is used, then a tri-axial accelerometer can be used and 3D motion of the structure measured at each test point.

In a SIMO test, one shaker is used and the shaker is the (fixed) reference. In a MIMO test, multiple shakers are used, and the shakers are the multiple references. When multiple

shakers are used, care must be taken to insure that the shaker signals are not completely correlated (the same signal). Furthermore, special matrix processing software is required to calculate FRFs from the multiple input APSs and XPSs resulting from tri-spectrum averaging.

### Step Sine, Slow Swept Sine

The sine wave excitation signal has been used since the early days of structural dynamic measurement. It was the only signal that could be effectively used with traditional analog instrumentation, as described earlier.

Even broad band testing methods (like impact testing), have been developed for use with FFT analyzers, sine wave excitation is still useful in some applications. The primary purpose for using a sine wave excitation signal is to put energy into a structure at a specific frequency. Slowly sweeping sine wave excitation is also useful for characterizing non-linearities in structures.

### Advantages of Sine Testing

Sine wave excitation has the following advantages,

- Best signal-to-noise and RMS-to-peak ratios of any signal.
- Controlled amplitude and bandwidth.
- Useful for characterizing non-linearities.
- Long history of use.

### Disadvantages of Sine Testing

The disadvantages of sine wave excitation are,

- Distortion due to over-excitation.
- Extremely slow for broad band measurements.

### Broad Band Excitation Signals

A variety of new *broad band* excitation signals have been developed for making shaker measurements with FFT analyzers. These signals include,

- Transient
- True Random
- Pseudo Random
- Periodic Random
- Burst Random
- Fast Sine Sweep (Chirp)
- Burst Chirp

Since the FFT provides a DFT over a broad band of frequencies (*0 to nearly half of the sampling frequency*), using a broad band excitation signal makes the measurement of broad band spectral measurements much faster than using a stepped or slowly sweeping sine wave. Nevertheless, sine wave excitation is still useful in some applications.

### Transient Signals

Using a transient signal in shaker testing provides the same leakage free measurements as impact testing, but with more controllability over the test. Application of the force is more

repeatable than impacting with a hand held hammer. However, this one advantage is usually outweighed by the disadvantages of using an impulsive force, when compared to the other broad band signals.

### True Random

Probably the most popular excitation signal used for shaker testing with an FFT analyzer is the random signal. When used in combination with spectrum averaging, random excitation randomly excites the non-linearities in a structure, which are then removed by spectrum averaging.

A true random signal is synthesized with a random number generator, and is an unending (non-repeating) random sequence. The main disadvantage of a true random signal is that it is *always non-periodic in the sampling window*. Therefore, a special time domain window (a Hanning window or one like it), must always be used with true random testing to minimize leakage. Typical true random signals are shown in Figure 11.

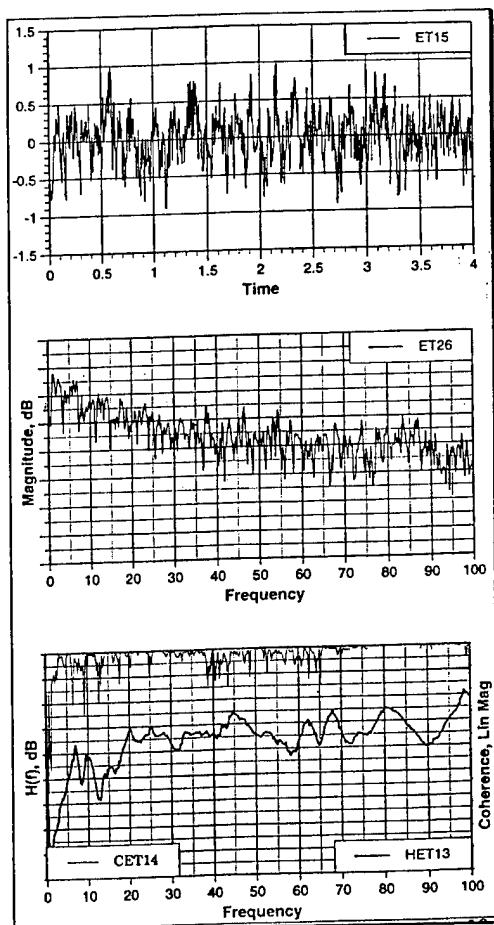


Figure 11. True Random Excitation (Time waveform, APS, FRF & Coherence).

### Advantages of True Random Excitation

The advantages of true random excitation are,

- Removes non-linear behavior when used with spectrum averaging.
- Fast measurement time.
- Leakage effects reduced with Zoom measurements.

### Disadvantages of True Random Excitation

The disadvantages of true random excitation are,

- Signals are non-periodic in the sampling window. Special windowing (Hanning, etc.) is needed to reduce leakage.
- Many averages are typically required.

### Pseudo Random

A pseudo random signal is specially synthesized within an FFT analyzer to coincide with the DFT measurement parameters. A typical random signal starts as a uniform (or shaped) magnitude and random phase signal, synthesized over the same frequency range and samples as the intended measurement. It is then *inverse FFT'd* to obtain a random time domain signal, which is subsequently output through a digital-to-analog converter as the shaker excitation signal.

During the measurement process, the measured force and response signals are sampled *over the same sampling time window* as the output of the excitation signal. This insures that the acquired signals are *periodic in the sampling window*, since the synthesized excitation signal is periodic in the window.

### Advantages of Pseudo Random Excitation

The advantages of pseudo random excitation are,

- Signals are periodic in the sampling window, so measurements are leakage free.
- Fast measurement time.
- The amplitude of excitation can be shaped for impedance mismatches between the shaker and structure.

### Disadvantages of Pseudo Random Excitation

The disadvantages of pseudo random excitation are,

- Doesn't remove non-linearities, because they are not excited randomly between spectrum averages.

### Periodic Random

Periodic random excitation is simply a different use a pseudo random signal, so that non-linearities can be removed with spectrum averaging. For periodic random testing, *a new pseudo random sequence is generated for each new spectrum average*. The advantage of this is that when multiple spectrum averages of different random signals are averaged together, randomly excited non-linearities are removed.

Although periodic random excitation overcomes the disadvantage of pseudo random excitation, it takes *at least three times longer* to make the same measurement. This extra

time is required between spectrum averages to allow the structure to reach a new steady-state response to the new random excitation signal.

### Advantages of Periodic Random Excitation

The advantages of periodic random excitation are,

- Signals are periodic in the sampling window, so measurements are leakage free.
- Removes non-linear behavior when used with spectrum averaging.
- The amplitude of excitation can be shaped for impedance mismatches between the shaker and structure.

### Disadvantages of Periodic Random Excitation

The disadvantages of periodic random excitation are,

- Slower than other random test methods.
- Special software required for implementation.

### Burst Random

Burst random excitation is similar to periodic random testing, but faster. In burst random testing, a true random signal can be used, but it is **turned off** prior to the end of the sampling

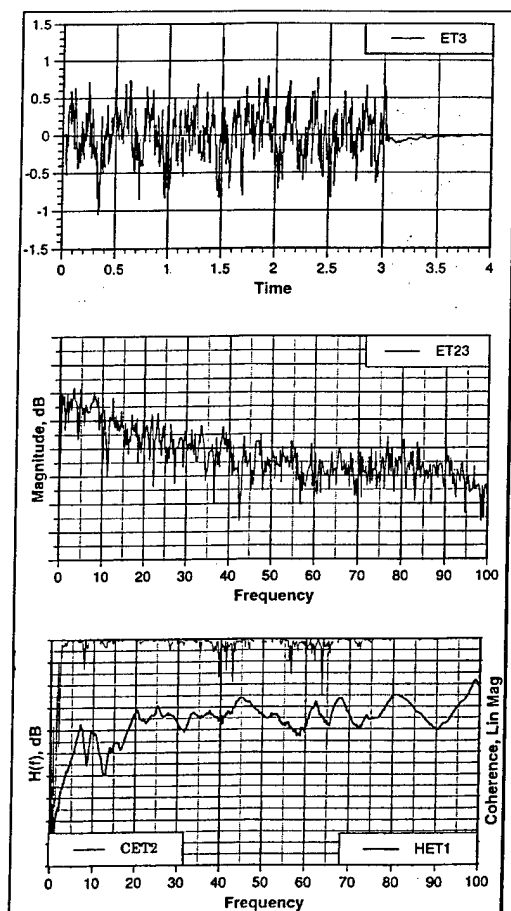


Figure 12. Burst Random Excitation (Time waveform, APS, FRF & Coherence).

window time period. This is done in order to allow the structural response to decay within the sampling window. This insures that both the excitation and response signals are completely contained within the sampling window. Hence, they are **periodic in the sampling window**.

Figure 12 shows a typical burst random signal. The random generator must be turned off early enough to allow the response to decay to zero (or nearly zero) before the end of the sampling window. Of course, the length of the decay period depends on the damping in the test structure.

Burst random must therefore be setup interactively on an FFT analyzer, after observing the free decay of the structure, following the removal of random excitation. Since a pure random signal can be used with burst random testing, it does not have the disadvantages of either pseudo random or periodic random testing.

### Advantages of Burst Random Excitation

The advantages of burst random excitation are,

- Signals are periodic in the sampling window, so measurements are leakage free.
- Removes non-linear behavior when used with spectrum averaging.
- Fast measurement time.

### Disadvantages of Burst Random Excitation

The disadvantages of true random excitation are,

- Special software required for implementation.

### Chirp & Burst Chirp

A swept sine excitation signal can also be synthesized in an FFT analyzer to coincide with the parameters of the sampling window, in a manner similar to the way a pseudo random signal is synthesized. Since the sine waves must sweep from the lowest to the highest frequency in the spectrum, over the relatively short sampling window time period ( $T$ ), this fast sine sweep often makes the test equipment sound like a bird chirping, hence the name chirp signal.

A burst chirp signal is the same as a chirp, except that it is **turned off** prior to the end of the sampling window, just like burst random. This is done to insure that the measured signals are **periodic in the window**. A typical burst chirp signal is shown in Figure 13.

The advantage of burst chirp over chirp is that the structure has returned to rest before the next average of data is taken. This insures that the measured response is only caused by the measured excitation, an important requirement for any multi-channel measurement such as a FRF.

### Advantages of Burst Chirp Excitation

The advantages of burst chirp excitation are,

- High signal-to-noise and RMS-to-peak ratios.

- Signals are periodic in the sampling window, so measurements are leakage free.
- Fast measurement time.

### Disadvantages of Burst Chirp Excitation

The disadvantages of burst chirp excitation are,

- Special software required for implementation.
- Doesn't remove non-linear behavior.

### Comparison of Excitation Signals

Ideally, all of the shaker signals that are leakage free (periodic in the window) should yield the same result. Figure 14 shows an overlay of two FRF magnitudes, one measured with a burst random and the other with a burst chirp signal. The two FRFs match very well at low frequencies, but show some disparity at high frequencies. This could possibly be due to a small amount of non-linear behavior in the structure, which burst chirp signal processing cannot remove through averaging.

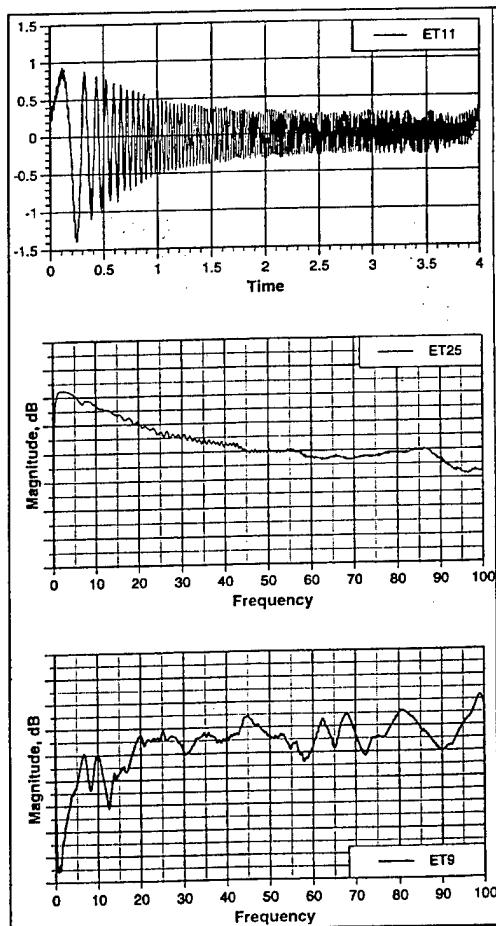


Figure 13. Burst Chirp Excitation (Time waveform, APS, FRF & Coherence)

Finally, all of the previously described test methods are compared in the table shown in Figure 15. Impact testing is by far the easiest method to implement. On the other hand, when impact testing cannot be used, or when multiple shakers are needed to provide sufficient excitation, then a variety of other implementation issues must be considered.

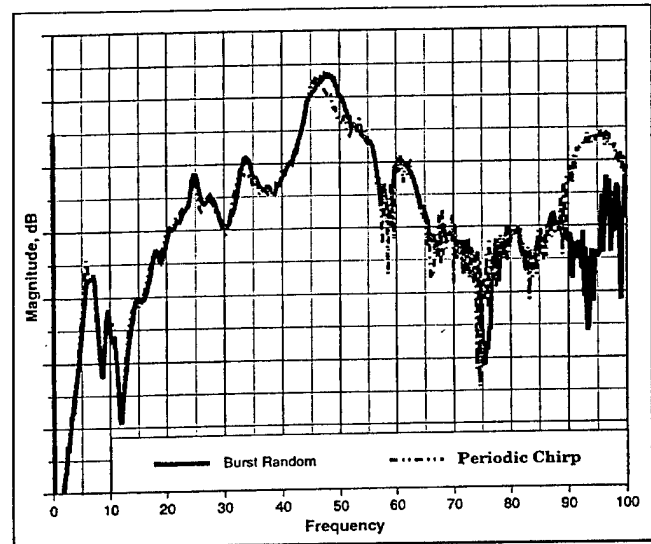


Figure 14. Burst Random Versus Burst Chirp FRF.

### Difficulty with FRF Measurements

Thus far, we have talked mostly about making FRF measurements. Mode shapes (part of a resonance condition) are normally obtained from a set of FRFs. Making an FRF measurement requires that *all of the excitation forces causing a response must be measured simultaneously with the response*. This can be difficult, if not impossible in many test situations.

FRFs usually cannot be measured on operating machinery or equipment where ambient forces (internally generated forces, acoustic excitation, etc.) are either unmeasured or unmeasurable. On the other hand, the vibration response caused by ambient forces can always be measured, no matter what forces are causing it.

### Difficulty With Operating Data Measurements

One key advantage of the FRF measurement is lost when operating data measurements are made. Without measuring the excitation forces, it is impossible to know precisely whether a peak in a response spectrum is due to a resonance or to the excitation forces. Nevertheless, valuable information can still be obtained from operating data.



	Impact	Sine	Swept Sine	True Random	Pseudo Random	Periodic Random	Burst Random	Burst Chirp
Periodic	YES	NO	YES	NO	YES	YES	YES	YES
Removes Noise	YES	NO	YES	YES	YES	YES	YES	YES
Removes Non-linearities	NO	NO	NO	YES	NO	YES	YES	NO
Test Time	FAST	SLOW	FAIR	FAIR	FAST	SLOW	FAST	FAST
SNR	LOW	HIGH	HIGH	FAIR	FAIR	FAIR	FAIR	HIGH
Frequency Control	SOME	YES	YES	YES	YES	YES	YES	YES

Figure 15. Comparison of Excitation Methods

## Operating Deflection Shapes

An Operating Deflection Shape (ODS) is defined as any *forced motion of two or more* DOFs on a structure. Specifying the motion of two or more DOFs defines a shape. Stated differently, a shape is the motion of one DOF relative to all others.

An ODS can be defined from any forced motion, either at a moment in time, or at a specific frequency. An ODS can be obtained from different types of time domain responses, be they random, impulsive, or sinusoidal. An ODS can also be obtained from many different types of frequency domain measurements, including linear spectra (FFTs), APS, XPSS, FRFs, transmissibilities, and a special type of measurement called an ODS FRF, described later.

### Mode Shapes and ODSs Contrasted

Mode shapes and ODSs are related to one another, but have different characteristics,

1. Modes are inherent properties of a structure. They don't depend on the forces or loads acting on the structure.
2. Modes will change if the material properties (mass, stiffness, damping properties), or boundary conditions (mountings) of the structure change.
3. Mode shapes don't have unique values, and hence don't have units associated with them.
4. Mode shapes are unique. That is, the motion of one DOF relative to all others at resonance is unique.
5. Modes are defined for linear, stationary systems.
6. Modes are only used to characterize resonant vibration.

ODSs have the following characteristics

1. ODSs depend on the forces or loads applied to a structure. They will change if the load changes.
2. ODSs also depend on the modes. ODSs will change if the modes change.

3. ODSs have unique values & units, typically displacement, velocity, or acceleration, or perhaps displacement per unit of excitation force.
4. ODSs can be used to answer the question, "*How much is the structure really moving, at a particular time or frequency?*"
5. ODSs can be defined for nonlinear or non-stationary structures.
6. ODSs can also be defined for structures that don't resonate.

### Modes From ODSs

Since all measurement data is forced response, whenever two or more measurements are taken spatially from two or more DOFs of a structure, this is an ODS measurement. Moreover,

- *All experimental modal parameters are obtained by post-processing ODS measurements!*

## Transmissibility Measurement

We have already seen that under the assumption of either a Repeatable (more restrictive) or a Steady State (less restrictive) measurement process, spectrum averaging can be accomplished, and multi-channel measurements made. When the excitation force cannot be measured, then a *reference response signal* can be used instead of the force.

A transmissibility measurement is calculated in the same way as an FRF, but with a reference response signal replacing the excitation force.

### Mode Shapes From Transmissibilities

A set of transmissibilities, calculated between multiple response DOFs and a single fixed reference response, can be used to find the mode shapes of structural resonances. The *values of the transmissibilities at each resonant frequency* is an *approximation to the mode shape*.

The difficulty with using a set of transmissibilities to determine mode shapes is that resonances correspond to "flat spots" instead of peaks in these measurements. Therefore, in addition to a set of transmissibilities, at least one APS is required in order to locate the resonance peaks.

## ODS FRF Measurement

A different cross channel measurement, called an ODS FRF, can be calculated from APS and XPS measurements [3]. An ODS FRF has two advantages over a transmissibility,

1. *It has peaks at resonant frequencies*, making it easier to locate resonances and identify mode shapes.
2. It has response units (G's, Mils, etc.). Therefore, operating deflection shapes taken from a set of ODS FRFs have these same units.

To calculate a set of ODS FRFs between multiple response DOFs and a single fixed *reference response*, a tri-spectrum averaging loop is used to estimate an APS for each response, and a XPS between each response and the reference response. When tri-spectrum averaging is completed, each ODS FRF is formed by *replacing the magnitude of the each XPS with the APS* of its corresponding response.

A set of ODS FRF measurements is useful for determining whether a structure or machine is simply undergoing excessive forced response, or whether a resonance is also being excited.

## Non-Steady State Operation

All of the foregoing measurements assumed that measurement process was either repeatable or steady state. However, many types of structures and machines undergo non-steady state operation. Automobiles and machine tools are common examples.

In fact, most rotating equipment is characterized by non-steady state operation. Measurements are typically made while sweeping the speed of the machine. These are called RPM sweeps. Since the measurement process is non-steady state, the spectra cannot be averaged together. Rather, they are plotted in a waterfall plot, or spectral map.

### Orders

Since the excitation forces in a rotating machine are primarily sinusoidal and usually cannot be measured, their response spectra will exhibit forced responses are peaks that vary in frequency with the speed of the machine. These peaks, called Orders, appear at frequencies that are fixed multiples of the machine speed.

Since machine speed continually changes (it is non-stationary), a portion of a rotating machine's response will also be non-stationary and exhibit peaks which "track" the cyclic forces. However, if a resonance is excited, it will al-

ways appear at its fixed (stationary) natural frequency in any spectral measurement.

## Conclusions

During the past 30 years, there has been a proliferation of new structural dynamics testing methods that are based upon the laboratory implementation of the FFT and related signal processing algorithms. The "parallel processing" nature of the FFT which yields the discrete frequency spectrum of a signal from one calculation, makes it a broad band tool. This created a fundamental departure from the traditional sine wave based, swept filter methods for testing structures.

For finding structural resonances, the FFT has made it convenient to excite structures using many different kinds of broad band signals. Not only are a variety of shaker signals now used, but impact testing has become very popular as a fast, convenient, and relatively low cost way of finding the mode shapes of structures.

## Acknowledgements

Many of the methods and ideas reviewed here were learned over the past 25 years during *on the job training* at Hewlett Packard, Structural Measurement Systems, Inc. and Vibrant Technology, Inc. Numerous co-workers, customers, and students have developed most of these methods, and have taught me.

Most of the graphics used in this article has been "stolen" from other sources, hopefully not illegally. Some of this material was developed in cooperation with Jim Steedman, NAVCON Engineering Network, for use in a modal training course.

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## What can Mechatronics do for Structural Dynamics?

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**Abstract.** Mechatronics, as a combination of mechanics, electronics and computer science opens new ways of designing and shaping the dynamics of mechanical systems. Concepts, methods and examples will be presented, and an outlook on potential future developments of intelligent machines will be given. Comparisons and extensions to structural dynamics, active dynamics or smart structures will be drawn.

### 1. What is Mechatronics?

Let us begin with a paradigm! In classical mechanics our standard question is: which motion will a body perform when a given force, for example gravity, is acting upon it? To answer this question we write down the equations of motion in a well-known way, and solve them. It is a problem of *analysis*.

This kind of question can be inverted, too, and then it runs: which force has to be exerted on the body such that it actually performs a certain motion? For example, how do the braking forces for a car have to be tuned so that the car can negotiate a curve safely even during braking? This is a problem of *synthesis*, and for the car example this actually has lead to the Anti-Blocking System (ABS), a safety feature installed in most cars today.

For solving such a synthesis task we of course need engineering mechanics, but in addition we need knowledge in systems theory and in control techniques. Further, if we want to realize the suggested solution we additionally have to use knowledge and methods from electrical engineering, electronics and computer science. The term mechatronics for such a synthesis task came from Japan in about 1980, having been coined, it is said, by an employee of the Yasukawa Company. This interconnection of disciplines is actually not new: in aerospace engineering especially it has been well known for a long time and has been successful. For some years now, however, this interdisciplinary field has been growing rapidly and it has developed a weight of its own. This has become possible by the availability of relatively cheap computational power and it is further supported by the rise of versatile power electronics.

Mechatronics characterizes a general trend resulting from the increasing importance of information processing in machinery and other products. Software has become a machine

element and an integral part of the product.

In recent years a number of different *definitions for mechatronics* have been suggested. At the ETH we have defined mechatronics in a way that clearly brings out the novel possibilities of combining different disciplines and the potential for machine intelligence [SCHWEITZER 96]. It is the extension and the completion of mechanical systems with sensors and microcomputers which is the most important aspect. The fact that such a system picks up changes in its environment by sensors, and reacts to their signals using the appropriate information processing, makes it different from conventional machines (Fig. 1).

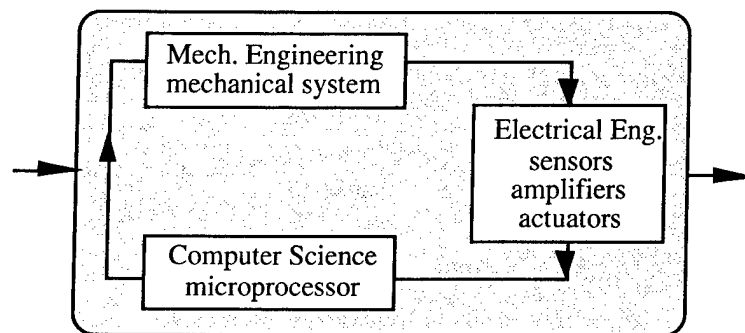


Fig. 1: Mechatronic system. The system picks up signals from its environment, processes them in an intelligent way and reacts, for example, with forces or motions. Methods for connecting the various areas of knowledge - mechanical, electrical engineering and computer science - are provided by the basic engineering sciences, system theory, control techniques and information processing

Examples of mechatronic systems are robots, digitally controlled combustion engines, machine tools with self-adaptive tools, contact-free magnetic bearings, automated guided vehicles, etc. Typical for such a product is the high amount of system knowledge and software that is necessary for its design. Furthermore, and this is most essential, software has become an integral part of the product itself, necessary for its function and operation. It is fully justified to say software has become an actual "machine element".

## 2. Relations of Mechatronics to Other Fields

The approach, to regard information processing and actuation as an integral part of a product, in addition to its obvious mechanical structure, may have been a kind of revelation in the technical area and a good reason to coin a special term for it such as mechatronics. In biology, however, there was never any doubt that a biological system needs more than just its physical-mechanical structure to make it work. Thus, an extension of mechatronics, or a technical derivation of biology are bio-inspired systems, characterizing artificial organs, insect-like robots, or the artificial eye or retina. Comparing technical and biological systems may lead to another interesting argument concerning the aspect of reliability. Mechatronics, it is said, will lead to very complex and therefore unreliable systems, an argument, which certainly could be backed up by numerous case histories. On the other side, information

processing in mechatronic devices is increasingly being used to make them safer and more reliable, just think of the dynamics control of modern cars. And this certainly is only the beginning towards intelligent machines [SCHWEITZER 96a]. Biological systems, even the simplest ones, are of a tremendous complexity, probably necessary to give them the capability for survival, or to support their ability to operate under uncertain conditions.

The number of new research fields and application areas with the appendix "...tronics" or with the attributes "active, smart, intelligent" is growing. In this way they are indicating the ability to process information within the system under consideration, and to react in an intelligent manner. Some of these new names are known as biotronics, structronics, adaptronics, thermotronics, active structures, active acoustics, active fluid-structure interaction, smart materials, smart products. Of course, this indicates as well, that the original fields are shifting their emphasis in research to new centers, they are adding new methods and they are opening new fields of applications [FULLER, et al. 96], [BOLLER 98]. What this may mean for structural dynamics will be discussed in the next section.

### **3. Mechatronic Approaches in Structural Dynamics**

Structural Dynamics is very strong in modeling techniques such as FEM and in simulation. Through modal analysis techniques experimental results can be related to simulations. The structure is characterized by modal parameters, the FE model is validated or refined by updating the parameters. In the first place all these techniques are serving the analysis.

This objective may be shifted under mechatronic aspects or even replaced. A major objective in mechatronics is to control the structure in such a way that the dynamics follow given specifications. The model where the control design is based on can be a non-parametric model, i.e. a set of frequency responses without detailed knowledge of the physical parameters involved. Section 4 will give an example of this. Of course, for the design phase of the structure itself an insight into the physics of the structural dynamics is still desirable, and for design modifications even a detailed model may be necessary. Methods for setting up structural models that are useful for control purposes and where control elements such as sensors and actuators can be integrated optimally have been discussed, for example by [SKELTON 95]. The observability and controllability of structural motions by placing sensors and actuators in suitable locations or distribute them optimally is being investigated.

Another mechatronics-inspired objective would be self-calibration and self-diagnosis of dynamic behavior, a feature that goes beyond mere monitoring. While the monitoring of structural properties such as condition monitoring in composite materials or in reinforced concrete is an actual research topic already, the step to actively investigate a structure by means of its control system has still to be taken. Once a control system has been incorporated into a structure, being an active structure now, a self-diagnosis consists in coming up with a suggestion for a fault, and to generate an internal control signal to verify

the suggestion or to modify it. This would require a strategy to compare expected behavior with measured behavior. One way to describe the expected behavior would be through a detailed model, which can accommodate the needs of such an active self-diagnosis.

#### 4. Example for Identification and Control of a Flexible Rotor

In a recent BRITE/EURAM project [MARS 96], part of the project was to support a flexible rotor with Active Magnetic Bearings [GAEHLER et al 96]. The test rig, which has been built at Darmstadt University, is shown in Fig. 2, two characteristic modes are presented in Fig. 3, and the main principle of an Active Magnetic Bearing (AMB) is indicated in Fig. 4.

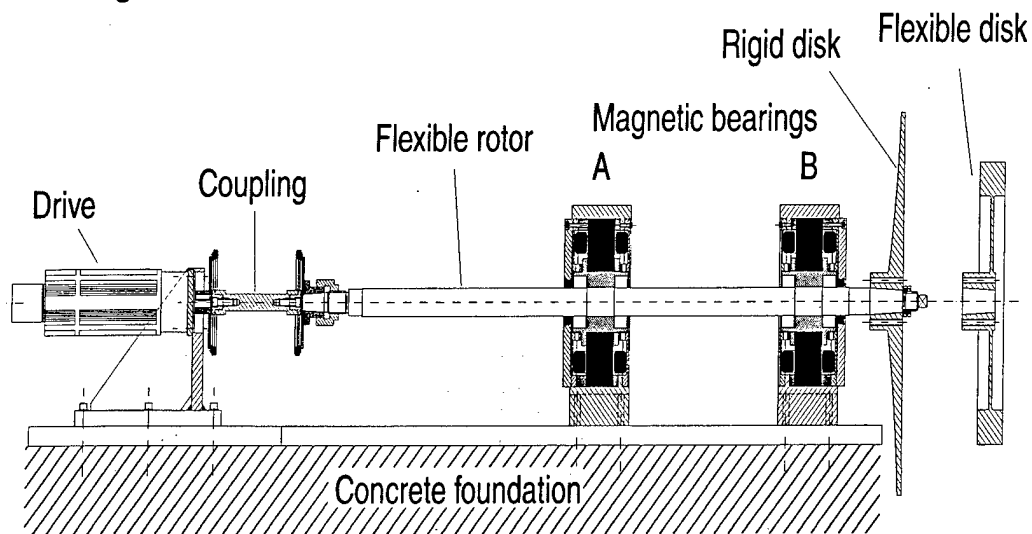


Fig. 2: Test rig configuration

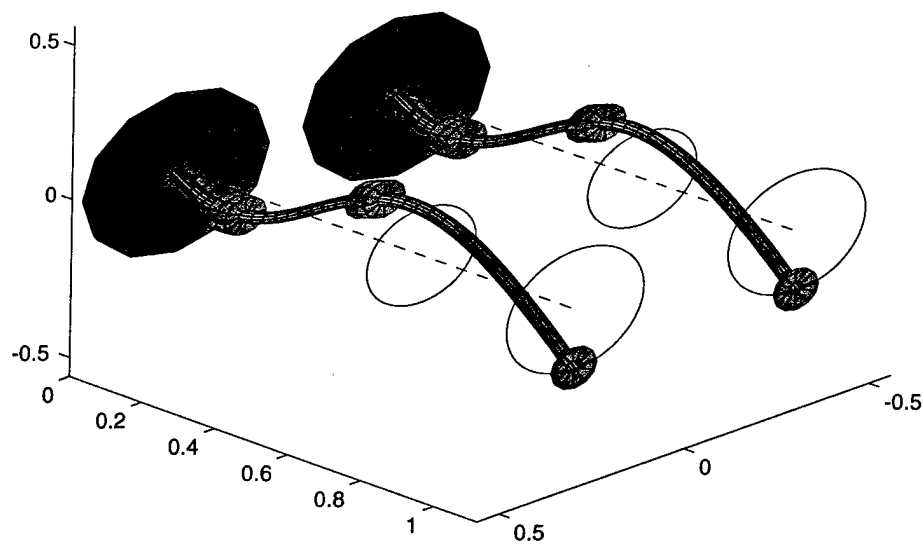


Fig. 3: A forward mode (left) and a backward mode (right) of the elastic rotor

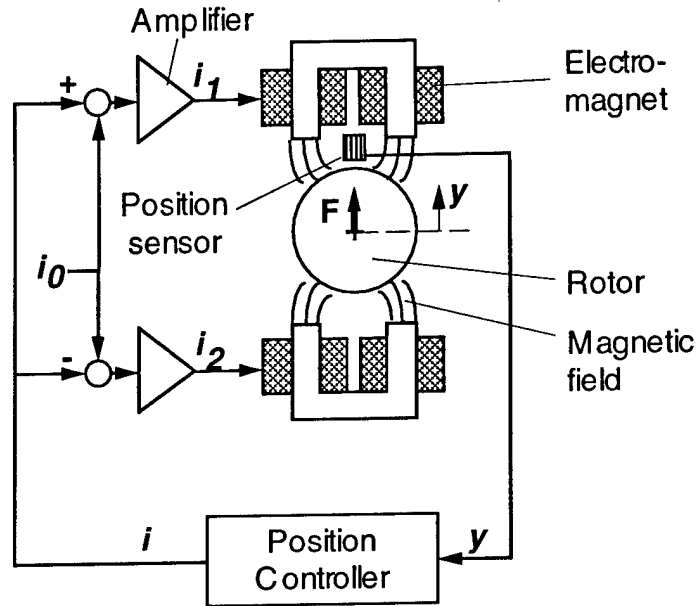


Fig. 4: Principle of a double-sided AMB for one degree of freedom

First, a simple model of the rotor bearing system was derived, and a robust control was designed that made the rotor just hover. Two ways to improve the performance were available. The model could have been refined using a modal analysis approach with updating the finite element model. This, however, would not have taken into account the needs of modeling the elements of the control loop as well, the sensor dynamics, the actuators and amplifiers. Therefore, a more direct approach was chosen. Within the closed control loop of the AMB the open loop frequency responses of the flexible rotor were measured. They describe its input/output behavior without prior knowledge of the internal structure of the rotor. The test signals were generated internally by the AMBs themselves and not by an external exciter. For the design of the controller a precise model is required and to this end the measured frequency responses were parameterized by rational functions. This identification was done in an iterative way. The controller determines to a large extent how large a model error can be accepted at a certain frequency. This means that not only the controller design depends on the model, but also the modeling depends on the controller. Thus, modeling and identification are no longer considered as two independent problems, but rather as one joint problem. The results are very satisfying. One element of the 4 by 4 transfer matrix is shown in Fig. 5.

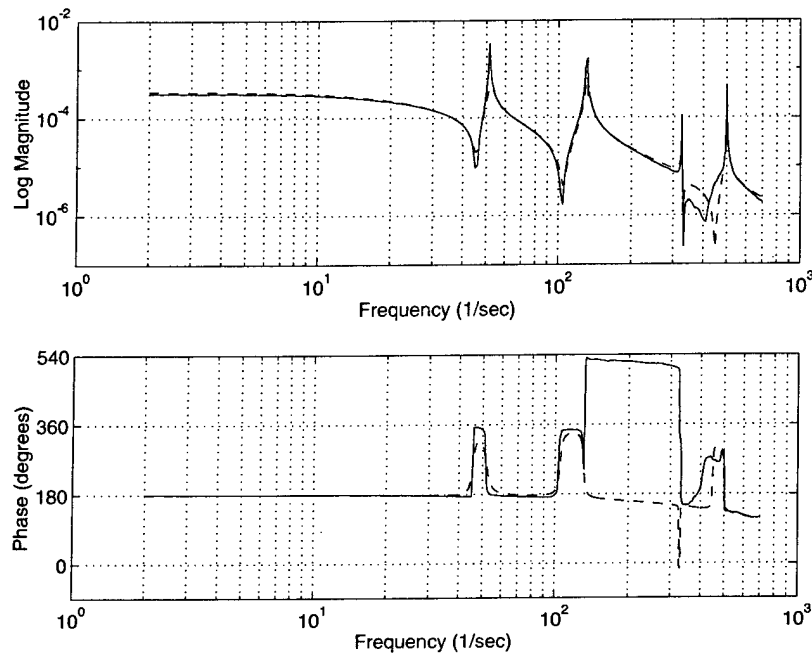


Fig. 5 Identification results. Solid: measured FRF; dashed: identified model (the larger differences in the phase are multiples of 360degrees)

## 5. Conclusions

Mechatronics has successfully developed concepts, methods and tools, which are already useful in other fields as well. It is expected that a strong relation between Structural Dynamics and Mechatronics will evolve, based on the following statements:

- The objective to build a system, product or structure to perform motions in a specified way leads to the task of synthesis. This goes beyond the classical task of modeling and understanding a system by proper analysis.
- The objective of specifying motions, in general, requires the introduction of a control loop, with means for sensing, information processing, and actuating. In particular, the integration of information processing will lead to intelligent products.
- The system models have to be extended and modified to accommodate the needs of control, too.
- By making use of the control, the procedures for monitoring and identification can be developed further into self-diagnostics and other features.

One of the consequences of this emerging trend may be that educational curricula in Structural Dynamics will have to consider control and related information processing as a basic method.



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# VIBROACOUSTICS BEYOND 2000: LOOKING FOR THE SOUND OF SILENCE

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## INTRODUCTION

The subject of vibro-acoustics concerns the interaction between structural motion and acoustic field. As such it involves a large range of frequencies, i.e. the bandwidth 20 - 20000 Hz of the human hearing. It is a custom and a necessity to divide vibro-acoustics into two different problems - external and internal, and subdivide each of them into three different parts: the low, medium and high frequency regions. The need for that relies on the different approaches that are used for the solution of each particular problem: the external problem is related to an infinite medium - typically characterized by fluid wave propagation, whilst the internal one is commonly described by acoustic eigenmodes or stationary waves. With respect to frequencies, the low frequency region involves large wavelengths or low eigenmodes, while the contrary is expected for high frequencies. And it is obvious that, from a numerical point of view, large wavelengths imply a coarse discretization while short wavelengths impel to use fine meshes. It should be evident that a strict bound among low, medium and high frequencies does not exist, in that these regions are related to the ratio between the dimensions of the acoustic medium and the involved structural-acoustic wavelengths.

Although the solution of each of these different problems requested a great amount of research until the end of the 80's, nowadays it can be stated that both the internal and external problems at low and low-medium frequencies can be quite easily solved by classical Finite Element methods (FEM) or integral formulations (Boundary Element (BEM) [1-3] or Succi's method [4]. The same statement cannot be stressed for the high frequency problems, for a number of reasons. Therefore the actual research in vibro-acoustics is concentrated on this high field, as it begun to be over thirty years ago. It is worthwhile to point out that we are here referring to the modal or wave approach to the problem. The ray approach, so far widely used for the internal acoustic problem of large closed spaces, is not considered here in that it cannot account for the interaction between vibrating structures and fluid, while it is commonly used for the design of large rooms, such as auditoriums, conference rooms, etc.

In the context of high frequency structural-acoustic problems there are two different arguments that deserve high consideration for the development of appropriate solution techniques. The first one is obviously connected to the definition of a suitable fine mesh that implies heavy numerical computations. While one could claim that the development of computers will make it possible to solve problems of larger and larger dimensions, it

should be stressed that the larger are the problem dimensions the lower is the solution accuracy, and the lower is the possibility of performing appropriate structural modifications to improve the acoustic response. The second item is related to the statistical behaviour of structural and/or acoustic systems at high frequencies: because of inherent uncertainties about the geometrical and physical parameters of the system, on the joints and/or end constraints, the vibrational-acoustic behaviour of any system differs unpredictable from similar ones when the modal density or modal overlap increases over a certain amount [5-8]. Therefore at high frequencies the possibility of obtaining accurate results by large computational models of complex systems is vain or illusory when many modes participate to the response. Figure 1, after Kompela et al. [9], clearly focuses this concept. One would keep in mind that an appropriate model for high frequency structural-acoustic problems should account for both these aspects: the need of decreasing the computational burden, related to a somehow reduction of the computational degrees of freedom, and the opportunity to create a model capable of providing a response in statistical sense.

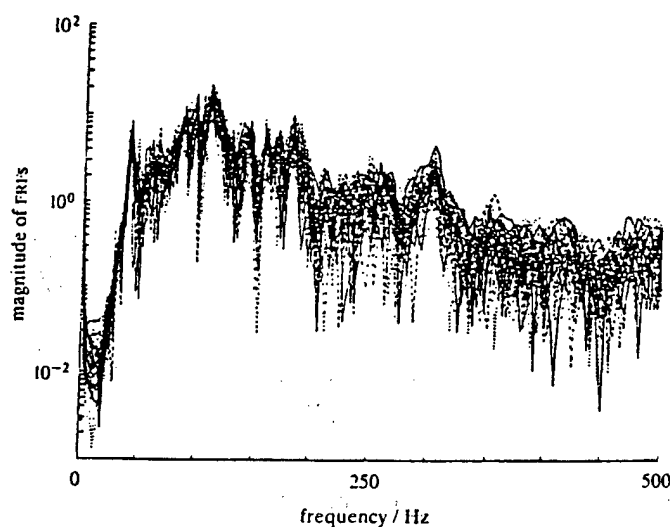


Figure 1: A series of structure-borne FRF amplitudes for pickup trucks for the driver microphone

It appears now incredible that the pioneers of the Statistical Energy Analysis (SEA), the most developed (philosophical) technique so far available for the solution of vibro-acoustic problems, had the intuition of accounting for both these aspects over thirty years ago when formulating the SEA approach. In fact SEA is supposed to be a statistical approach not accounting for the vibro-acoustical behaviour of a particular system but rather for the "ensemble-average" behaviour of an entire population. Although it is questionable that SEA is an actual statistical approach because neither an average response is in fact determined nor any kind of variance is provided nor some probabilistic distribution is required or assumed to solve the problem in SEA terms, there are smart elements in SEA capable of provide some kind of statistical response. SEA is a "minimum system descriptor necessary for a prediction of ensemble-average behaviours ... and a gross parameter model. ....SEA does not provide the ensemble-mean energy response function for a population but rather it estimates the frequency-average value of the energy

*response functions of individual archetypal subsystems over interval of frequencies*" [8]. Then, which are the "statistical" elements in SEA?

- the evaluation of modal densities by general geometrical descriptors such as the area (volume) of the structural (acoustic) subsystem, instead of its real dimensions, without accounting for the specific boundary conditions;
- the evaluation of damping and coupling loss factors without referring specifically to the subsystem under analysis;
- the use of a global space descriptor (mean-pressure or mean square velocity averaged in space) instead of a local descriptor which smooths away unavoidable uncertainties and space fluctuations;
- the use, as global descriptor, of the time-averaged energy that is phase independent;
- the possibility of overcoming one of the basic and critical SEA assumptions, the weak coupling between modal subsystems, by increasing the number of subsystems in the model, thus averaging and lowering the intercoupling effects among subsystems. This justifies the increasing use of SEA in many transport industries: a large number of subsystems are used in recent applications of SEA, and the results, that previously were almost unreliable, begin to match better with the experimental results [10].

However, notwithstanding a much larger comprehension of the SEA basic assumptions, the big amount of theoretical and experimental work developed on the subject in over thirty years, the availability of more friendly user and less expensive SEA codes, the miss of other valid alternative methods, SEA is not still considered a reliable approach to use at a design stage.

Therefore, since the end of the 70's, but more systematically since the end of the 80's, an important piece of work has been developed to overcome some SEA limitations. Some studies were particularly devoted to fill the frequency gap between the high frequency limitation of deterministic finite element methods related to mesh refinement (implying a high computational burden), necessary for a description of the short wavelengths involved, and the low frequency bound of Statistical Energy Analysis, due to an insufficient number of modes in the modal subsystems. However, most of the work was addressed to the appealing possibility of providing a more rich description of the vibro-acoustic response along the considered systems. In this framework the energy flow formulations and the envelope formulations gain a particular role: the first because in recent years it was often assumed that the high frequency vibrations could be modelled by a vibrational conductivity equation analogous to the steady equation governing the heat flow, which is an extension of SEA laws into differential terms. On the contrary, the envelope formulations disregard the thermal analogy while try to use a new description of the dynamic response by a suitable transformation that would be capable, as the thermal methods, to provide a smooth or quasi-static solution for the high frequency oscillating field.

In section 2 an overlook of the SEA state of the art with regard to its developments, limitations and perspectives is presented.

In section 3 some interesting formulations are discussed: they provide a deep insight on some basic SEA assumptions and throw a bridge toward some recent and important developments that promise to be very useful for the practical solution of structural-acoustic problems. In the same session the energy flow formulations and envelope formulations are presented.

In figure 2, a block diagram on the connection among these different approaches is

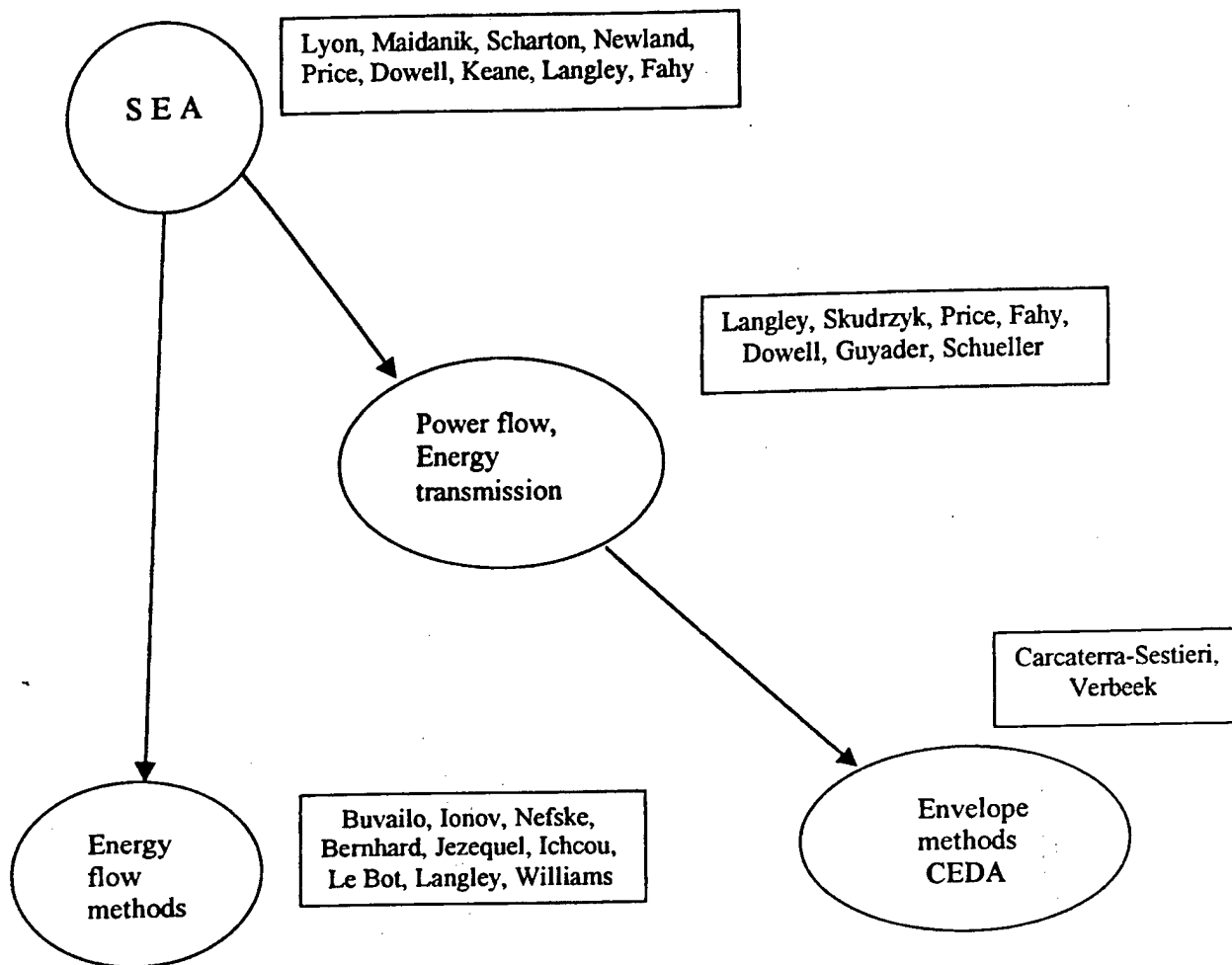


Figure 2: Block diagram representing the high frequency vibro-acoustics research

shown, with reference to some researchers that contributed to their developments.

## STATE OF THE ART ON SEA

The pioneeristic works on SEA were developed by a group of scientists (Lyon, Maidanik, Smith, Heckl, Noiseux) that met together in 1961 to discuss how to predict the rocket noise and vibrations of satellite launch vehicles ([11]). In 1962 Lyon and Maidanik published a first systematic work on SEA [12]: they consider the average power flow between two simple harmonic oscillators excited by random force, discovering that the power flow is proportional to the difference of kinetic energy between the two oscillators, i.e. the energy (more precisely the modal energy) of each system is equivalent to the temperature in thermal systems. Then they extend this result to two coupled multimodal subsystems and introduce the first restrictive assumption on

the type of coupling between subsystems that is requested to be weak, in the sense that the interaction forces between them must be much more small than the internal forces within each subsystem. In that paper they provide the concept of modal groups as the systems interchanging their energy, and that of equipartition of energy to specify that all the modes of a group contribute equally to the total energy of the subsystem. Each modal group can be regarded as a collection of simple oscillators. Furthermore, they assume that the energy exchange between the two multimodal systems can be obtained by superposition of the energy exchange among the elementary oscillators of each group.

Further developments in this area are due to **Lyon and Scharton** [13], **Kakar** [14], **Lyon and Eichler** [15], that contributed, until 1975, to a better comprehension of SEA bases and limitations.

An important work to understand the limits of validity of SEA is due to **Woodhouse** in 1981 [16]. He disregards the mathematical formulation developed by **Lyon and Maidanik** for the coupled system and proposes a general approach for studying the power flow among mechanical systems by using a technique similar to a previous one developed by **Lord Rayleigh**. In this way he reaches important and original results. He finds a condition of energetic reciprocity that is a necessary condition so that modes belonging to two different groups be coupled in the way suggested by **Lyon**. This condition is not easily verified, excluded the case of two elementary single dof oscillators coupled together and the case of a set of homogeneous resonators (**Lyon and Scharton** in 1968). With this condition he proves quite simply most of the previous obtained results, but shows that, in general, the **Lyon's** approach is not rigorous for three or more coupled oscillators. Finally he proves that the necessary condition is also sufficient under the assumptions of i) statistical independence of the random forces applied to each oscillator, ii) weak coupling and iii) small damping.

In 1983-85 **Dowell and Kubota** [17] developed an original and efficient approach to study high frequency vibration problems. It is a systematic simplification of modal analysis and it can be considered valid when the structural (or acoustic) response is characterized by a high modal density: thus his approach is called Asymptotic Modal Analysis. Either white random forces or harmonic forces can be used, the last case not being permitted by SEA. Two types of averages are performed on the response: the first one is a spatial average on the whole structure, the second one is a frequency average on the band of excitation. In this way **Dowell** obtains several classical results of SEA for both the structural coupling and the structural-acoustic coupling, providing a rigorous interpretation of SEA assumptions and, possibly more important, presents a new systematic approach for the theoretical analysis of high frequency problems.

Further insight into the SEA theoretical bases are provided by **Keane and Price** in 1987 and **Langley** in 1989.

**Keane and Price** [18] give a number of restrictive hypotheses under which the proportional link between power flow and energy exchange is valid:

- the forces acting on each modal subsystem must be statistically independent;
- power flow and energy must be averaged on rather narrow frequency bands, provided that they contain several modes;
- the subsystems' damping must be proportional;
- the coupling must be weak and conservative.

However they also stress that the weak coupling assumption can be eliminated pro-

vided that the overall response of each subsystem is not dominated by a single mode of that group: in fact this general assessment can be obtained by either a weak coupling or by assuming the presence of many interacting modes.

Langley's approach [19] provides a convincing theoretical analysis on SEA bases. He starts from the motion equations of a continuous elastic system and assumes that the exciting forces are random and spatially uncorrelated, while the coupling is conservative. He finds that the general expression of the power flow can be reduced to the proportional form proposed by Lyon when the following conditions hold:

- uniform mass density;
- damping proportional to the mass;
- energy averaged over a population of random systems;
- weak coupling, where by weak coupling he means that the Green function of any isolated component does not vary sensibly when it is coupled to the other subsystems.

In 1990 is still Maidanik to reformulate the theoretical bases of SEA [20], re-determining the governing SEA equations by means of a wave approach to the dynamic problem. The basic assumptions are:

- the subsystems energies must be averaged over space and statistically, i.e. with reference to a population of structures;
- the total energy must be considered as a simple superposition of energies associated to each elementary wave propagating within the medium.

Although limited to continuous one-dimensional systems (simple coupled waveguides) his approach is more general and an extension to complex system can be likely expected. His rigorous formulation clearly emphasizes the assumptions that must be established to obtain the SEA formulation from the general formulation of energy exchange. However, if this is true from a mathematical point of view, his formal assumptions cannot be easily interpreted in physical terms.

As a conclusion of this presentation, it is worthwhile to point out some general comments on the past and future developments of SEA.

When thinking to the governing equations and even accepting the validity of the fundamental relationships between energies and power flows (although it was shown that this result cannot be considered valid in any situation), it is very difficult to provide a valid SEA model of the dynamic system. If the SEA equations are quite simple, the effective unknowns are not only the subsystems' energies but also the coupling and internal loss factors, modal density and input powers. On such quantities the analyst does not know anything a priori. The effective simplification of SEA relies on the elimination of the space dependence (through the spatial average) and the frequency dependence (by the ensemble-average that is in fact substituted by the frequency-average). However the crucial problem is the evaluation of the coupling loss factors that are dependent on the frequency band excited by the input(s). SEA does not provide any criterion to determine these factors, although they can be evaluated experimentally or theoretically by different methods related to modal analysis, wave approach or mobility functions, or even using finite element techniques. Moreover the input power is never practically known, and its estimate open complex problems that were partly analysed by Pinnington and White [21]. Finally the definition of appropriate modal groups is not necessarily a simple task, in that modal groups do not generally coincide with structural components.

Another point that is critical and has been seriously considered in several alternative

methods to SEA is the frequency limitation of the SEA approach. In fact the limitation concerns the frequency bandwidth where the number of modes is too high for finite element analysis but too low to provide accurate averages for SEA.

To conclude it can be stressed that while the input requirements are rather heavy (input power, coupling loss factors, loss factors, modal densities), on the contrary the output is rather poor since any local information is lost as well as any resonant behaviour. Although this is inherent in the same soul of SEA that, in this way, would provide a "statistical" response, no doubt that it is felt as a limitation, and, possibly, it can explain why SEA has not yet reached the important role that it would deserve in structural dynamics and vibro-acoustics.

## FROM SEA TOWARD ENERGY FLOW AND ENVELOPE FORMULATIONS

After the definition of the concept of power flow or structure-borne intensity by Noiseux in 1969 [22], related to the product of force and velocity (or stress and velocity), many basic experimental and theoretical studies were developed by several authors.

Based on these power flow concepts, especially during the 80's new approaches were studied and new formulations proposed to provide a more deep insight into SEA philosophy, fundamentals and implications, to avoid the uncertainties of SEA results and to provide a vibrational energy distribution rather than an overall mean energy as in SEA. Although only few of these formulations are really convenient for the solution of vibro-acoustic problems, they gave an important stimulus to new research in high frequency vibro-acoustics.

### Alternative approaches to SEA

Certainly the Asymptotic Modal Analysis of Dowell, described in the previous section, belongs to this category, being contemporary a procedure capable of explaining many of the SEA assumptions, while providing a systematic analysis for problems at high frequencies.

Important contributions to the comprehension of some SEA results, but not necessarily related to this goal were given by Newland who, as one of the maximum experts in random vibrations, developed studies on coupled oscillators subjected to random forces. In particular in [23] a perturbation method is proposed for calculating the statistics of energy transfer between weakly coupled oscillators, showing that the first order approximation for the mean power flow coincides with the classical SEA result. Moreover it is stressed that this approach can provide important statistics and more accurate results than SEA when second and higher order approximations are considered.

Skudrzyk in 1980 [24] studied an interesting and basic approach to predict the dynamic response of complex coupled systems. Unlike SEA, the mean-value method of Skudrzyk is not a statistical theory, although it shares with SEA a basic task in that it tries to eliminate unnecessary details to describe the dynamic response of complex and coupled structural systems. By a limited knowledge of few structural parameters, such as the mass and the density of resonances, and by some information on the excitation and the response location, the mean-value method predicts a mean-line through the



frequency response curve of the vibrating system and the envelopes of the resonant and antiresonance peaks in almost the whole frequency range of interest, thus not showing the classical limitations of FEM and SEA.

In trying to eliminate the frequency gap between FEA and SEA, and proceeding along the line shown by Skudrzyk, Cuschieri uses the power flow concept as complementary to FEA and SEA [25], by considering the mean level of the transfer function mobilities: he shows that the mean responses are independent on the exact geometries but depend only on the general structural characteristics. He also stresses that, at high frequencies, the power flow method converges to SEA results, provided that frequency averages are suitably performed.

A very important work in this framework is the Wave Intensity Analysis (WIA) proposed by Langley in 1992 [26]. In the WIA Langley regards the displacement field as the result of a superposition of waves travelling along any direction, with proper amplitudes and phases. If the phase dependences are neglected, the effects of resonances and antiresonances, related to wave interactions in correspondence to in-phase and out-of-phase effects, are eliminated. From a modal point of view, this corresponds to perform modal averages on the response. Once the phase dependence is eliminated, only an energetic beam is associated to each wave, corresponding to the mean energy of the travelling wave. The general beam which is dependent on the spatial coordinates, is expanded into Fourier series. An energy balance equation is then written for the interacting beams and the coefficients of the Fourier series are determined by a Galérkin procedure. If the series expansion is arrested to the first linear term, Langley shows that the energy balance exactly corresponds to the SEA equations. In this context WIA is a generalization of SEA, and SEA becomes an approximate formulation in which:

- the phase effects of the travelling waves are eliminated, while the reflection, dissipation and transmission effects are still accounted for;
- the energetic field is approximated by the superposition of the mean energy (first term of the series expansion) transported by each wave.

Thus Langley determines more general equations than the classical SEA and proves that, in this way, better results than SEA are usually obtained.

### The thermal analogy and related approaches

Particular mention deserves, in the framework of methods alternative to SEA for the analysis of vibro-acoustic systems, a set of similar procedures generally known as heat conductivity methods or thermal analogy approaches. In fact, they represent a very attractive development, promising to overcome SEA limitations while providing results of higher informative content. However, recently several arguments have been discussed that contrast the validity of the thermal analogy bases.

In 1977 three soviet researchers (Belov, Rybak and Tartakovski [27]) and later on in 1979 Buvailo and Ionov [28] proposed an extension of the SEA laws into differential terms. More precisely they assumed that elemental volumes within any vibrating elastic medium exchange energy in direct proportion to the difference of their energy levels. Thus, by using the same relationship proposed in SEA for the dissipated power, they

establish a local energy balance as:

$$\frac{\partial E}{\partial t} = -\nabla \bar{q} - \Pi_{diss}$$

where  $E$  is the energy and  $\bar{q}$  the power flux obtained as the product of the stress tensor by the velocity vector. Then, thanks to the thermal assumption ( $\bar{q} = -\mu \text{grad} E$ ) and to the hypothesis that the dissipated power  $\Pi_{diss} = \alpha E$ , where  $\mu$  and  $\alpha$  are constants depending on the material and excitation frequency, they obtain, for a stationary process:

$$\nabla^2 E - \beta^2 E = 0 \quad (1)$$

This equation is formally equivalent to the heat diffusion equation, as it is expected in that it is obtained by an extension of the macroscopic thermal SEA law into differential terms.

With respect to the classic equation of motion described by some form of hyperbolic differential wave equation, the previous equation presents outstanding advantages from a numerical point of view. In fact, the heat equation is a parabolic equation describing a diffusion phenomenon and admits solutions exponentially decaying from the source, without oscillations. On the contrary, the wave equation describes a propagation phenomenon and has oscillating solutions in space, whose wavenumbers increase in direct proportion to the exciting frequency. This difference permits to solve the parabolic equation with a coarse mesh that is usually frequency independent while the wave equation requires a mesh that becomes more and more demanding as the frequency increase and the space passes from one to two and three-dimensions. This finally implies a prohibitive computational burden for any computer.

Probably because published in Russian, the works by the soviets were almost ignored until Nefske and Sung [29] reformulated the problem in 1987, and proposed, for the first time, some interesting applications on beam structures excited by harmonic concentrated loads. The comparison with modal analysis results show that the solution of the thermal equation provides a kind of average trend of the energy density along the beam (figure 3, after Nefske and Sung [29]). The agreement is good, even for coupled system of beams, although it represents an heuristic solution providing acceptable results for one-dimensional systems only. Notwithstanding the poor theoretical assumptions of this approach, some groups begun to work on it, by proposing interesting applications to coupled beams (Palmer, Williams, Fox [30, 31]) that confirm the good results for one-dimensional systems but show severe limitations when extended to two-dimensional plates. In this framework, Bouthier, Wolhever and Bernhard developed theoretical considerations related first to one-dimensional systems, then to plates [32, 33]. They state that the diffusion equation can be considered valid under the hypotheses that:

- the excitation is concentrated and harmonic;
- the reference energy is the total energy per unit length (sum of kinetic and potential energy) averaged in time over a period of oscillation; moreover some kind of local average must be performed to eliminate harmonic terms appearing in the reference energy;
- the near field contribution is neglected, i.e. the energy field must be considered far away (more than a wavelength) from the source.

Under these assumptions the results they obtain are identical to those of Nefske and Sung, although they attribute different meanings to the energy variable in the thermal

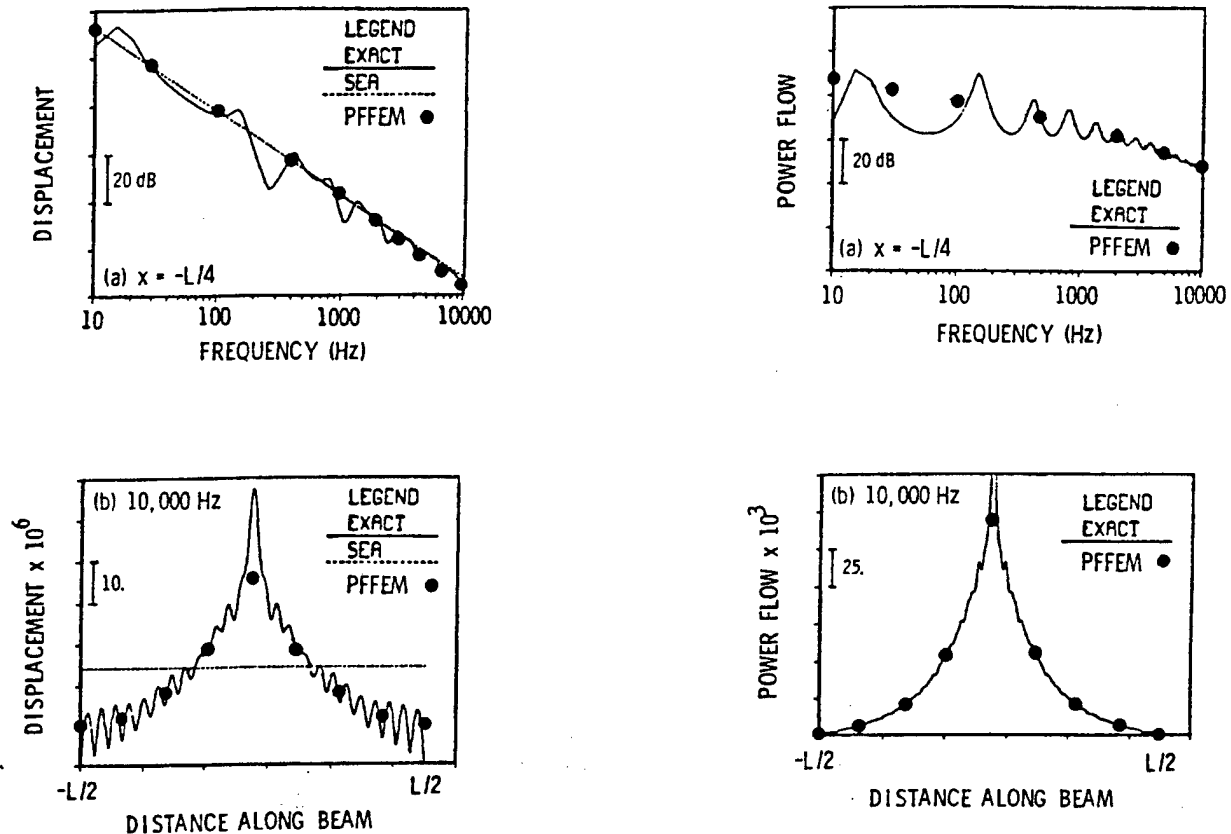


Figure 3: Comparison of Nefske-Sung results with exact modal solution and SEA

equation. Nefske and Sung interpret it as local energy averaged on time and then on some appropriate frequency band (SEA inspiration); on the contrary Wolhever and Bernhard [32] give to it the meaning of local energy averaged on time and space, in such way eliminating the harmonic energy terms. Although their analysis refers rigorously to one-dimensional systems only, successively Bouthier and the previous authors assume the general conviction that if time and spatial energy averages are performed, the resulting energy variable satisfies a diffusive equation. Under this subjective assumption they extend the thermal approach to two-dimensional systems [33].

At the beginning of the 90's **Le Bot, Jezequel** and others began to study energy propagation in structures with the aim of developing alternatives procedures to SEA at high frequencies. Originally [34] they developed a General Energy Formulation (GEF) for describing the spatial distribution of the time-average energy densities and lagrangian of continuous structures, and determined a pair of exact linear differential equations of eighth order. Because of the much more difficult mathematical problem than the original physical equation of motion in terms of displacement, it is obvious that these equations are meaningless for a solution procedure of vibro-acoustic problems.

Incidentally it can be stressed on this purpose that although the energy is a quite appealing variable for studying vibro-acoustic problems in that both the sound and the structural vibrations can be expressed in terms of it, difficulties arise when looking for mathematical matching conditions between the structural and acoustic fields because

they usually have different energy values. Moreover, the structural analyst is not generally interested to know the energetic state of vibrations because it is not the energy level responsible only of the noise radiated by the structure or of the fatigue life of a structural component, that rather depend on the frequency or the stress level in the system.

To overcome the numerical difficulties related to the GEF formulation, **Le Bot and Jezequel** [35] produced for the total energy a simpler equation (Simplified Energy Formulation: SEF) by performing spatial averages on the single wavelengths. At least for one-dimensional systems, it corresponds to the heat equation, giving more credit to the thermal approach. Successively **Le Bot and Luzzato** [36] try to extend the SEF model to two-dimensional systems, but they do not succeed in determining a heat equation for them. The non validity of the thermal analogy for two-dimensional structures was previously obtained by **Le Bot and Jezequel** [34], when examining the case of circular membranes excited by a harmonic force, by numerical simulations and theoretical arguments.

In 1996 **Ichchou, Jezequel and Lase** [37, 38] propose a wave approach to build up an energy description of some dynamic systems, in contrast with the approach followed by **Bernhard *et al.*** in that the concept of space average used by them is not considered generally acceptable. However, using the same hypotheses proposed by the previous authors and adding the assumption of neglecting the interference among propagation waves (by means of a statistical average between two general propagation directions, as explained later in this section) to generalize the space average concept, **Ichchou and Jezequel** derive and confirm the same heat diffusion equation, that they stress to be valid for one as well for two-dimensional systems. In particular the energy model derived by them for plane wave fields is written in terms of the group velocity of the plane waves as:

$$-\left(\frac{c_g^2}{\eta\omega}\right) \nabla^2 \langle \bar{e} \rangle + \eta\omega \langle \bar{e} \rangle = 0$$

where  $\langle \bar{e} \rangle$  is the time and space average of the energy density, and  $\eta$  is the loss factor. This result is quite important because it would confirm that the thermal analogy can be justified by different approaches, by introducing only few new assumptions: in this case the elimination of wave interference in the high frequency range. Wave interference is an important concept considered also by **Langley** in developing his wave intensity technique, but also by **Carcattera and Adamo** to justify the non-validity of the thermal approach, especially for two-dimensional systems: this point will be discussed later on. In any case, these authors, by their wave approach, claim for a generalization of the thermal analogy, and in fact in successive papers they present their model for one-dimensional systems (bars and beams) [37] and then for plates and general two-dimensional structures [38].

### Contributions to a better understanding of energy flow in structures

An important group of papers in the 90's deeply analyse the problem of energy flow in structures: most of these papers originate from a critical review of the thermal analogy.

A first analysis on the subject was given by **Carcattera and Sestieri** and later on by **Langley, Smith, Xing and Price, Carcattera and Adamo, Le Bot, Orefice, Cacciolati and Guyader** by theoretical and experimental developments. A work that can be appropriately inserted in this context, although having a different background

derived from the dynamics of stochastic systems, has been recently presented by Pradlwarter and Schuëller.

As already stressed, the thermal analogy tries to describe the mechanism of mechanical power propagation within a structure, by assuming that the power flow is only proportional to the gradient of the energy density. In [39] the following results are shown by theoretical considerations.

- For longitudinal rods, the thermal analogy is valid provided that no dissipative effects are introduced into the system. In presence of dissipative effects the thermal analogy can be obtained only if some kind of spatial average is performed so that the time-average potential energy is equal to half the total time-average energy.

- For flexural beams, the heat equation as determined in [29] is not valid even if the near field contributions are neglected: in fact, the time-average far field energy has two terms that are not thermal. Moreover, in presence of dissipative effects, it is not possible to derive a time-average energy equation from the displacement equation of motion.

- For flexural plates, the time-average energy equation differs from the heat equation.

On the bases of these conclusions, further arguments can be developed to contrast the validity of the thermal analogy.

While SEA states its thermal propagation law for the mechanical energy with reference to systems of finite dimensions, the thermal analogy tries to extend this law into differential terms, thus referring to elemental systems. Although SEA equations can be obtained by different approaches, the SEA transmission law relies on the following points: i) each subsystem is equipped with modes; ii) the number of modes in the bandwidth must not be too small; iii) the modal groups must be weakly coupled. On the contrary the elemental elements of an elastic medium do not present a modal behaviour and are strongly coupled because the interaction forces are of the same order of magnitude of the internal forces. Therefore there seems to be a methodological error in extending the SEA laws into differential terms, and it would be inappropriate to describe the power flow in structural problems by the thermal analogy: a more complex mechanism would be required.

Another important contribution to a better understanding of the energy propagation mechanism is given by Carcaterra and Adamo in [40]: this work gives a special insight into the thermal wave approach, and can be conveniently correlated to the works by Langley [26, 41] and Ichchou and Jezequel. In [40] the authors highlight different scale laws in the mechanical energy transmission, controlled by a suitable  $\mu$  parameter related to the ratio between a characteristic space-average length and the excitation wavelength. For  $\mu < 1$  (small scale) the vibrational conductivity fails, while for  $\mu > 1$  the asymptotic thermal analogy is valid for rods and beams but not for plates or other two-dimensional systems. The difference is explained by considering two different types of energy interaction between wave energies: the coincident wave energy (c.w.e.) determined by waves propagating in the same direction and the interference wave energy (i.w.e.) depending on waves propagating along different directions. It is shown that their asymptotic features are totally different: while the c.w.e. tends to satisfy asymptotically the energy balance in thermal form for  $\mu > 1$ , the i.w.e. exhibits a complex behaviour showing a non thermal contribution even for  $\mu > 1$ . It is focused by asymptotic considerations that the non thermal component is largely dominant in the whole medium scale range. The authors clearly show by theoretical and experimental tests performed by a scanner

laser that while the one-dimensional systems tend to behave thermally in the large scale because of the c.w.e mechanism, two and higher dimensional systems are dominated by the i.w.e. mechanism, that inhibits the thermal behaviour.

Actually the interference problem is often considered by a statistical point of view. Ichchou and Jezequel in the mentioned papers, but also Langley in his WIA, assume that the expected value between two wave phasors associated to two general directions of propagation  $\theta_i$  and  $\theta_j$  are uncorrelated, i.e.

$$E\{A(\theta_i)A^*(\theta_j)\} = 0 \quad (2)$$

Carcattera and Adamo suggest in [40] that equation (2) would hold for reverberant fields while, for a direct field, an "isotropic condition" ( $E\{A(\theta_i)A^*(\theta_j)\} = \text{const}$ ) should be rather considered. Therefore the thermal analogy would be valid for reverberant fields but not for direct fields: thus the sum of these two contributions, total field, would not be thermal.

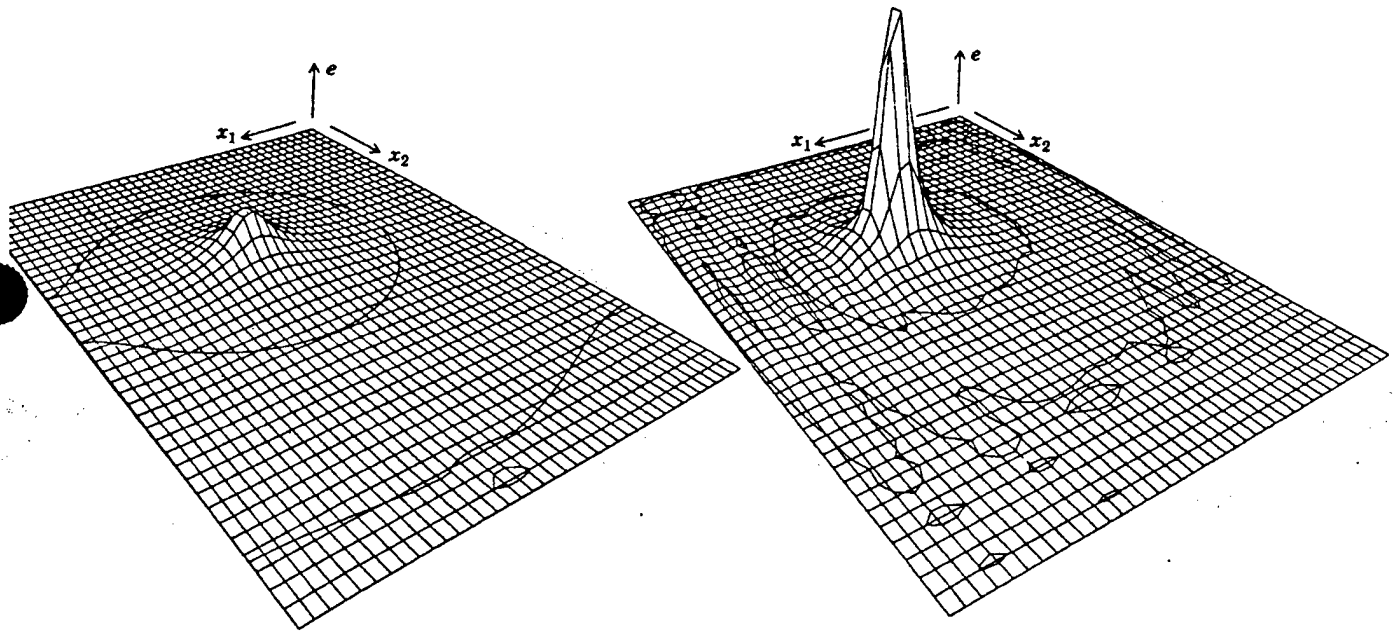


Figure 4: Comparison between the surface plot of the energy density obtained by the vibrational conductivity equation (left), and exact analysis (right)

Similar concepts are considered by Langley in [41]. Particularly he shows that:

- for axi-symmetrical wave fields, the wave interference cannot be neglected so that the thermal analogy fails. More specifically he states that the conditions under which the vibrational conductivity equation holds are not met by the direct field arising from a point load: the heat equation predicts a far field energy density which decays in proportion to  $1/\sqrt{r}$ , while the exact analysis yields  $1/r$

- for two-dimensional structures such plates, the vibrational conductivity approach yields an energy distribution which is more spatially uniform than the true result (see figure 4 after Langley, [41]).

Addressed to the same goal is the paper by Smith [42], who tries to reformulate the thermal analogy by separating the direct and reverberant fields, so obtaining better results than those related to the standard thermal analogy.

In the same line of the previous papers is the work by Xing and Price [43], that develop a mathematical model describing the energy flow associated with the dynamics of a continuum: in this way they derive the energy flow equations of dynamically excited systems, and analyse the particular cases of rods, beams and plates, under the point of view of the heat conductivity. They state that necessary and sufficient condition for the existence of an energy flow potential, that can justify the proportionality of the energy flow to the gradient of the energy density, is that the energy flow vector is irrotational. By analysing the case of a simple rod, they show that a similarity between the energy transmission along the rod and the flow of thermal energy in a heat conduction problem does not exist, and conclude by stressing that the development of any hypothesis or modelling based on such analogy is of limited value.

A convincing work for the study of high frequency problems based on the energy formulation seems to be a recent paper by Le Bot [44]. It represents the last evolution of previous works by the same author, Jezequel and Guyader, and summarizes many of the works on the energy flow by Bernhard. It could be said that it keeps the wealthy part of all the energy formulations, while eliminating most of the controversial parts. He considers the wave field generated by a point source in a free field and determines a relationship between the intensity vector and the energetic field: in this way he finds a simple energy balance equation that, in general, is not thermal anymore, thus confirming several results determined in [39, 40].

Some analogies with this last work can be found in a paper by Orefice, Guyader and Cacciolati [45]. They introduce a power/energy transfer function, and discuss some important properties of it. Particularly, it is discussed whether and when the energetic cross-terms (wave interference) are negligible, making quite clear the limitations of this assumption, especially related to the stochastic characteristics of the input power.

Another work [46], addressed to the comprehension of energy transmission phenomena in vibro-acoustics, is a paper by Pradlwarter and Schuëller. It has a different origin from those so far analysed in that it refers to the dynamics of stochastic systems. Actually this should not wonder because SEA itself finds a basic support on this branch of mechanics: Morse, Ingard and Lyon have several common connections, and Morse, a well known physicist, developed a lot of work on both atomic physics and acoustics, finding analogies between the acoustic propagation and the wave theory in atomic physics (see Morse: *Vibration and Sound*, McGraw Hill, 1948).

Pradlwarter and Schuëller in [46] determine energy equations for dynamic systems similar to those obtained by SEA, valid even for strong non-conservative coupling. They start their analysis from the Lyapunov equation describing the evolution of the output covariance matrix for a linear system under random excitation. This seems to be a correct starting point for the energy analysis because the covariance matrix represents:

- the simplest statistical indicator for the system's response;
- the simplest indicator of energy distribution among the system's degrees of freedom.

In particular its diagonal terms represent the expected values of the energetic levels for each degrees of freedom.

The analysis developed in [46] shows how the energy is stored and transmitted within

mechanical systems. While the method is not useful to predict the structural behaviour at high frequencies because the knowledge of the eigenparameters participating to the response is required, however it works at low frequencies where the number of participating modes are few: in this case the energy distribution of the system is determined as well as the coupling loss factors.

### The envelope methods

The envelope models developed by Carcaterra and Sestieri try to overcome the theoretical limitations encountered by the thermal methods. In the years three different methods were successively proposed: the envelope energy model (EEM) [47], the envelope-phase energy model (EPHEM) [48] and the complex envelope displacement analysis (CEDA) [49].

In the EEM an energy variable is used (the kinetic energy density), obtained by an envelope energy definition that uses the Hilbert transform averaged over time and space. In the EPHEM an energy variable is still used that is exactly defined as in the EEM, but the structural response is also characterized by a second variable, the phase, introduced to recover energy jumps at the discontinuities. With this model the physical dynamic response can be reconstructed combining together the envelope energy and the local phase: a main drawback is related to its nonlinear character.

Completely different is the CEDA approach. The new variable is not anymore the energy but rather a variable directly related to the physical displacement. The dynamic response is determined by the local displacement without performing any kind of average. Unlike EEM and EPHEM, neither the knowledge of transmission or reflection coefficients nor the input power are necessary, because the forcing term is a complex envelope force directly related to the excitation force only.

The complex envelope displacement theory relies on a suitable variable transformation, achieved through the action of the envelope operator  $\mathbf{E}$  on the displacement  $w$ , defined as:

$$\mathbf{E}(\cdot) = [\mathbf{I} + j\mathbf{H}(\cdot)]e^{-jk_0x}$$

where  $\mathbf{H}$  and  $\mathbf{I}$  are the Hilbert and identity transformations, respectively,  $k_0 = \omega_0/c$  is the carrier wavenumber, corresponding to the harmonic excitation frequency  $\omega_0$ , and  $c$  is the phase wave speed in the considered system. Therefore for the new variable - complex envelope displacement  $\hat{w}$  - one has the relationship:

$$\hat{w} = \mathbf{E}[w] = [w + j\tilde{w}]e^{-jk_0x} = \hat{w}e^{-jk_0x}$$

where  $\hat{w}$  is the analytic displacement  $\hat{w} = w + j\tilde{w}$ ,  $\tilde{w}$  the Hilbert transform of  $w$  and  $j$  the imaginary unit.

By this definition a very convenient transformation is obtained, at least for one-dimensional systems. It can be easily shown that, if the physical displacement spectrum is band limited around the carrier wavenumber  $k_0$ , the complex envelope displacement is band limited around the wavenumbers' origin. In fact, for a band limited spectrum around  $k_0$ , the Fourier transform  $W(k)$  of the physical displacement is concentrated within two limited regions around  $\pm k_0$  of bandwidth  $\Delta k$  (figure 5a). The Fourier transform of the analytical displacement  $\hat{w}(x)$  is then given by:

$$\hat{W}(k) = W(k) + j\tilde{W}(k)$$



and it can be verified that it provides:

$$\widehat{W}(k) = W(k) + \text{sign}(k)W(k)$$

i.e. the negative wavenumber contribution of  $W(k)$  is deleted, and the positive one is doubled (figure 5b). Finally the Fourier transform of the complex envelope displacement provides  $\widehat{W}(k) = \widehat{W}(k + k_0)$ , implying a shift of the positive wavenumber contribution of  $\widehat{W}(k)$  towards the origin of wavenumbers (figure 5 c). According to the Nyquist criterion, this property suggests that, unlike the physical displacement requiring a space sampling that increases with the excitation frequency  $\omega_0$ , the complex envelope displacement, which is a low wavenumber function, can be correctly described by a limited number of samples. Therefore, provided that a suitable governing equation is obtained in terms of this new variable - which is actually available [49], the dynamic response of the structure can be obtained by a coarse set of sampled points even for high frequency excitation. The physical displacement rapidly oscillating in space is transformed into a slowly oscillating solution that presents the further advantage of admitting an inverse transformation so that, if needed, the physical displacement can be recovered from the envelope displacement. Similar developments can be obtained for the pressure field in acoustic problems.

In figures 6a and 6b the complex envelope displacement and the recovered physical displacement for the flexural beam of figure 7 are presented, showing the advantages of this approach.

Notwithstanding these promising results, it must be stressed that some drawbacks should be overcome before a complete extension to two-dimensional systems and to coupled structural-acoustic problem can be reached. In fact the extension of this approach to two-dimensional systems is not immediate in that it implies the elimination of a spurious solution in the CEDA formulation and a more complex treatment of the wavenumber spectrum shift.

With reference to the first point it is worthwhile to point out that the spurious solution is easily eliminated in the one-dimensional problem by a suitable choice of the boundary conditions, but a similar procedure is not applicable for higher dimensional cases: therefore the problem of the spurious solution has been reconsidered from its origin, and it is expected that the way for its elimination can be reached soon.

With respect to the response spectrum, it must be underlined that, for two-dimensional systems, the spectrum of the analytic signal is not anymore concentrated around the wavenumber  $k_0$  but rather it occupies an approximate ring-shaped zone in the two-dimensional wavenumber spectrum around the radius  $k_0$ , that is much more difficult to shift into the wavenumbers' origin. Different approaches to overcome this difficulty have been studied and compared, and a definite answer is expected shortly.

Momentarily an hybrid approach has been developed to provide a solution to two-dimensional problems [50] (Adamo, Sestieri and Carcaterra). Although the original ideas of this method rely on the CEDA procedure, it is not anymore an envelope approach, so that it cannot be considered in the framework of the envelope methods.

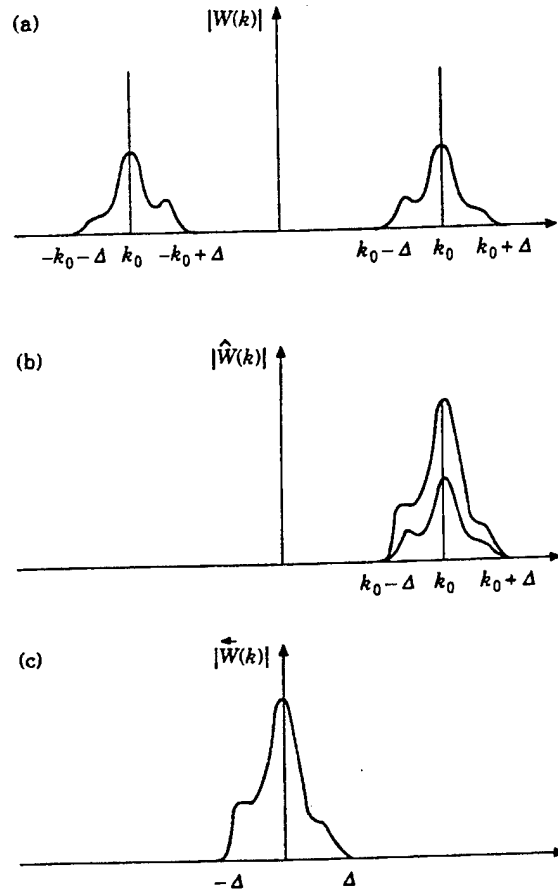


Figure 5: Fourier transforms of the physical displacement (a), analytic displacement (b), complex envelope displacement (c)

### Comments on the field variables and governing equations

Any high frequency approach uses, instead of the physical displacement, a different field variable: typically some kind of energy average is introduced. Consequently the formulation of any technique is implicitly performed in two fundamental steps:

- the definition of a new descriptor;
- the determination of related governing equations.

The definition of a new variable  $\xi$  is obtained by the action of a transform operator  $\mathbf{T}$  on the physical displacement  $w$ :  $\xi = \mathbf{T}w$ .

To determine the equation governing the new field variable it would be necessary to start from the equation of motion  $\mathbf{L}[w] = p$ , where  $\mathbf{L}$  is a structural dynamic operator and  $p$  the external load. The new equation is determined as:

$$\mathbf{G}(\xi, p) = 0$$

Two operators are then introduced: a transform operator  $\mathbf{T}$  and a governing operator  $\mathbf{G}$ . To understand advantages and limitations of each formulation it is of paramount importance the nature of the transform operator and, particularly, the existence of the inverse transformation  $\mathbf{T}^{-1}$ , provided that new governing equation  $\mathbf{G} = 0$  can be solved numerically at a much lower cost than the original equation of motion.

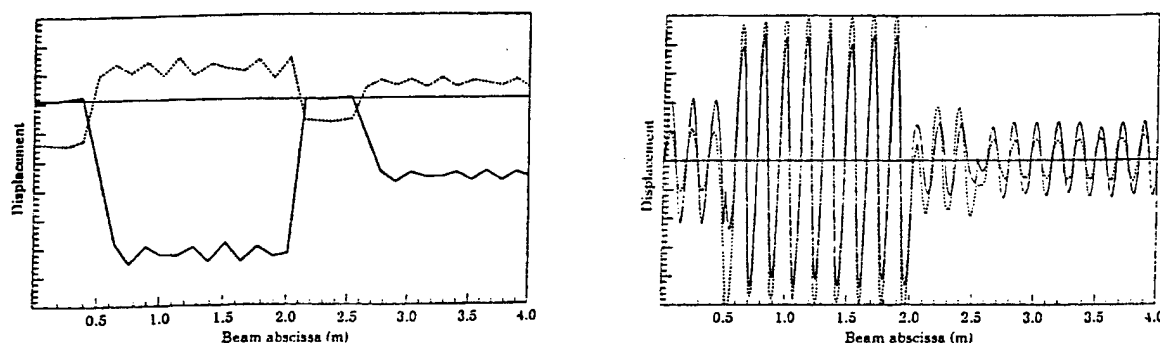


Figure 6: Complex envelope displacement - real (—) and imaginary (- - -) components - and comparison between physical (—) and recovered solution from CEDA (- - -)

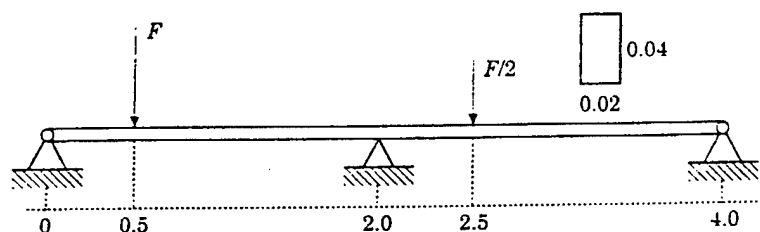


Figure 7: Flexural beam

When the existence of the inverse operator fails, serious drawbacks arise:

- the use of  $\xi$  instead of  $w$  produces a loss of information. This is a typical situation with any energy approach. The knowledge of an average energy does not allow to recover the physical displacement, and any information on the local response is lost;
- the existence of  $G$  itself is not guaranteed, and, even if it exists, its determination is not simple;
- the evaluation of the boundary and joint conditions for the new variable is a difficult task. Very often, as in SEA and thermal approaches, different experimental and numerical techniques are required to determine appropriate coefficients that are necessary to assemble the systems together;
- even when  $G$  exists, the forcing term cannot be determined in function of the physical load  $p$  alone, but rather depends on both  $p$  and  $w$ . This is typical of all energy approaches, where the forcing term is the input average power.

These limitations seem to be the price to pay for the use of the new variable  $\xi$ , presenting a lower informative content than  $w$ , although, for the same reason, the solution of  $G(\xi, p) = 0$  implies a lower computation cost than  $L[w] = p$ .

However this is not a general rule: provided that the inverse of  $T$  exists, sometimes it is possible to obtain a simple solution for the problems mentioned above.

Although a common definition of the energy variables is not encountered in the thermal methods, so that a non unique definition of the transform operator can be provided, in general for them the inverse of  $T$  does not exist, with consequent problems for the joint conditions, the impossibility of recovering the physical solution and the difficulty of computing  $G$ .

On the contrary, for the envelope methods it is shown in that the envelope operator admits an inverse, i.e.  $E^{-1} = \text{Re}\{(\cdot)e^{jk_0x}\}$ . In particular for the CEDA, the transform

operator is the envelope operator, which is invertible, and the form of  $G$  is easily determined by the relationship  $G = ELE^{-1}$ . This circumstances would suggest that the CEDA approach can provide reliable results under good general conditions: it is possible to recover the physical displacement, it is easy to express the boundary and joint conditions in terms of the physical conditions and, last but not least, the forcing term is a simple function of the physical load.

## COMMENTS AND CONCLUSIONS

Vibro-acoustics covers the human hearing bandwidth between 20 and 20000 Hz. Only for the low range reliable and efficient solutions exist that can be obtained by classical finite element methods or boundary elements or the Succi's approach. For the medium or high ranges, SEA, energy methods and envelope methods have been proposed to provide solutions that can be used at a design stage. Unfortunately neither SEA has reached full acceptance nor the other methods are still considered valid alternatives.

Notwithstanding the large amount of work developed, there is still a real theoretical and practical interest - and a large space - on this field that should push researchers to turn their interest toward this argument. Industrial requirements would be a convincing reason, but I would say that the subject per se is really fascinating involving interdisciplinary topics ranging from theoretical physics, statistics, acoustics and classical structural vibrations.

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# SD2000

## STATE OF THE ART REVIEW: DAMPING

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*This review article in its present form does not contain any equations, diagrams or references. However, the final version for the meeting will be complete.*

### 1. Introduction

Damping is a physical process which dissipates energy through the conversion of work into heat. In engineering structures it is present in several forms: internal hysteresis in materials, friction via the rubbing action of surfaces or particles, viscous friction in fluids, radiation damping, electromagnetic effects.

Damping plays a very important role in structural dynamics and damping augmentation is the primary means by which resonant amplitudes are controlled, enhancing life cycle behaviour, durability, quality of life and cost reduction.

The methodologies for introducing damping into structures can include combinations of passive, semi-active, active, adaptive techniques. These make use of viscoelastic materials, ferroelectric and ferromagnetic materials, ER/MR fluids, SMAs, auxetic polymers, liquid crystals and are often employed with electric circuits (shunts) or non-linear control laws for improving structural damping.

Damping models and damping prediction/optimisation methods tend to be somewhat fragmented and limited in their ability to relate to reality. Generally linearity is assumed and the physics of the dissipation process is sometimes unclear (eg particle damping). Material models tend to be at a macro level and often employ assumptions that are clearly violated in practise. Many damping materials have extreme limitations with regard to temperature, frequency and creep behaviour and may undergo phase transformations that are excluded in the modelling process.

This paper will attempt to address the key issues relating to damping in structural dynamics.

## 2. Passive Damping

Passive damping is, in general, simple to implement and cost effective, requiring no on-line control. The most well known form of passive damping is the use of a viscoelastic material in either a constrained layer or a free layer configuration.

Constrained layer damping (CLD) works by dissipating cyclic shear energy in a thin constrained layer of viscoelastic material of high shear modulus, thus the material should be added in regions where the surface strain (or the curvature) is a maximum.

Free layer damping (FLD) works by dissipating extensional strain energy and the modal loss factor is proportional to the loss modulus ( $E\eta$ ), thus a high Young's modulus is advantageous.

The importance differences between FLD and CLD are related to the effective stiffness of the viscoelastic material (FLD requiring a high stiffness material) and to the fact that the loss factor for CLD is frequency dependent whereas for FLD it is independent of frequency.

Viscoelastic materials are frequency and temperature dependent. Their properties are usually described by Master Curves. These properties can be employed directly with FE codes for forced harmonic response predictions or with the Relaxation curves, which are basically the inverse Fourier transforms of the frequency domain Master Curves, in time response studies.

The important parameters for optimising the damping of CLD treatments are the viscoelastic layer modulus, the thickness of the layer and the flexural stiffness of the constraining layer.

It has been shown that in some instances a compromise has to be made between a high or a low viscoelastic layer modulus. For example, extensional modes require a lower constrained material modulus than flexural modes, which are dependent on the shear in the viscoelastic layer.

One idea proposed to 'smooth' the damping between the two different modes of vibration is to use an anisotropic viscoelastic material which effectively reduces the transverse Young's modulus by the ratio of  $E(\text{anisotropic})/E(\text{isotropic})$  resulting in an improved damping performance over a wide frequency range.

These damping techniques are, in principle, well established. However, from an applications and modelling point of view the performance of such treatments in rotating systems, where hydrodynamic loads or creep effects arising from centrifugal forces occur, is not fully understood.

Composite laminated designs in which a thin layer(s) of viscoelastic material is incorporated in the manufacturing lay-up have indicated the potential to offer significant damping without significant reduction in the stiffness characteristics of the



structure. Ultra-thin finite element models of viscoelastic materials have recently been developed and have shown good correspondence with experimental results.

Recent work on the use of magnetic constraining layers (referred to as electromagnetic damping) in configurations which utilise the opposing magnetic fields to induce cyclic shear in the viscoelastic constrained layer show promise and FE models have been developed to predict the performance of these systems.

Several important issues exist with respect to using viscoelastic materials for damping purposes. From the materials point of view, the limited temperature and frequency range tend to offer high damping in a narrow bandwidth, thus some form of optimisation is essential. The standard approach for 'optimising' the location of the CLD is to carry out modal strain energy studies and to locate the damping materials in these areas. An alternative to this using an evolutionary method has been recently reported which can automatically locate the damping material to give a vibration reduction over a range of modes of vibration.

Models for predicting the performance of viscoelastic damping materials vary from the simplistic such as the Modal Strain Energy (MSE) method to the more advanced models such as the Golla, Hughes, McTavish (GHM), Anelastic Displacement Fields (ADF) or Augmenting Thermodynamic Fields (ATF) models. Unfortunately these advanced models tend to be research tools and are not commercially available. The simple and most commonly used approach, namely the MSE method, needs to be used with caution since the method does not currently take account of the fact that complex eigenvalues and complex modes need to be considered when significant damping is added to a structure. This tends to result in an overestimate of the damping and an underestimate in the resonant frequencies.

Approaches using conformal surface coatings are also under consideration. These can be in the form of a high modulus constraining layer deposited on a thin viscoelastic coating. Numerical studies have indicated that for a 100 micron constraining layer of modulus 300GPa on a 10 micron viscoelastic layer of modulus 3GPa, loss factors in excess of 5% are possible. The use of hard coatings as a damping treatment involving plasma deposition techniques is a relatively new research area. Depending on the plasma process the microstructure of the coating can be controlled, which may give different damping mechanisms. These coatings appear to behave in a 'free-layer' configuration and test results on simple coated beam specimens have displayed good damping properties.

Research into impact and particle dampers has become topical as a result of the need for high temperature damping applications. Impact damper models are based on friction and momentum losses and may include single or multiple impacting bodies. These models are still under development and at present are not capable of accurately predicting damping performance levels.

Particle dampers have shown a capability to damp vibration over a range of frequencies. However, the dynamics of particle dampers are more complex than the impact dampers because their mode of operation can vary from a solid phase through a convection phase to a gas phase, depending upon the acceleration levels

that the particles are subjected to. Experimental data is available on the performance of these devices but no prediction or design models are currently available. Impact and particle damping devices are less effective under the action of centrifugal forces due to the particle motion being constrained and research results into these effects have only been recently reported.

Interface friction damping of general structural joints, such as bolted, riveted or clamped joints, is far from maturity. Stick-slip, micro and macro models have not been fully formulated and are still under development. Zero thickness friction finite elements have been developed and many commercial codes include simple contact elements. However, the prediction tools for dry friction mechanisms has still some way to go.

### **3. Damping Via Active/Electronic Methods**

Passive CLD treatments are being re-engineered in the context of smart materials and structures by using active materials such as ferroelectrics (piezoelectric/electrostrictive materials) and magnetostrictives. In these applications the active materials may be used to enhance the energy dissipation mechanism by substituting the passive constrained layer with an active layer, referred to as Active Constrained Layer Damping (ACL D).

Research has focused on aspects such as the control algorithms, ACL D configurations, optimisation of the placement and sizing of the actuators, solution methods (analytical/numerical), segmented systems, noise control, boundary conditions and viscoelastic damping models. Configurations range from the active element or patch (the actuator) being attached to the host structure directly (active control) to the actuator bonded to the thin constraining layer or the actuator being bonded directly to the viscoelastic layer. Devices in which the active piezoelectric constraining layer acts as a sensor have also been reported. Numerical and experimental studies on simple beam and ring type structures have indicated good results, with the ACL D treatment outperforming the active and the direct passive method.

Developments in active intercalated embedded piezoelectric fibres or embedded magnetostrictive layers/fibres in composite structures has shown that good damping can be achieved without a reduction in the integrity of the structure. This is a major step forward that has potential in aerospace applications.

Electronic damping techniques in which a passive piezo patch bonded to the vibrating host structure dissipates energy via an electronic shunting circuit (usually composed of RLC components) has been shown to offer modal damping which is comparable to that obtained from passive CLD treatment. Tests on a plate with the CLD being full coverage returned similar damping properties to the electronic damper which used only a relatively small coverage. Such devices act as tuned damped vibration absorbers and to damp several modes it is necessary to use an array of these.

Recent developments in single crystal materials with high internal loss factors offer promise for improved levels of damping. The ability to utilise integrated electronics to create electronic 'damping packages' with these and other materials indicate promise for electronic damping technologies.

#### **4. Semi-Active Damping**

Semi-active damping technology combines active control to vary the properties of passive elements or materials. Semi-active friction damping via the use of an active joint composed of a piezoelectric stack replacing the washer in a bolted configuration has been reported and applied to truss structures.

Compared to fully active control which is seen as more complex and costly, semi-active methods are being employed with materials such as electro(ER) and magneto-rheological(MR) fluids to control damping levels in a number of application areas such as suspension systems, seismic response.

ER fluids comprise a mixture of semi-conducting particles in a dielectric carrier liquid. The fluid is activated by a high electric field (of the order of 8kV per mm) using a relatively simple electrode arrangement with time constants of the order of milliseconds. On application of the electric field the particles form chain-like structures aligned nominally parallel to the applied field, producing an effective shear stress characteristic. When this is combined with an appropriate geometry, damping devices can be designed.

MR fluids consist of magnetically permeable particles typically 3 to 5 microns in size suspended in a non-magnetic medium. The material is activated via a magnetic field which causes a similar effect to the ER fluids. As with the ER fluids these exhibit enhanced shear stress characteristics, which can be converted into effective damping devices.

MR solids have also been developed using magnetic particles in elastomer media which result in field-induced dependent moduli, offering the potential to control the dynamic properties of systems such as vibration isolation mounts.

Several important aspects relating to the use of these materials as dampers requiring further research are:

- lack of a coherent control strategy,
- improved models for the dynamic material properties,
- speed of response,
- practical issues regarding the durability and life cycle behaviour of the materials.

#### **5. Conclusions and Recommendations**

Approaches to the modelling of damping would benefit from expanding the scale of understanding from the molecular, meso, micro to macro models. In addition, thermodynamic models, probabilistic and non-linear models may provide a better insight into energy dissipation capabilities, robustness and adaptability that would lead to improved design-in capabilities rather than the retro-fit approach.

In order to extend the state-of-the-art in damping technologies that would lead to the design-in of damping as opposed to retro-fitting, and reduce some of the limitations discussed in the above sections, the following topics may provide the way forward:

- introduce anisotropy into materials to enhance their damping performance,
- change the chemistry of the materials to optimise the damping performance over a wider temperature and frequency range,
- model damping at a molecular level to gain a deep understanding of the mechanisms involved,
- develop procedures for characterising the dynamic Poisson's ratio of damping materials (leading to better models),
- gain a better understanding of damping materials under high strain, large displacement conditions (non-linear behaviour),
- solve the inverse problem of defining damping requirements by designing the optimum materials/devices to achieve this,
- devise efficient and robust FE and BE models incorporating stochastic methods to accommodate variability of input data,
- quantify errors in analytical, computational and numerical damping models,
- develop end-user damping design methods and toolboxes,
- set goals for damping levels based on realistic measurements and establish damping targets for given applications,
- devise reference test methods and techniques that will offer confidence in measured data of material damping properties,
- develop 'best practise' guidelines for damping measurements,
- improve and 'keep simple', where possible, the models and methods for damping designs and predictions,
- enhance damping via active/electronic means and improve electronic damping packaging systems,
- exploit non-linearity to enhance damping properties.

## **Acknowledgements**

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# Grand Challenge: MINIMISATION OF NOISE EMISSION FOR ENGINEERING STRUCTURES

## Challenge Statement:

*Accurate prediction and minimisation of noise for engineering structures*

## Background:

*The ability to minimise the noise produced by engineering structures at the earliest possible design stage is a major requirement throughout the aerospace, civil, mechanical and marine engineering communities. Current noise prediction methods are either based on experimental techniques that require trial-and-error adjustments to an existing prototype, or consist of semi-analytical techniques that can only deal with a limited number of simplified geometries. The subject area is truly multi disciplinary as it is not always possible to distinguish between various origins of noise: structural vibration through a multitude of transmission paths, aero-acoustic effects, fluid-structure interaction, electro-magnetic effects, propagation of noise in air, water or other media, etc. Typical examples include train noise for passengers and the environment, aircraft engine noise for landing and take-off, submarine noise, noise generated by everyday tools that have rotating parts, etc.*

*The most difficult challenge is the formulation of accurate and representative models that can contain all required ingredients: structural vibration, unsteady aerodynamics, fluid-structure interaction, propagation of noise in compressible and incompressible flows. Currently, some of the required analytical/numerical tools are available but huge gaps exist between the various disciplines involved.*

## What makes this a Grand Challenge?:

- 1. Noise prediction and minimisation is required by all portions of our technical infrastructure.*
- 2. The rules regarding noise emission are becoming more and more stringent.*
- 3. For general geometries, there are no clear theoretical links between structural vibration and structure-borne sound.*
- 4. The structural (FE) models are not accurate for predicting higher modes of vibration and for dealing with damping.*
- 5. There are no established rules for ranking similar designs.*
- 6. It is not clear if statistical methods or large numerical models should be used.*
- 7. This problem has been worked on for many years and, most likely, will not be solved in the next 5-10 years*
- 8. The amount of detail that must be incorporated into the numerical models is not known.*
- 9. The noise source and the required location of the prediction can be separated by large distances.*
- 10. This problem requires a multi-disciplinary approach to its solution. (linear and non-linear vibration analysis, fluid-structure interaction, unsteady aerodynamics, sound propagation in air, water, etc.)*

## Goals Demonstrating that the Challenge Has Been Met:

*The permissible noise levels for engineering products (aeroengines, car exhausts, lawnmowers, submarines, etc) are reviewed almost every year. A reduction of about 1 dB per year is becoming the expected norm.*

# Grand Challenge: Structural Damage Identification

## Challenge Statement:

Perform robust global vibration-based damage assessment of engineering systems

## Background:

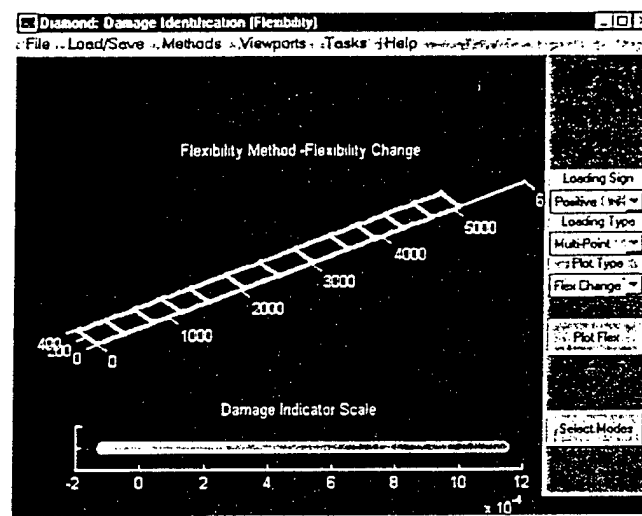
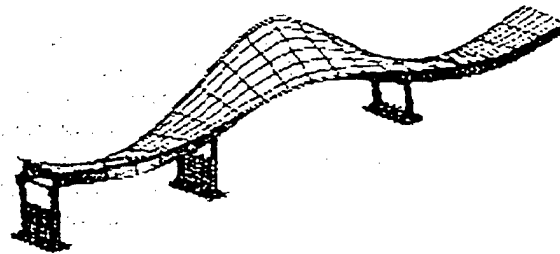
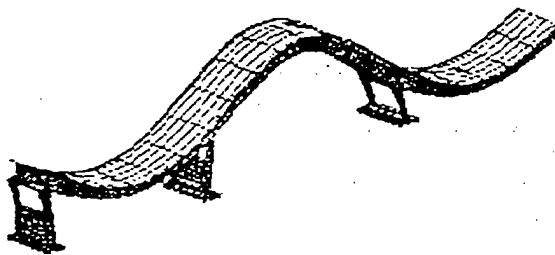
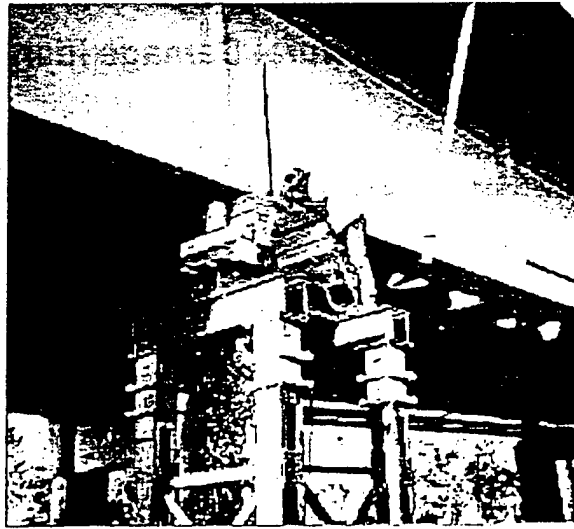
The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the aerospace, civil and mechanical engineering communities. Current damage-detection methods are either visual or localized experimental methods that require the vicinity of the damage to be known a priori and that the portion of the structure being inspected is readily accessible. The need for quantitative global damage detection methods that can be applied to complex structures has led to research of methods that examine changes in the vibration characteristics of the structure. The basic premise of vibration-based damage detection is that the damage will significantly alter the stiffness, mass or energy dissipation properties of a system, which, in turn, will alter the measured dynamic response of that system. Although the basis for vibration-based damage detection appears intuitive, its actual application poses any significant technical challenges. The most fundamental challenge is the fact that damage is typically a local phenomenon and may not significantly influence the lower-frequency global response of structures that is typically measured during vibration tests. This challenge is supplemented by many practical issues associated with making accurate and repeatable vibration measurements at a limited number of locations on structures often operating in adverse environments. Over the last thirty years global vibration-based damage detection has been applied to numerous aerospace, civil and mechanical structures as part of research studies. However, to date, only in the rotating machinery industry has this technology made the transition from a topic of research to actual implementation as a standard diagnostic tool.

## What makes this a Grand Challenge?:

1. Damage detection is required by all portions of our technical infrastructure.
2. Successful development of this technology will have tremendous economic impact by reducing unscheduled down time of manufacturing equipment, making damage assessment after earthquakes a quantifiable process and maintaining our transportation infrastructure in operating order.
3. Early detection of damage in systems such as bridges and aircraft will have positive life safety implications.
4. This problem has been worked on for many years and, most likely, will not be solved in the next 2-3 yrs.
5. This problem requires a multi-disciplinary approach to its solution. (vibration analysis (linear and nonlinear); vibration measurement; signal processing; sensor development; statistical analysis; and remote data acquisition, processing and transmission)
6. Current measurement and data analysis technology does not allow for sufficiently precise quantification of damage-sensitive dynamic properties.

## Goals Demonstrating that the Challenge Has Been Met:

1. Within fifteen years the state of California mandates that every new building requiring strong motion instrumentation is also fitted with a vibration-based structural health monitoring system.
2. Within ten years the micro-electronic manufacturing industry can reduce plant costs by eliminating 50% their redundant mechanical equipment.
3. Within fifteen years annual scheduled maintenance costs of commercial aircraft are reduced 10% because inspection intervals have been increase as the result of in-service structural health monitoring.





## SD2000--Grand Challenge: "Planet Earth Seismic Array"

### Challenge Statement:

Measure, monitor, image and analyze the dynamics of planet earth, its seismic fault systems and its important infrastructures using a vast array of seismic and vibration sensors which are interconnected using the Internet.

### Background:

This project involves measuring and modeling the dynamics of planet earth. Historically, there have been three major groups who have examined this problem. The first group is primarily basic scientists who are trying to understand the physics and dynamics of the earth. They are probing the interior of the earth with seismic waves to better understand the physics of the core, the geology of the mantle, etc. This group utilizes an array of weak wave seismic monitoring stations for monitoring seismic waves. These stations are used for basic studies and for locating seismic events and nuclear tests. The second group is primarily concerned with monitoring earthquakes utilizing strong wave monitoring stations. The third group is involved with monitoring infrastructures. There are considerable overlaps between the three groups, but in general the measurement systems are independent.

One of the main objectives of this project is to develop a common measurement, data management and computational system for addressing the needs of these different disciplines. The measurement system will be distributed worldwide and consist of massive arrays of seismic sensors, primarily concentrated around important seismic sites.

The data is collected using the Internet and distributed to computational sites located on the Internet. This project will take advantage of the rapid changes which have taken place in the last decade in the areas of measurement, signal processing, computing and networking. Unlike most past scientific technological advancements, this project will be made possible by advancements in consumer products. The personal computer revolution, digital music, wireless communications, and the Internet are all key factors in making this Grand Challenge experiment possible.

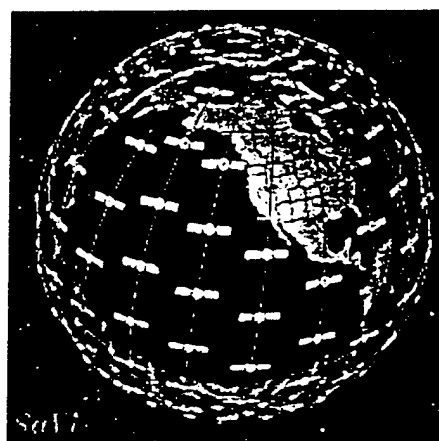
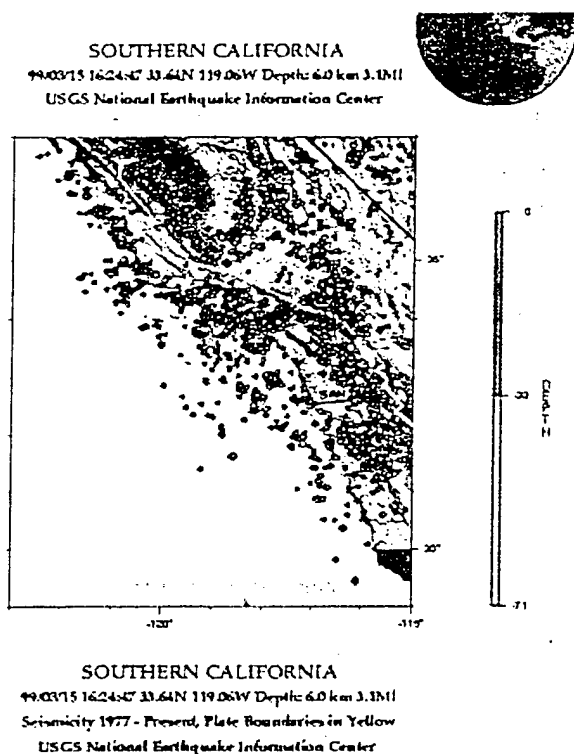
### What makes this a Grand Challenge?:

- The integration of number of interconnected disciplines to help solve a basic science problem, which is to understand the dynamics of the planet.
- This project involves using a massive application of making distributed measurement and computations over the Internet. The recent advances in computers, networking, data acquisition, and consumer electronics make this possible.
- Installation of a vast array of seismic measurement nodes (100,000 or more) which can measure and image the geology of the planet.
- Utilization of low earth orbit satellite systems to synchronize and locate elements in the array (GPS) and to collect the vast amount of data and distribute it over the Internet (Teledesic Satellite System).
- Development of a series of inexpensive seismic measurement nodes (basic 4 channel node less than \$1000) which includes:
  - Multiple Sensors (accelerometers, strain gages, tilt sensors, etc)
  - Data Acquisition Module (24 bit dynamic range)
  - Re-circulating Digital Memory (128 Mbytes)
  - DSP Chip
  - GPS Timing and Position Module
  - Internet Communication Module (Wireless – Teledesic Satellite System)
- New inexpensive multi-element seismic sensors which can measure with nano G resolution ( $10^{-9}$ g's to 10g's)
- Distributed measurements for infrastructures using local area networks interfaced with master seismic node.
- Data distributed using the Ring Buffer Network Bus (RBNB) over the Internet
- Distributed computing using the Internet
- Development of new beam-forming and imaging algorithms for analyzing data from large seismic arrays. These algorithms need to be developed for parallel processing using a large number of computers distributed along the Internet. These algorithms need to be optimized for network communication.
- New computational algorithms for condition monitoring of infrastructures.

## Goals Demonstrating that the Challenge Has Been Met:

This Challenge is a major science project. The final goal of this project is the development of a measurement system utilizing a vast array of seismic sensors which can be used to measure the dynamics characteristics of the planet, its seismic faults systems, and its influences on infrastructure. This vast array tremendously improves the resolution and sensitivity of the current systems. It also will allow a much larger database to be collected during the large rare events where nonlinearities and other effects are present which cannot be interpolated from a linear model based upon small events.

- Since seismic waves are the only practical type of energy which can be used to probe the core of the earth, a system which can drastically expand the capabilities of existing systems to measure and image seismic waves is being developed. The primary goal of the basic science experiment is imaging the core of the earth and its mantle and developing models which can explain its motion, magnetic properties, etc.
- A practical and more immediate goal concerns the imaging and monitoring of seismic fault systems and predicting the influence of these faults systems on important infrastructures. Understanding seismic fault systems has significant social and financial impact on societies that are located in areas where there are active seismic faults systems.
- There will be significant scientific gain from this project:
  - Contributions to distributed measurement systems
  - Contributions to distributed computation systems.
  - Massive data management and distribution.
  - Measurement and signal processing which can be applied to many other types of health monitoring systems. Manufacturing, process control, and energy distribution systems are a few of the applications where there would be immediate impacts.
- The scope and size of this project rivals other scientific projects for exploring the universe.



Low earth orbit satellite systems to synchronize and locate elements in the array (GPS) and to collect the vast amount of data and distribute it over the Internet (Teledesic Satellite System). These satellite systems allow the system to be rapidly reconfigured to study local hot spots. For example, Mount St Helens would have been a good example where this type of system would have been useful.

Expanding the existing capabilities for imaging earthquake fault system. Massive arrays of seismic sensors can use minute seismic events to better image these fault zones in the presence of high seismic background noise and to record a large database for rare large events.

Unlike most past scientific technological advancements, this project will be made possible by advancements in consumer products. The personal computer revolution, digital music, wireless communications, and the Internet are all key factors in making this Grand Challenge experiment possible.

## Micro Electro Mechanical Systems

### Challenge Statement:

Develop tools and methods for studying the dynamics of Micro Electro Mechanical Systems (MEMS)

### Background:

The human kind often saw the greatest achievements of engineering in systems and structures having extreme size and proportions. During the 20<sup>th</sup> century huge buildings, bridges and machines were built, almost to an extent where size seems to have reached the largest reasonable proportions. New challenges arise in the other extreme namely, very small vibrating systems. Such systems although quite similar at a first glance to ordinary structures have special features attributed to their size as compared to atomic scale. Micro electro mechanical systems (MEMS) are being used as sensors, microphones and actuating devices, all requiring a thorough understanding of their dynamical behavior. A small sensing device having a capacitive measuring device, for example, must be modeled as a coupled electro-dynamic system rather than treating the elastic and electrical parts separately. In micron scale, the electrostatic forces, the damping due to viscosity of the surrounding fluid and other effects, which could be neglected when dealing with ordinary sized structures, become very important. These tiny devices are very attractive due to their inexpensive manufacturing process (developed for micro-electronics) and the potential to include sensors and actuators in places they were never considered before.

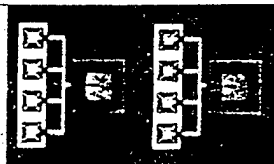
Due to lack of knowledge, practitioners use very simple and inadequate models which postpone the appearance of MEMS in many aspects of our lives. The challenge in this field is the formulation representative models and experimental verification of each mechanisms, e.g. vibration, damping, fluid-structure interaction, influence of electrostatic forces.

### What makes this a Grand Challenge?:

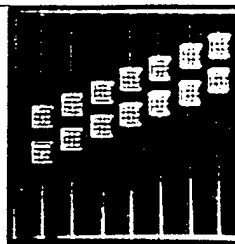
- Micro mechanical systems (MEMS) have a great commercial potential
- The dynamic response of MEMS is one of the most important factors in their performance
- New effects that exist in small scale (electro-static), e.g. loading due to non-contacting sensing devices must be understood
- Small scale makes experimental verification very difficult
- The validity of elasticity which forms the basis for vibration theory must be validated

### Goals Demonstrating that the Challenge Has Been Met:

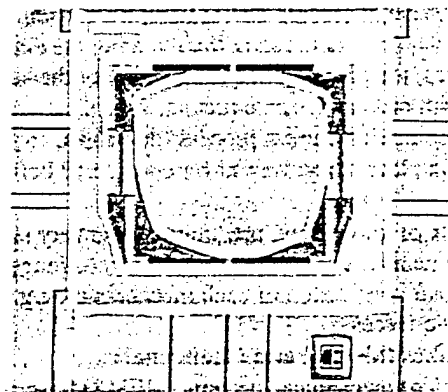
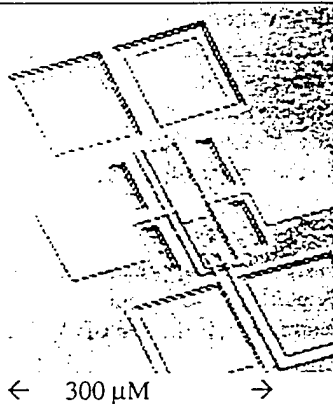
- 1) Progress in this field will create a large range of products which will affect products from cars to medical instruments within 2-5 years
- 2) In 2-4 years Every car would have a MEMS 5-10 rate gyros which costs \$4 and control the stability of motion, acceleration and be a navigation aid.



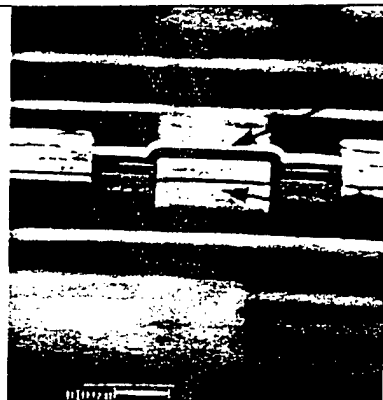
Microphone array



Vibration sensor for several frequency ranges



Rate gyro (made of 2 DOF vibrating structure utilizing the coriolis effect)



Part of a vibration sensor

## **Dr. Peter Avitabile**

University of Massachusetts-Lowell, USA

### **Q1: Making Sophisticated Structural Dynamic Testing/Analysis Procedures Commonplace in Everyday Design**

About 20 to 30 years ago, finite element modeling (FEM) was reserved for sophisticated engineering applications that had extensive time and budgets. FEM analysis was reserved for the knowledgeable and capable analysis engineers working in critical applications that required its use. During that period of time, there was extensive development and new concepts/ideas in the formulation of elements and models flourished. Today, FEM is a commonplace structural design tool in almost every industry and application.

Over the past 10 to 15 years, there has been tremendous growth and development in the structural dynamics technology area from both a testing and analysis perspective. Over that period of time, more sophisticated users have been involved in the generation of very elaborate test and analysis endeavors - most of which have required extensive time and budgets.

As we move into the next century, how do we make these newly developed techniques and methodologies more readily available to the more commonplace design of everyday commercial products so that these techniques are affordable for less sophisticated industries where time and budgets are not plentiful?

Specific examples:

Computer cabinet/disk drive design must be produced rapidly with little or no time for in-depth test/analysis to be performed. Cabinets need to be designed to accommodate a wide variety of different peripherals (especially disk drives) which must perform flawlessly in their environment. Drives need to be subjected to qualification tests that are reflective of their actual installed environment (rigidly affixing the drives to a shaker for qualification may not be sufficient or proper to confirm performance). Different disk drives (with different dynamic characteristics) need to be substituted into the design configuration and perform equally well. Accuracy of the model requires that modeling techniques employed are confirmed through the use of experimentally measured data. Cabinet designs are typically constructed such that nonlinear behavior is anticipated and can have an important effect on performance. Noise and vibration (creature comfort) are issues that need to be addressed. The only time when significant money is allocated for evaluation is when problems exist and a "fire" needs to be put out.

Another example is the design of a commercial clothes dryer. Expensive tooling must be committed early in the design with the expectation that the engineering design is correct and will produce an acceptable configuration. Fairly large models are developed of the overall configuration - modal density is populated with local modes of the flimsy cabinet configuration. Test/analysis/correlation of the model is difficult due to the large number of modes. Model accuracy is important to the prediction of system performance for both noise and vibration issues.

Both of the above are excellent examples of design situations that are not much different in technical content than those of the space station design, aerospace program, or automotive designs with one big difference - there is typically very little time or budget to undertake the extremely involved techniques/methodologies required to address the designs properly. The two situations above embrace all of the aspects of structural dynamic modeling techniques required to address the problem. Development of accurate models that are validated through experimentally acquired data. Ability to address the design which is likely to include nonlinear joints, connections and component interrelations. Development of system models that are made up of a variety of different components - using a combination of analytical and experimental components which are interconnected with both translation and rotation effects. Ability to address a large number of modes (both global and local) over a wide frequency range to predict system performance due to operating conditions. Development of an accurate model that can be used for noise and vibration predictions.

## Q2: Transfer of Technology - Educating Today's Engineer

Structural dynamics reaches into many areas of design today as we try to make all of our designs "lighter, quieter, more efficient, easier to manufacture, etc.". However, most graduating engineers receive little education in these applications. How do we effectively educate the working engineer to be cognizant of the approaches to be used in a realistic manner? Many of the techniques currently available (and still being developed) require "significant" background to understand the subject matter. How do we train and impart this knowledge in a usable format for the practicing engineer?

There currently exists a Shock and Vibration Handbook which is an excellent resource of information. Do we need to develop a Handbook of Structural Dynamic Modeling Techniques which addresses all the areas which should be of concern for the design of structural dynamic systems? Do we need to provide guidelines or back of the envelop techniques to identify if problems are expected to exist? Do we need to identify what types of tools are useful for different types of problems? Just how are we going to educate the engineer in these newly developed techniques and methodologies that are required and useful for general use in a wide variety of design scenarios?

The need for "Educating Today's Engineer" stems from some typical (uneducated) statements, questions and misconceptions that I expect that we all have heard at one time or another - some examples are:

- Automotive brake engineer believes that correlation/updating of an analytical model will reduce brake squeal.
- Engineers believe that model updating is a "push button" technology; salesman sold them a "bill of goods".
- Operating data provides a true picture of the "actual response"; but what do I have to do to modify the design?
- Performing a modal test will describe the dynamics of the system; but without a forcing function, what good is it?

And the list just goes on and on. How do we educate relatively new engineers into all the detailed technology that exists today, in a practical sense?

## **Dr. Etienne Balmes**

Ecole Centrale de Paris, France

### **Q1: Bias and variance in identification methods**

Estimating the quality of identification results would be of significant help in assessing and improving model correlation or simply to perform robustness studies.

Typical identification algorithms estimate the poles while over-specifying model order. A major reason for doing so is that out of band modes have a strong influence on structural dynamics, so that additional computational modes are needed to account for these contributions. The representation of residual contributions by computational modes or simple asymptotic terms clearly induces bias in the estimated poles. The bias on pole estimates is usually quite small but this is not true for modeshapes (in particular for the phase of complex residues).

In this light, is it possible to determine if the (small) variations in poles estimates are representative of variance in the estimate or simply of changes in the bias effects when considering various model orders. Can something be said for the larger variations found for modeshape estimates.

### **Q2: Design objectives for non-linear dynamics**

Many criteria have been developed to judge the performance of linear structures. One thus minimizes the RMS response or gives a frequency domain envelope for acceptable response spectra. The assumption of linearity leads to two major simplifications. The characterization of the excitation can be limited to simple statistical representations. The relation between the excitation and the response is simply characterized by transfer functions which are easily computed using modal analysis.

With the objective of judging the dynamic performance of non-linear structures, tools seem to be missing to characterize broadband excitations with quantities that can be propagated into response characteristics in a reasonable time. Extending modal analysis notions to non-linear structures seems key to the propagation problem but this will still leave the input characterization issue open.

## **Dr. Izhak Bucher**

Technion Institute of Technology, Israel

### **Q1: Should deterministic or statistical models of structures be used for structures having high modal density?**

Large structures could have tens to even hundreds of modes (natural frequencies) in the frequency range of interest. Common inaccuracies of 1-10% in some modal parameters and even 40%-200% in damping may render such a model useless for deterministic prediction of response levels. The question arises whether dynamic prediction should be re-directed towards statistical estimation of response level based upon statistical models of the structures? Do deterministic models have some merit even in such cases that cannot be overlooked?

This question puts in question the validity of modal-testing in the cases where it should be mostly required.

### **Q2: Recommended Signal of excitation: which type excitation should be used for linear structures and which one is preferred in the slightly nonlinear ones. Sine-sweep, sine-step, random, burst, impact?**

In dynamic testing of structures many excitation methods were used and it seems that the 'fashion' changes every few years and there is no agreement upon the 'best' method for each particular application.

Is it better to use an excitation that 'linearises' the identified structure (random) or an excitation that exposes the non-linear nature of the structure (sine, impulse)? Should non-linear behavior be treated in the excitation phase, signal processing phase or model-fitting stage?

### **Q3: Should linear, non-linear or Floquet (periodically varying coefficient systems) be applied to rotating machines?**

Rotating structures exhibit a very rich response spectrum which is caused by the complex nature of the various phenomena co-existing in a rotating machine. A rotating structure is excited by external forces such as fluid-structure interaction, cutting forces (in saw blades) or by speed variation and parametric excitation (e.g. modulation of the response by stiffness and mass depending upon the instantaneous angle of rotation).

It has been shown (A. Bendat) that in some cases non-linear systems possess the same input-output behavior as hypothetical linear systems with periodically varying coefficients.

Inspecting the spectrum of the vibration measured on a rotating machine one can always notice numerous harmonics of the frequency of rotation as well as other frequencies. These harmonics could be attributed to the nonlinear nature of the supports (bearings) or to parametric excitation due to time varying parameters. The parametric excitation is caused by the imperfect rotating structure (not perfectly isotropic). This effect could potentially be explained or even more complicated phenomena (coupled bending torsion in shafts?).

How this problem should be approached and resolved? How could these phenomena be separated from measurements and how should they be modeled? Is this important or is any suitable 'black box' model that relates forces to response could be used?



## **Dr. Bernd Caesar**

Chrysler Daimler Aerospace, Germany

### **Q1: Transfer of CAD into Mathematical Models:**

Could straight forward procedures be developed to establish mathematical models directly from the design of complex and/or assembled/integrated structures?

Actually, automated procedures exist to transfer single structures into FE-meshes under certain restrictions as simplicity of the structure design or use of tetrahedral elements. As soon as structures become more complex or there is the need of coupling several structures together still the skill of structural engineers is required to transfer the reality into the simplification of the mathematical models.

### **Q2: In addition to the example question "Question 2: Coupling Macroscopic to Microscopic Analyses":**

Mechanisms especially high precision ones can be described by actual available mathematical models only on the basis of empirically derived data. Mostly the mathematical model is sufficiently accurate not before tuned to the hardware behavior. This causes high development risks in costs and schedule. The feasibility of a design cannot be predicted.

### **Q3: Coupling of Control and Structure Mechanics**

How should software be designed to couple control sensor and actor design with structure behavior?

Actually the coupling is performed via standardized data package and/or model transfer from one to the other independent S/W-tool. Application example: ESA, NASA, ESO and others are working on high precision space and earth born telescopes with interferometric optics under the task "Next Generation Space Telescope" and "Very Large Telescope" for which pointing mechanisms in the delay line with accuracies in the range of 1 nm are required. Such extreme accuracy demands require new strategies.

### **Q4: Damping Prediction**

How can we solve the old problem of damping prediction?

### **Q5: Medium and High Frequency Range Dynamics Analyses**

With which approach shall the dynamics in the medium and high frequency range be treated?

The medium and high frequency range is most often important in acoustic and shock excitation. Several not fully satisfying approaches are available, one is the refinement of discretization (refinement of FE-mesh), another is the statistical energy analysis SEA. Solving this problem would help e.g. to predict more accurate stresses and test levels for equipment on structures under acoustic or random loads, or to reduced structure radiated noise.

## **Dr. Scott Cogan**

University of Franche-Comté, France

**Q1 : What criteria can be used to judge the usability of a model for a given application, that is to say, whether model-based decisions can be integrated with confidence into an engineering activity. Is it really predictive precision we are after or is it robustness of model-based design decisions to ambient uncertainties ?**

Model quality has become a major concern in the structural dynamics community. The availability of attractive and highly developed software for creating mathematical models of complex behaviors (due to both local behavior laws and intricate topologies) has lured (to be provocative) the industrial community into their purchase and use. These tools have produced undeniable success stories and their usefulness is difficult to contest. However, the influence of diverse sources of uncertainties ranging from the simplifying concepts used to analyze a given problem to the various measurement errors tend to create an incompressible distance between analytical predictions and the reference behavior observed in the field. It is certainly a tribute to engineering savoir-faire (and a healthy dose of safety factors) that planes fly and nuclear reactors remain contained given the abiding inaccuracies in the mathematical models of complex technological structures.

Model-based decisions are useful only in so far as they maintain a certain fidelity with respect to real prototyping, that is to say, that they represent true pseudo-tests. The goal of modelling, measurement and updating is simply to convince ourselves that we are indeed in the presence of such a model. The question is just how this can be done in a reliable way ? Just when is a model precise enough ? Is it really 'precision' that we are looking for or is it 'robustness' of our design decisions with respect to the known (and unknown or unexpected) sources of uncertainty in both the model(s) and prototypes. A model might well provide very precise predictions with respect to a given parameterization and for a set of measurement data. These predictions however might be extremely sensitive to uncertainties in both model parameters and to the fact that a set of apparently identical prototypes present a dispersion in their behaviors. Conversely, model predictions may be relatively imprecise while the design decision in question may well be relatively robust with respect to these prediction errors . Clearly a methodology is required to allow these concerns to be addressed in a comprehensive way.

## **Dr. Jonathan Cooper**

University of Manchester, UK

### **Q1: Are there developments in the signal processing and numerical analysis fields waiting to be applied to vibration analysis ?**

Great use is currently made of tools such as the Fast Fourier Transform for vibration testing and standard analysis approaches such as Least Squares. Other recent developments have included the use of the Singular Value Decomposition (SVD) and Time/Frequency methods (e.g. wavelets). Any major developments in the field from the algorithmic viewpoint will depend upon the use of new techniques. Do such tools exist, and if so, how do we apply them to vibration analysis ?

### **Q2: Is linear modal analysis solved ?**

The process of identifying frequencies, damping ratios and mode shapes from modal test data is nowadays taken as being fairly straightforward. However, most commercial implementations of the methods rely a great deal upon engineering judgement of the user (how many modes, which modes should I select from my stability plot? How to deal with scatter in the results, Etc.) The amount of research devoted to investigating new approaches has diminished and most of these (e.g. subspace methods) are viewed as research tools rather than appearing in commercially available software. Has the problem been solved totally or should further effort be devoted towards providing industry with "intelligent" modal testing that removes the need for a large amount of user interaction. (PS. When is it not OK to treat a structure as linear ?)

### **Q3: How should structures in the mid-frequency range be modelled ?**

Conventional FE tends to be used for low frequencies whereas Statistical Energy Analysis (SEA) is applicable for very high frequencies. Little work has been undertaken to address the frequency ranges in the middle where the accuracy of FE breaks down. How should the Structural Dynamics community tackle this problem ?

### **Q4: How should structures with non-modal looking FRFs be identified ?**

When a structure with a very high modal density and relatively large damping is tested, the FRFs do not look modal and instead look very flat. Analysis using conventional curve-fitting methods does not produce very good results. Should black-box time domain approaches be used, or should some other methodology be used ?

### **Q5: How should non-linearities on real structures be identified ?**

There has been a great deal of work devoted to the modelling and identification of non-linear systems (e.g. NARMAX, restoring force surfaces, higher order FRFs, etc.) Whereas these methods can deal with simulated systems and small purpose built laboratory experiments, their use on real structures (e.g. automotive, aerospace, civil structures) is problematic. Is the correct approach being used with improvements in the technology being made very slowly, or is a radically different methodology required to tackle the problems of detecting, quantifying and identifying non-linearities in real structures.

## **Prof. Patricia Davies**

Purdue University, USA

**Q1: How can the classical single frequency excitation perturbation analysis of non-linear systems analysis become a useful tool for structural dynamicists trouble shooting problems in structures?**

When trying to understand the conditions necessary for modal interactions interactions to occur in weakly non-linear structures, a two-stage analysis is usually performed. First, for the conditions under which the structure is operating, a reduced order modal model of the structure is derived, and secondly, the steady-state (or slowly varying) response of the modes of this model for various forms of stationary (or slowly-varying) harmonic excitation is examined. Analysis of these response versus model parameter (excitation level, natural frequency, excitation frequency, etc.) functions allow determination of regions where multiple solutions may exist, or where no stable steady-state solutions exist and energy sharing between modes may occur. Multiple solutions co-existing for the same excitation will mean that in structural testing, the response observed will be a function of initial conditions and perturbations to the system, a phenomenon not present in linear systems.

As structures become thinner and deflections become larger the role of non-linearities must be considered, and in frequency regions where many modes are

present, the probability of modal interactions is high. When excitations come from rotating machinery, it is not unrealistic to consider harmonic excitations, or to assume that only a few of the structures' modes are directly excited. Therefore, this classical nonlinear dynamics analysis is suitable for practical situations. However, while the tools are used by people studying non-linear systems in their research, they are not widely used in the structural dynamics community. There are also some interesting issues that need to be addressed. Here are a few: (1) Is it possible to probe the structure through testing to determine with some certainty whether this modal coupling is present, and thus to justify the non-linear modal analysis? (2) Given a particular excitation, how can we systematically determine which modes to include in the model? (3) Since the response curves are highly dependant on the modal parameters, how can the parameters be estimated using excitations that will only cause the modes in the model to respond? (4) Can the analysis techniques be extended to multiple harmonic excitations? (5) Does the structural dynamicist need to be an expert in non-linear systems analysis to use these tools?

**Q2: It is often possible with computer control of manufacturing to tailor the chemical content of materials during manufacture. How can we optimize the in-application macroscopic behavior of materials using these manufacturing capabilities?**

This is really a problem of linking the material and chemical level modeling with the structural dynamics models. There is a need to feed the basic material properties into the structural model, or if this is not possible to do directly, to understand how fundamental changes in material microstructure affect the terms in the macroscopic structural model. Robust system identification techniques are needed to determine the types of terms, and the parameters in those terms, that are necessary to model the macroscopic behavior of the material in the application of interest. These terms may be, for example, stiffness, viscous damping, hysteretic damping, dry friction, or viscoelastic terms. Many interesting problems arise here in system identification, not only because many of the material models will be nonlinear, but also because materials such as polyurethane foams (used in transportation seating) have both fast- and slow-time behavior and are sensitive to environmental factors such as temperature and humidity, making repeatable experimentation as well as parameter estimation challenging. The interaction between the terms in the structural model will add an additional complexity to the identification, perhaps requiring the need for excitation techniques that allow for isolation of particular types of behavior.

Development of robust system identification techniques to identify structural dynamics model parameters related to material properties, and derivation of functional relationships between these parameters and the

material manufacturing parameters, is only the first stage. We then have to take this understanding and use it to design the structure, working backwards from the desired response to known excitation, to the space of effective structure dynamics model parameters, and finally to the materials manufacturing parameters required.

## **Dr. Scott W. Doebling**

Los Alamos National Laboratory, USA

### **Q1: What will be the appropriate relationship between simulation and experimentation in the early 21st century?**

Historically, in structural dynamics as in other fields, experimentation has been used to both PROVE and IMPROVE: (a) PROVE that an engineered system meets its design criteria by performing a "proof" or "qualification" test and (b) IMPROVE the mathematical models of the engineered system to better predict the response of the system under different initial and/or boundary conditions. Some engineers would claim that the IMPROVE experiments provide more useful information for engineering purposes than do the PROVE experiments, but nevertheless the PROVE experiments receive much higher priority in terms of resources (i.e. money) than do the IMPROVE experiments.

GIVEN the extreme decrease in the price-to-performance ratio of computing power over the last 30 years, is it possible that structural dynamicists can "substitute" computational simulations for many of the PROVE experiments, so that resources can be concentrated on IMPROVE experiments to actually make our modeling and analysis capabilities better?

### **Q2: How can analysis of structural dynamics issues be used to decrease "time to market" ?**

Many industries, such as those with aerospace, automotive, and civil infrastructure applications, are inherently reliant upon analysis of structural dynamics issues to ensure that the engineered systems perform their functions within specifications. However, many other industries that have high-volume, low-cost, quick-time-to-market business constraints do not have a direct requirement for structural dynamics analysis and therefore forego the expense in favor of quick turnaround time in product development and deployment. What many engineers and managers in these industries fail to consider, however, is the improvement in product quality that can be had with the use of structural dynamics in the product design and deployment phase. For example, consider the portable CD player. While undoubtedly such a product undergoes rigorous testing during the design phase, consider how much more robust the design can be through an equally rigorous application of analysis. However, the application of analysis in such instances is typically limited because it "slows down the design process."

GIVEN the potentially huge market in the consumer products (and other) industries, how can the structural dynamics community make a case for structural dynamics analysis as a tool to improve product quality while not impeding the "quick time to market" ?

### **Q3: What is the relevance of structural dynamics in the information age?**

It is generally agreed upon that the "industrial age" is largely behind us and the "information age" is fully upon us as the 21st century approaches. Structural dynamics has undoubtedly enjoyed a prime period of growth and increase in resources as the industrial age has matured. Structural dynamics is traditionally associated with the "heavy" industries such as aerospace, automotive, robotics, and civil infrastructure. But what is the relevance of such industries in the information age? An example of such relevance would be increased demand for satellite production as worldwide wireless networks are increasingly in demand. A demand for more "features" on a smaller, more lightweight satellite will certainly involve new structural dynamics issues. But aside from such "new tricks" for the "old dogs" of the industrial age, does the information age present any new opportunities for structural dynamics?

GIVEN that the majority of the economic growth in the early 21st century will most certainly come in the field of information technology, rather than industrial technology, what can we as the structural dynamics community do to "get a piece" of the information age "action" ?

## **Dr. Michael Friswell**

University of Wales - Swansea, UK

### **Q1: Has modelling structures using finite element analysis reached the limits of accuracy? Is a new paradigm required for design and analysis of structures?**

Many engineers believed that any structure may be modelled to arbitrary accuracy merely by increasing the finite element mesh density. These refined meshes are able to model the geometry of the structure more accurately, but uncertain parameters, for example geometric tolerances or joint dynamic stiffnesses, mean that modelling errors will never be resolved using this approach. Model updating can help, although for complex structures the incompleteness of the measured data makes it impossible to identify a physically meaningful model. Thus there are definite accuracy limits for modelling a single structure, and also for modelling the variability of batches of components because of manufacturing tolerances.

Perhaps we should be looking at different ways of analysing the dynamics of structures. Methods based on uncertainty (defined either as a statistical or convex model of uncertainty) could be used to optimise structures as part of a robust design scheme. Control engineers are more advanced in this respect and robust controller design, based on an uncertain plant model and influenced by uncertain disturbances, is a mature technology. Could any of the robust control methods be transferred to the design activity in structural dynamics? Could the design process be tailored to produce a structure where the known uncertainty in the structure has the minimum effect on its dynamic response? Is this a useful thing to do?

### **Q2: Do uncertainties in the model, measurement noise and the changes due to environmental effects, make it impossible to robustly locate damage in structures using low frequency vibration measurements?**

The area of damage detection and location using measured low frequency vibration data has attracted considerable attention recently. The great advantage is that low frequency data is global, and thus only a limited number of sensors, located remotely from the potential damage sites, are required. However there are considerable difficulties to be overcome before the approach is robust enough to be used in practice. The question is whether the inherent difficulties render all the proposed methods unusable in the field.

In terms of methods, the only viable approaches are based on forward identification procedures which recognise the fact that damage generally only changes a limited number of parameters. Methods that try to identify a large number of parameters simultaneously produce ill-conditioned equations, whose regularised solution destroys any capability to identify damage location.

Even forward identification methods have considerable difficulty. Is the finite element model accurate? How accurate does it need to be? How does the dynamics of the structure change with environmental conditions, for example temperature and humidity? Can these effects be modelled with sufficient accuracy so that their influence may be removed? How large are the changes in a structure's dynamics due to typical faults? Is this level of change likely to be detected reliably, given the signal to noise ratio, the changes due to environmental effects and the modelling errors? Even if damage can be detected, is the 'effective wavelength' of the low frequency modes too large to make localisation of the damage sufficient to be useful?

Given all these significant difficulties that are based on the physics of the problem not the methods used, what is the prognosis for damage detection and location using low frequency vibration?

### **Q3: Is state space methodology state of the art for damped structures and rotating systems?**

Often damping is ignored in the analysis of dynamic models of structures, but recently there has been a resurgence of interest in the analysis of damped structures. The standard approach to analysing such

structures is the transformation to a first order system (the state space formulation). The increased effort for the eigensystem computation required to progress from an undamped system with  $N$  degrees of freedom, consisting of real eigenvalues and eigenvectors, to a damped system represented by a first order system with  $2N$  states and complex eigenvalues and eigenvectors, is significant. In terms of understanding, as soon as the state space matrices are formed, the constraint on the states (where half are displacements and half the corresponding velocities) is lost. Indeed there is also a constraint between the real and imaginary part of the complex eigenvectors that is not used or enforced in a state space analysis. Are we missing something that could enhance our fundamental understanding of the damped system? Should we, for example, make sure these constraints for a second order system are enforced in experimental modal analysis?

What application does Geometric Algebra (for example Clifford Algebra) have in structural dynamics? Many areas of physics have benefited from the unifying force of Clifford Algebra. Most engineers are happy to use complex numbers in the analysis of dynamic systems (particularly in state space form); they afford tremendous insight and compactness of notation. Could Clifford Algebra do a similar job for second order systems, helping both understanding and the computation burden? In rotordynamics it is possible to encode multiple uses for complex numbers (for example, using complex numbers for both the eigenvalues and to encode displacements or rotations at a given node in orthogonal directions). This area is only just beginning to develop and the benefits have not as yet been fully investigated. Is this the future?



## **Dr. François Hemez**

Los Alamos National Laboratory, USA

### **Q1: Developing Accelerated Testing Procedures**

Could experimental procedures be developed that would simulate the aging of a structural system and help assessing its performance after a life-span of, say, 30 years without having to wait that long to take the measurements?

For many structural dynamics applications, it is important to estimate during the design phase what the remaining performances of a system will be after it has been used for a long period of time (20 to 30 years). This is especially critical with aerospace applications for which modelling the effects of aging is almost impossible because very little data are available to validate the simulations (we are just starting to witness on a large scale the effects that aging has on aircraft systems). Another example would be civil engineering structures such as bridges that often stay operational for longer periods of time that they had been originally designed for.

Finite element simulations can be used for estimating the performance of a system after a series of load cycles. However, our current modelling capabilities do not allow us to handle complicated structural systems for which residual strength assessment still relies, to a great extent, on testing. Are experimental techniques available (or could they be developed) for reducing 30 years worth of structural dynamics in a few tests?

### **Q2: Coupling Macroscopic to Microscopic Analyses**

Rather than attempting to develop "useless" mathematical models for accounting for complicated dynamics (such as energy dissipation), could the structural dynamics community focus more on the means of coupling the conventional, mechanics-based representation of structural dynamics with a localised, physics-based description of the mechanics at the microscopic (if not, nanoscopic) level?

The motivation for this question is best illustrated with a (provocative) example. Many research efforts currently focus on developing damping models starting from the equations of mechanics and a description of the material behaviour at the macroscopic level. For the past ten years or so, it seems that this research has delivered no significant improvement of our modelling capabilities. It can be argued that the reason is that most damping sources originate from the localised contact/friction between two surfaces or from the dissipation of energy in the material at the microscopic level. To model this phenomenon (and many others such as contact), it seems therefore critical to account for surface irregularity, material imperfection and anisotropy, if not molecular interactions!

It seems like it would be more productive to study how macroscopic properties could be coupled to a characterisation of the material at the microscopic scale and, conversely, how microscopic state variables could be averaged to produce, for example, "global" displacement or stress components. The goal here would be to enable a fully coupled macro/micro analysis in order to improve our predictive capability by taking advantage of a physics-based representation of the dynamics, whenever necessary. Of course, such an endeavour would generate significant computational challenges.

### **Q3: Predictability of Structural Dynamics Models**

Should the structural dynamics community put a strong emphasis on developing tools for assessing the predictive quality of a given numerical or experimental model? If so, which technologies need to be developed?

No practical tools are currently available to assess the predictability of a structural dynamics model. The quality of an individual finite element mesh can be assessed, the quality of test data can be characterised statistically but the structural dynamics community lacks the capability of assessing the degree of predictability of a given model with respect to a given objective. (For example, what is the accuracy associated with the response obtained from a given finite element model knowing that the objective is to minimise the distance between test data and numerical simulations?) Developing such a capability would probably require the integration of statistical analysis, multiple-model representation and inverse problem solving (for comparing the predictions of a model or family of models to test data).

#### **Q4: How do we improve the image of Structural Dynamics?**

All too often structural dynamics is considered in the context of failure rather than performance. If we compare this with research in say Fluids or Materials, the emphasis of these activities has been consistently related to product enhancement rather than deterioration. Historically, we have a legacy of troubleshooting, when dynamics has been used as a corrective measure "after the event" – whatever that might have been. This culture - with the notable exception of aeroelasticity – leaves dynamicists fighting a rearguard action. Inevitably this puts the issue of dynamics towards the back of the queue in the design process. Even worse the issue of dynamics may only be considered retrospectively.

The education process in dynamics often reinforces the role of dynamics as a discipline associated with structural failure. We all use the "great disasters we have known" anecdotes in our teaching – this may be doing us no favours. The challenge in teaching therefore, is to maintain the interest level imbued by eye catching events, but portrayed in a more positive light. Perhaps the dynamics should be included seamlessly in the design elements of engineering courses?

The challenge for structural dynamics research is more demanding in this respect. Industry frequently views structural dynamics as necessary evil. The commercial case for retrospective dynamics is not strong. With the increasing representation of industrialists on government funding bodies, we need to convince industry of the benefits of dynamics research. What are these?

If we consider the apparent success Computational Fluid Dynamics in attracting funding, it is evident this has arisen through effective marketing on the basis of reduced cost and increased performance. Dynamics research strives to do the same and yet does not attract the same enthusiasm from industry. How do we address the publicity issue?

Perhaps the role of dynamics has become entrenched by its applications? When we consider dynamics we immediately think of bridges, aircraft and cars. These products are established, and do look forward to new growth technologies. We only have to consider the burgeoning Nasdaq to see where technology is expected to develop – dynamics is not evidence. The question in this case is, how do we associate ourselves with products and technologies that will take us forward from the entrenched applications we work on today?

## **Dr. Mehmet Imregun**

Imperial College, UK

### **Q1: Modal Updating – Is this the end of the road ?**

For the last 20 years or so, significant research effort has been devoted to improving mathematical models using vibration test data. However, there are still no established and universally-applicable methods that can produce an updated model that can satisfy the following criterion:

The (true) experimental and updated models must have the same modal and response properties within the measurement frequency range.

The updated model must be able to predict the effects of further changes.

It is debatable whether the corrections to the updated model should have a physical meaning. Most updating work is focussed on FE modelling, which may or may not be the way to go. Is it time to accept defeat and have a fundamental review of the modelling techniques ?

### **Q2: Damage detection – Given the relative lack of success with model updating methods, can we expect to do better ?**

In the general case where the damage location is not known, is it possible to develop detection techniques using numerical base models without having solved the model updating problem first ?

### **Q3: Should we develop model updating methods on small-size models and assume that these are equally applicable to large-size models ?**

It is common practice to develop updating methodologies on small-size models, say with 10-1000 DOFs. When dealing with such models, numerical accuracy, computational feasibility in terms of CPU effort and storage are taken for granted. However, there are many computational problems when working with large-size models, say 30,000-200,000 DOFs. For instance, it is not a trivial task at all to obtain the first 5,000 modes of a 100,000 DOF model. Not only different FE codes will give different results, various eigensolution extraction routes within the same code will also yield different results. Such problems will probably go unnoticed when computing the first 50 modes of a 1,000 DOF system.

Another point to remember is that the size of the model is often dictated by the amount of geometric detail that needs to be incorporated. For modelling simplicity, can we ignore such detail and hope to be able to update the model ? In other words, how can we determine the minimum amount of modelling detail that is required ?

**Dr. Nick Lieven**

University of Bristol, UK

**Q1: How do we improve the image of Structural Dynamics?**

All too often structural dynamics is considered in the context of failure rather than performance. If we compare this with research in say Fluids or Materials, the emphasis of these activities has been consistently related to product enhancement rather than deterioration. Historically, we have a legacy of troubleshooting, when dynamics has been used as a corrective measure "after the event" – whatever that might have been. This culture - with the notable exception of aeroelasticity – leaves dynamicists fighting a rearguard action. Inevitably this puts the issue of dynamics towards the back of the queue in the design process. Even worse the issue of dynamics may only be considered retrospectively.

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# **Mr. Charles Pickrel**

**The Boeing Company, USA**

## **Q1: Confidence Intervals**

Can we estimate confidence intervals on structural dynamic parameters which affect the design and performance of our products?

In a more perfect world we would assess variability in manufacture of products, test measurements, model predictions, and the assessment of operating environments. Many products are engineered with conservatism to compensate for ignorance of these confidence limits. Improved assessment of variance and confidence intervals would lead to improved performance or reduced cost of these products or designs.

## **Q2: "Open Source" Research Model**

Following the "open source model" (Linux software development) could a few structural dynamics problems be placed on an open web site(s), facilitating collaboration and making data and solutions widely available?

Some important problems in structural dynamics involve multiple aspects of our discipline, such as modeling, testing, and controls to name a few. Seldom does a researcher have the resources to accommodate all of these aspects of the problem. An open web site could bring together contributions from different researchers and institutions. The "publishing cycle" could be reduced from months/years down to minutes/days, accelerating the solution process. Examples of the information which might accumulate for a given problem include: drawings of structure, test plans, test data, models and predictions, modal parameter estimates, model updates, damage detection, structural modifications, solutions, discussion (questions, answers, comments and assessments).

Obviously, many issues arise, and not all research will be conducted this way any time soon. The Linux software community has found the "open source" and "release early and often" approach to be a powerful method for rapidly developing robust software. Perhaps the same would be true for the solution of multifaceted problems in structural dynamics?

## **Q3: Dynamics in Engineering Education**

How can we increase the emphasis on dynamics in engineering education? It seems that structural dynamics has played an increasingly important roll in product design in recent years. Yet the average engineer is sorely lacking in appreciation of dynamics fundamentals. The dynamics community would benefit from a broader awareness and accessibility of dynamics fundamentals on the part of decision-makers.

## **Q4: Mid-Frequency Problems**

How do we approach mid-frequency and high frequency problems where modal analysis is not practical? Modal analysis provides us with a robust approach to "low frequency" problems in structural dynamics, about which much has been written. Is there a consensus for an approach to problems involving higher frequencies where the modal approach breaks down?

## **Q5: Dynamics and Controls**

Can anything be done to increase the communication and integration between the Structural Dynamics community and the Controls community?

Increasingly, problems arise which require the feedback control of flexible structures. It seems this requires the participation of engineers from two different organizations (in a larger company), who graduated from different departments, read different journals, and talk different "language."

## **Prof. Stephen Shaw**

Michigan State University, USA

### **Q1: Development of techniques for building reduced-order models**

Several methods exist for reducing the size of computer-based, linear models of structural systems. These techniques are very useful for substantially reducing the number of degrees of freedom required for high-fidelity models over a certain frequency range. However, when nonlinearities are present, this type of model reduction becomes much more difficult. Can some basic, useful model-reduction theories be developed for this class of problems? Can these be developed into useful computer codes? Can one develop specialized methods for cases in which the nonlinearities are strictly local in nature, say at joints, or due to attached components? Or, similarly, for the case when the nonlinearities are inherently distributed, for example, due to kinematic effects?

### **Q2: Structural vibration control: deciding on passive, semi-active, or active control**

Tremendous advances have recently been made in the general field of structural vibration control. We now have a large knowledge base about the effects of damping treatments, semi-active control techniques, and active control methods. Research has typically been carried out in each of these areas in a largely isolated manner. While such an approach is necessary for advances to be made in these fields, a practicing engineer faces his/her vibration problem in a very different way. They are faced with a problem that requires attention, and they must choose from among the many options available. While there are tremendous variations in the nature of these vibration problems, it would be worthwhile to step back and consider the features of a given situation that would influence one's choices in these matters. Under what general conditions are passive measures effective and appropriate? What is typical of a situation in which one would recommend using a more expensive type of feedback control that requires sensors, actuators, and computing?

### **Q3: Relevance to industry**

This is a problem that faces many areas of engineering research. Industry is typically slow to make use of the latest results from any basic and/or applied research that is done outside of their labs. In terms of structural dynamics, the aerospace industry has probably been one of the most receptive groups in this regard. However, other industries, such as the automotive companies, are slow to respond. They are often aware of advances, but are reluctant to try new ideas. How can we be more effective in making inroads into these industries?

### **Q4: Development of effective damping models and their inclusion into FE codes**

This is, of course, an old and well-known problem, but it may deserve attention at this forum.

### **Q5: Structural dynamics and control at small scales**

An extremely important area for the future of structural dynamics is that related to micro, and even smaller, devices. There are new exciting challenges waiting for the dynamicist here, and there are opportunities to be involved with cutting-edge technology.

## **Prof. David C. Zimmerman**

University of Houston, USA

### **Q1: Where does Structural Dynamics fit in?**

Recently, several traditional United States funding agencies have publicized their plans for research initiatives in the 21st century. In a recent DARPA briefing, it was stated that nearly 50% of the DARPA budget will be focussed on biotechnology (including bioengineering) within the next three years. The NSF and several other agencies (DOD, DOE, NASA, NIH and NOAA), working under the auspices of the National Science and Technology Council (NSTC), has developed a long-term research plan for Information Technology in the 21st century. At the same time, the NSF announced that it's second major focus in the 21st century will be in "Biocomplexity in the Environment". The change in the funding picture is changing rapidly ... how should the structural dynamics field react?

### **Q2: Is there an "Economic" Based Grand Challenge Problem in Structural Dynamics?**

The quick rise of Computational Fluid Dynamics research and subsequent commercial software can be traced back to President Reagan's call for the High-Speed Transport (or as he called it the Orient Express). It was estimated that after the initiation of this program, nearly 90% of all supercomputer resources in the country were associated with CFD development. Although the High-Speed transport never did materialize (in fact, NASA Langley, the NASA lead in high speed transport is ending this program in FY00), it did serve as a catalyst to the CFD research and application community. This focussed effort was sold to the public and to Congress based on the "economic impact" and need for such transports in the "new global economy". Look at another major Grand Challenge: The Supercollider. This project folded several years ago because the physicists were unable to sell the public and congress on the economic impact of their research. Is there a Grand Challenge problem which has structural dynamics in the lead role AND has a major economic impact?

### **Q3: Design for Reliability**

As we are always reminded, we all will (or already) have Cray computing capability sitting on each and every desk of an engineer in the near future. Have we as structural dynamicists stayed in the safe "linear" world, and away from the nonlinear and stochastic real world, based on our thoughts that the computational power needed to properly perform design is not available?



## **Dr. Peter Avitabile**

**University of Massachusetts-Lowell, USA**

Peter Avitabile - BS, MS and Doctorate in Mechanical Engineering from Manhattan College, University of Rhode Island, University of Massachusetts, respectively. Currently, Manager, Modal Analysis and Controls Laboratory at the University of Massachusetts Lowell (1985) supporting testing/research contracts and teaching for Mechanical Engineering Modal Analysis Graduate Program. Previously, Westinghouse Bettis Atomic Power Laboratory (1974) reactor vessel structural analysis for severe accident conditions, thermal effects and brittle fracture analyses. ITT Grinnell R&D (1976) structural/dynamic testing/analysis of nuclear vessels/piping components. General Electric Turbine Technology Laboratory (1983) structural/dynamic testing/analysis submarine propulsion systems.

Over 25 years experience in design, analysis, finite element modeling and experimental modal testing. Main area of research is structural dynamics specializing in the areas of modeling, testing and correlation of analytical and experimental models. Research, testing and consulting performed for automotive, aerospace, defense and computer/consumer related products for companies such as General Motors R&D, Southwest Research, GTE Technical Systems, General Electric, Canadian Space Agency, Martin Marietta, Apple Computer, Velcro, Sun Microsystems, Harley Davidson, Whirlpool, Instron, Brown & Sharpe, Digital Equipment, Newport News Shipbuilding, Cambridge Acoustical Associates.

Written over 50 technical papers and given numerous seminars in the areas of experimental modal analysis, structural dynamics, vibration fixture design, and modeling and correlation. Developed the Young Engineer Program for the International Modal Analysis Conference and currently authoring a series of articles for SEM Experimental Techniques "Modal Space - Back to Basics". Awarded the Dr. Irwin Vigness Memorial Award in 1996 at the 42nd Institute of Environmental Sciences' National Conference in recognition of his outstanding technical guidance in the development of the IES Boston Chapter's Vibration Fixture Seminar Program.

Founder and President of Dynamic Decisions, a consulting company specializing in structural dynamic applications as well as technical training and technology transfer. Developed the multimedia format Modal Handbook on CD (a computer based training/reference guide which addresses the practical aspects of experimental modal testing).

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## **Dr. Etienne Balmes**

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Etienne Balmès, Habilitation à Diriger des Recherches (UPMC Paris-6 1997), Ph.D. (Aeronautics and Astronautics, MIT, 1993), MS (MIT, 1991), Ancien Elève de l'Ecole Polytechnique (Paris, 1989).

- 1997-present : Assistant professor Ecole Centrale Paris (Laboratory of the Mechanics of Soils Structures and Materials MSSMat). Teaching in experimental modal analysis, continuous and analytical mechanics. Sponsored research on test/analysis correlation and model updating, variability of the dynamic behaviour, prediction of the damped behaviour of viscoelastic structures.
- 1993-1997 : ONERA (Office National d'Etudes et de Recherche Aéronautique) with the ground vibration testing team doing tests for Aérospatiale, Eurocopter and small aircraft manufactures and research on force appropriation methods, identification methods, test/analysis correlation, parametrised reduced models for use in optimization and model updating. Technical supervision of the GARTEUR Structures and Materials Action Group 19 on ground vibration testing methods (a round Robin exercise on modal analysis methods).
- 1991-present : Developer of the Structural Dynamics Toolbox for MATLAB (<http://www.sdtools.com>) which combines experimental modal analysis, finite element modelling and test/analysis correlation within a single open package.

Research interest are experimental modal analysis, test/analysis correlation, modelling of damping properties, use of iterative reduction methods for the verification, design, or optimisation of structural dynamic properties.

## **Prof. Lawrence A. Bergman**

University of Illinois at Urbana-Champaign, USA

Lawrence A. Bergman received the B.S. in Mechanical Engineering from the Stevens Institute of Technology (1966), and the M.S. in Civil Engineering (1978) and Ph.D. in Engineering Mechanics (1980) from Case Western Reserve University. Before returning to graduate school, he was on the technical staff of TRW, Inc. and Lord Corporation. His research is primarily in the areas of structural dynamics and control, system identification and state estimation, applied stochastic processes, random vibration and structural reliability, and computational methods. He is the author of more than 90 articles and chapters in archival journals and books and 70 papers in conference proceedings, has edited or co-edited 5 volumes and holds 4 United States patents. He is an Associate Fellow of AIAA and a member of ASCE and ASME. He was the co-recipient of the State of the Art in Civil Engineering Award from ASCE (1983), was an associate editor of the ASME Journal of Vibration and Acoustics (1989-1997), is currently associate editor of the Shock and Vibration Digest (1997- ) and a member of the editorial board of the Journal of Vibration and Control (1996- ), and was the general chair of the 1997 biennial ASME Conference on Vibration and Noise in Sacramento. He has been at the University of Illinois at Urbana-Champaign since 1979, where he is a professor in the Aeronautical and Astronautical Engineering Dept., an affiliate of the Mechanical and Industrial Engineering Dept., and served as assistant dean of the college of engineering for the 1996-97 academic year. He has been principal investigator on a number of externally funded research projects over the past 20 years, primarily addressing topics in stochastic structural dynamics and reliability, vibration and control of distributed parameter systems, system identification, and civil infrastructural systems. These have been funded by various organizations, including NSF, NASA, US Army Corps of Engineers, Sandia National Labs, and Lockheed Missiles and Space Co.

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## **Prof. David Brown**

**University of Cincinnati, USA**

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- 1961 to present University of Cincinnati
- 1961-66 Research Assistant Aerospace Engineering
  - Hypersonic flow and viscous boundary layer
  - General Electric Co – Hypersonic ramjet development
- 1966-69 Research Assistant – Mechanical Engineering Structural Dynamics Research Laboratory
  - Machine tool dynamics
- 1969-72 Research Associate - Structural Dynamics Research Laboratory
  - Digital signal processing, modal analysis, acoustics
- 1972-present Director – Structural Dynamics Research Laboratory
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## **Dr Izhak Bucher**

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Izhak Bucher, DSc., MSc, BSc (Mechanical Engineering, 1993), Currently, a faculty member in the Mechanical Engineering Department, Technion ISRAEL. Previously, Research Associate at Imperial College, participating in a European community funded project (MARS) (1993-1996) dealing with dynamics of rotating machinery.

- Experimental Structural Dynamics
- Model identification
- Fault detection using a probing excitation
- Improving the quality of machining processes using high-frequency travelling waves
- Modelling and identification of Micro electromechanical
- Dynamics of rotating structures
- Modal testing and identification of shafts, discs, foundation fault (cracks) detection
- Active control (magnetic bearings) of shafts and discs, non-contacting measurement and signal processing in frequency/speed/space (scanning laser)
- Mechatronics
- Microprocessor - based product design.

## **Dr. Bernd Caesar**

**Daimler Chrysler Aerospace, Germany**

Study of Civil Engineering at the Technische Hochschule Darmstadt (Technical University of Darmstadt, Germany) with emphasis on Light Weight Structures. Since 1973 working for Dornier (Daimler Chrysler Aerospace) in the Space Business. Development of structural dynamic computer programmes, various studies for the German and European Space Agency e.g. "Updating Dynamic Mathematical Models on the Basis of Vibration Test Data", "Derivation of Spacecraft and Components Flight Loads", "Clamp Band Dynamics", "Development of a Force Measurement Device for Spacecraft (S/C) in Vibration Tests". Mechanical and System Engineering for S/C, S/C Subsystems and Space Station MicroVibration Payloads (Scientific S/C: GEOS measuring magnetosphere and sun wind phenomena, GIOTTO for flyby of Halley comet, ROSAT for deep space x-ray source images, CLUSTER, ENVISAT and others. Instruments and Payloads: CRISTA high atmosphere gas detector; Antenna subsystems for communication S/C and others). Since 1987 Head of the Structural Mechanics Department at Dornier Satellitensysteme GmbH, a department with specialists in acoustic-structure interaction, multibody dynamics, shock analysis, fibre reinforced plastics with emphasis on high stability structures, adaptive structures, fracture mechanics, mechanical testing.

## **Dr Scott Cogan**

**University of Franche-Comté, France**

Scott COGAN, Ph.D. (Mechanical Engineering Sciences, University of Franche-Comté, 1990), MS, BS (Mechanical Engineering, University of Michigan, 1985,1984). Currently, Senior Research Fellow, CNRS, at the Raymond Chaleat Applied Mechanics Laboratory, University of Franche-Comté in Besançon, France.

### **Research Interests**

- Structural Dynamic Analysis
- Model-based optimal test design
- Correlation of experimental data with analytical models
- Simplification, condensation and reconstitution of large order models
- Prediction of modified structural behavior based on measured data
- Model Usability
- Characterizing the implications of uncertainties in test and analysis
- Evaluating the robustness of model-based decisions to ambient uncertainties in model parameters and excitation space
- Microdynamics
- Application of MEMS to control the submicronic quasi-static behavior at mechanical interfaces
- Development of Engineering Software Tools
- Co-inventor of PROTO-Dynamique – a MATLAB based dynamic analysis platform for optimal test preparation, modal analysis, model correlation and updating.

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## **Dr. Jonathan Cooper**

**University of Manchester, UK**

Jonathan E. Cooper BSc (Univ London, 1983) PhD (Univ London, 1988)

Currently Senior Lecturer in Aerospace Engineering - University of Manchester (since 1997). Head of Dynamics and Aeroelasticity Research Group. Previously Lecturer (1989 - 1997), Senior Research Fellow - Royal Aerospace Establishment, Farnborough (1986 - 1989). Visiting Professor - KU Leuven, Belgium 1995. Royal Academy of Engineering Industrial Secondment - British Aerospace Military Aircraft and Aerostructures 1998. Member of Royal Aeronautical Society and C.Eng(1992), Member of American Institute of Aeronautics and Astronautics (1995). Author of over 70 technical papers. Research interests include: system identification (phase separation and phase resonance methods, output only methods, on-line identification, non-linear identification), aeroelasticity (modelling and prediction of non-linear aeroelastic and aeroservoelastic behaviour) and optimisation of aerospace structures (traditional and evolutionary methods).

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## **Dr. Patricia Davies**

**Purdue University, USA**

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### **EDUCATION AND EMPLOYMENT**

Ph.D. 1985 Sound and Vibration, University of Southampton, Britain.

M.Sc. 1981 Sound and Vibration, University of Southampton, Britain.

B.Sc 1977 Mathematics, University of Bristol, Britain.

1987- School of Mechanical Engineering, Purdue University, West Lafayette, Indiana. Associate Professor since 1994.

1981-1986 Institute of Sound and Vibration Research, University of Southampton, Britain. Research Assistant

(1981-1984), Research Fellow (1984-1986).

1980-1981 London Research Station, British Gas Corporation, London. Scientific Officer II.

1977-1979 Jada Teachers College, Jada, Gongola State, Nigeria. (Voluntary service.)

### **TEACHING EXPERIENCE**

Class Teaching:

Purdue: Dynamics, Statics, Signal Processing, Digital Control, Measurements, EPICS.

Southampton: Signal Processing, Fortran Programming. Nigeria: Mathematics (High School)

Short Courses:

Signal Processing: 2 days (Caterpillar: Sept. 1997; Purdue: July 1990, June 1988); 5 days (Purdue: 6x;

Southampton: 7x). Sound

Quality: 1 day (Bruel and Kjaer: 3x).

### **SOCIETY ACTIVITY**

Member of the: Institute of Noise Control Engineering, Institute of Electrical and Electronic Engineering, Acoustical Society of America, and American Society for Engineering Education.

Chair for the Engineering Acoustics and Vibrations Committee of the ASEE, 1994-1996.

Co-Chair of ANSI subcommittee on Sound Quality, ANSI S12/WG36, Summer 1995-.

## **Dr. Scott W. Doebling**

**Los Alamos National Laboratory, USA**

Scott W. Doebling, Ph.D. Ph.D. (Aerospace Engineering Sciences, University of Colorado, 1995), MS, BS (Aeronautical and Astronautical Engineering, Purdue University, 1991, 1989). Currently, Technical Staff Member at Los Alamos National Laboratory since 1996; Previously, Postdoctoral Research Associate at Los Alamos National Laboratory (1995-1996). Also employed at Sandia National Laboratories (1993), The Boeing Company (1990), and General Electric Aircraft Engines (1989). Member of AIAA, SEM, Associate Member of ASME. Member of ASCE Committee on Structural Identification of Constructed Facilities (1996-1999). Co-author and presenter of short courses on Vibration-Based Damage Identification (1997-1999). Co-founder and Vice President of Los Alamos Dynamics, a structural dynamics consulting company (not affiliated with Los Alamos National Laboratory).

Research Interests include:

- Experimental Structural Dynamics
  - Vibration-based damage identification in civil and mechanical structures
  - Statistical identification of modal parameters from experimental data
  - Field vibration testing of highway bridge structures
- Analysis of Structures and Dynamic Systems
  - Correlation of experimental data with analytical models
  - Modeling and simulation of rigid and flexible mechanical systems
  - Modeling of flexible satellite mast deployment
- Development of Engineering Software
  - Statistical refinement of finite element model parameters
  - DIAMOND – a graphical interface software program for:
    - Graphical presentation of vibration-based damage identification results
    - Statistically based modal parameter estimation
    - Implementation of finite element model correlation and refinement algorithms

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## **Prof. David J. Ewins**

**Imperial College, UK**

David J. Ewins F.Eng, DSc(Eng) (Univ. London 1983); PhD (Univ Cambridge, 1967); BSc(Eng) Univ London 1963). Currently, Professor of Vibration Engineering at Imperial College, London since 1983; also, Head of Dynamics Section, Mechanical Engineering Dept., Director of Centre of Vibration Engineering, and Director of the Rolls-Royce University Technology Centre (UTC) for Vibration at Imperial College. Previously, Reader (1977-83) and Lecturer (1967-77) at Imperial College. Honorary Professor, Nanjing Aeronautical Institute, and Shandong Polytechnic Institute, China. Has spent periods as Visiting Professor at: Nanyang Technological University, Singapore (1994, 1997); Institut National Polytechnique de Grenoble (INPG), France (1990); ETH, Zurich, Switzerland (1986); Virginia Polytechnic and State University, USA, (1981) and as Maitre de Conferences Associes, INSA, Lyon, France (1974-75) and Senior Lecturer, Chulalongkorn University, Bangkok, Thailand (1968-69). Elected to the Royal Academy of Engineers in 1995; Fellow of the Institution of Mechanical Engineers, UK (1990); Member of ASME (1982), SEM (1987). Recipient of Society of Experimental Mechanics D J DeMichele Award, 1993. Founding Chairman of the Dynamic Testing Agency (UK, 1990). Author of "Modal Testing: Theory & Practice" first edition, 1984; 9th reprint 1996; Co- editor of "Encyclopedia of Vibration" (to appear, 2000); contributor to 3 books; author and co-author of over 200 technical papers on vibration-related topics. Editorial Board Member of: Mechanical Systems and Signal Processing; Journal of Mechanical Engineering Science (IMEchE); Inverse Problems in Engineering; Shock and Vibration Digest Founder and organiser of series of International Conferences on Structural Dynamic Modelling (UK - 1993; UK - 1996, UK - 2000). Partner, ICATS (founded 1989). Author and principal presenter of over 100 short courses on Modal Testing in 20 countries. Research Interests cover a wide range of Structural Dynamics topics and focus on (i) Modal Testing and (ii) Rotating Machinery dynamics. Within these areas, special interests relate to: virtual testing and test planning; novel measurement technologies, including the laser vibrometers and testing of rotating structures and non-linear structures; application of modal test data to theoretical model validation, structural optimisation, structural assembly analysis and damage detection on a wide range of different structures, from offshore to automotive to aerospace; also, on blade and disc vibration characteristics, rotor-stator interactions with reference to various application areas - turbomachine rotors, brakes discs, computer discs, etc. Current research projects are sponsored by EU, Rolls-Royce, Bosch, Fiat, Air Products, and other companies. Current collaborative projects exist with TU Darmstadt; ETH, Zurich, Universities of Ancona and Trieste (Italy), Gothenburg (Sweden), Lisbon (Portugal). Supervisor of over 30 PhD graduates.

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## **Dr. Charles R. Farrar**

Los Alamos National Laboratory, USA

Charles (Chuck) R. Farrar, Ph.D., P.E., Ph.D., MS (Civil Engineering, University of New Mexico, 1988, 1982), BS, (Civil Engineering, Michigan Technological University, 1979). Currently, Material Behavior Team Leader in the Engineering Analysis Group at Los Alamos National Laboratory since 1997. Previously, Engineering Mechanics Team Leader (1992 -1995), Technical Staff Member (1983-1992, 1995-1996). Also employed as Adjunct Professor, Depts. of Mechanical and Civil Engineering, University of New Mexico (1989-present); Engineer in the Nuclear Engineering Department at Norfolk Naval Shipyard (1979-1981). Co-founder and President of Los Alamos Dynamics, a structural dynamics consulting company (not affiliated with Los Alamos National Laboratory). Co-author and presenter of short courses on Vibration-Based Damage Identification (1997-1999). New Mexico registered professional engineer (1985-present). Member of the American Society of Civil Engineers (1980-present), ASCE Committee on Structural Identification of Constructed Facilities (1996-present), ASCE Dynamic Analysis Subcommittee of ASCE Nuclear Standards Committee (1987-1995), ASCE Technical Committee on Lifeline Earthquake Engineering Post-Earthquake Investigations Committee (1989-present, Chairman, 1993-1996). Member of the Society of Experimental Mechanics (1988-present), Earthquake Engineering Research Institute (1986-1996).

### Research Interests:

- Vibration-based damage detection
- Scale-modeling and similitude of dynamic systems
- Earthquake engineering
- Applications of statistical pattern recognition to dynamics problems
- Environmental testing
- Field testing large structural systems

### Past Projects:

- Nonlinear analysis of elevated temperature piping
- Seismic response of nuclear power plant structures using scale models
- Seismic buckling analysis of reactor containment structures
- I-40 Bridge experimental/analytical damage detection study
- Applications of statistical pattern recognition to damage detection in weapon systems
- Random vibration analyses of laser fusion mirrors

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## **Dr. Michael I. Friswell**

**University of Wales-Swansea, UK**

Michael I. Friswell. PhD (Vibration Engineering, Aston University 1991), MSc (Mathematics, Open University, 1988), BA (Mathematics, Oxford University, 1982, MA, 1987). Currently Reader in Mechanical Engineering, University of Wales Swansea, UK. Previously, Senior Lecturer (1995-1998) and Lecturer (1993-1995), University of Wales Swansea; Lecturer, Aston University (1987-1993); Scientific Officer / Higher Scientific Officer, Admiralty Research Establishment Portland (now part of DERA) (1982-1987). Awarded Engineering and Physical Sciences Research Council Advanced Fellowship (1996-2001). Fellow, The Institute of Mathematics and its Applications (1990); Member, Institute of Physics (1993); Member, American Society of Mechanical Engineers (1995); Member, Society for Experimental Mechanics (1990). Associate Editor, ASME Journal of Vibration and Acoustics (1999-2001). Member of the Editorial Boards of: Journal of Vibration and Control (since 1995); Inverse Problems in Engineering (since 1996); Sports Engineering (since 1998). First author of 'Finite Element Model Updating in Structural Dynamics' (with John Mottershead, 1995). Organiser of conferences on 'Identification in Engineering Systems' (1996, 1999). Research Interests cover a wide range in structure dynamics, control and smart structures: model updating (particularly parameterisation and regularisation), calculation of eigensystem sensitivities, structural health monitoring, fault diagnosis, model reduction, squeal in brake disks, rotordynamics (particularly foundation modelling and identification), damped structures, vibration control in smart structures, modelling structures with viscoelastic components, geometric algebra in engineering dynamics.

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## **Prof. Lothar Gaul**

University of Stuttgart, Germany

Lothar Gaul, Dr.-Ing. habil. 1980, Dr.-Ing. 1976, M Eng 1973 (University of Hanover), Degree in Welding Engg. 1969 (SLV Berlin), Degree in Mech. Engg 1969 (FHS Wilhelmshaven), is Full Professor of Mechanics at University of Stuttgart since 1993; also, Director Institute A of Mechanics, Techn. Cybernetics Dept., Director of CAE + CAT Consulting Prof. L. Gaul, Visiting Professor at Florida Atlantic University.

Previously, Chief Engr and Lecturer, 1973 – 1981 at University of Hanover, Full Professor and Head Institute of Mechanics Univ. Federal Armed Forces Hamburg, 1981 – 1993, Dean of Engg. 1991 – 1993, Offer 1st Chair of Mechanics, Ruhr Univ. Bochum 1985, Offer Chair A of Mechanics and Material Testing Lab. Techn. Univ. Munich 1996, elected Reviewer German Research Society (DFG) since 1996, Member German Engg. Society (VDI) Board Vibration Engineering since 1990, Member VDI, DVS, IABM, ISBE, SEM, GAMM (Vice Secr. 1995), Recipient Hon. Ring German Engg. Society 1985. Author of "Methode der Randelemente in Statik und Dynamik" with C. Fiedler 1997; contributor to 3 books; author and co-author of 200 technical papers on continuum mechanics, waves and vibrations and discretization methods. Editorial Board Member of: Mechanical Systems and Signal Processing; Boundary Element Communications; Engineering Analysis with Boundary Elements.

Patentee seal with polyon profile, active joint.

Author and presenter of short courses on Boundary Elements and Finite Elements.

Principal investigator in DFG collaborative research projects on Adaptive Aerospace and Lightweight Structures, Influence of Ultrasound on Metal Forming Processes, Multifield Problems in Continuum Mechanics, Computer Aided Modelling and Simulation of Processes in Chemical Engineering. Current research projects are sponsored by DFG, Fraunhofer Society, Bosch, Daimler Chrysler, Voith and other institutions.

Research interests cover a wide range of topics in Numerical Simulation and Computer-Aided Testing (CAT) of Mechanical and Adaptive Structures. Within these areas, special interests relate to: development of Boundary Element and Finite Element Formulations for Fluid-Structure Interaction, Soil-Structure Interaction, Machine Dynamics, Contact Dynamics of Structural Joints and Brakes, Piezoelectrics; implementation of Generalized Damping Description and Uncertain Parameters by Fuzzy Arithmetic;

CAT with novel measurement technologies, including 3-D Electronic Speckle Pattern Interferometry (CW-ESPI, Pulse-ESPI), Non-contact Measurement for Modal Testing, Wave Propagation Techniques for Structural Health Monitoring.

Research in Adaptive Structures includes: Damping Control by Active Joints, Active Vibration Isolation, Active Structural Acoustic Control.

Supervisor of more than 20 DEng Graduates and 5 Habilitation Projects.

## **Prof. J.K. Hammond**

**University of Southampton, UK**

Professor J.K. Hammond is the Director of the Institute of Sound and Vibration Research, University of Southampton, and Professor of Signal Analysis. He has a B.Sc. in Aeronautical and Astronautical Engineering and a Ph.D., both from the University of Southampton. His research interests are in the areas of applied digital signal processing and the theory and application of time series analysis, with particular application to problems in acoustics and dynamics. He has supervised 20 M.Sc. project students and 25 Ph.D. students to successful completion and has published 48 papers in refereed journals and 106 papers in conference proceedings. He lectures regularly on specialist courses on signal processing in the UK, USA, France and Korea. He is on the Editorial Boards of Mechanical Systems and Signal Processing and Acta Acoustica.

## **Dr. François M. Hemez**

**Los Alamos National Laboratory, USA**

Ph.D. (1993), University of Colorado at Boulder; MS (1991), University of Colorado at Boulder; Graduate from Ecole Centrale Paris, France (1989).

1998-present, Technical Staff Member, ESA-EA, Los Alamos National Laboratory:

- Engineering work in structural design, structural analysis of nonlinear, dynamic systems. Applied research work in Bayesian parameter estimation, inverse problem solving for time-domain, transient, nonlinear systems. Test-analysis reconciliation and finite element model updating. Development of engineering software for nonlinear finite element modeling and updating. Development of massively parallel inverse solvers for linear dynamics.

1994-1997, Research Associate of the French National Center for Scientific Research (CNRS) at Ecole Centrale Paris, France:

- Teaching courses in analytical and experimental structural dynamics, applied mathematics for mechanical engineering. Applied research in the areas of structural and topology optimization, experimental modal analysis, test-analysis reconciliation and finite element model updating. Advising of four-month research projects in test-analysis correlation and co-advising Ph.D. students in model updating and structural optimization.

### **Work Interests:**

- Structural design, analysis and computational dynamics; Test-analysis reconciliation, finite element model updating; Statistical, direct and inverse finite element analysis; Nonlinear dynamics; Topology and structural optimization; Numerical analysis and engineering software.

### **Contact Information:**

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## Mr. Norman F. Hunter

Los Alamos National Laboratory, USA

MS (Electrical Engineering, University of New Mexico, 1968), BS, (Electrical Engineering, Mississippi State University, 1964).

Currently, Staff Member in the Measurement Technology Group, Engineering Sciences Division, Los Alamos National Laboratory. Previously, Environmental Testing Team Leader (1983 –1990), Technical Staff Member at Sandia National Laboratories (1964-1975). Co-author and presenter of short courses on Nonlinear Random Vibrations, Neural Networks, Bootstrap Statistical Techniques, and Chaotic Systems. Background in vibration testing, analysis of nonlinear dynamic systems, vibration data analysis.

### Research Interests:

- Nonlinear vibration analysis and characterization of nonlinear dynamic systems.
- Vibration-based damage detection
- Data Analysis Algorithms, especially as applied to vibration or modal test data.
- Applications of statistical methods to quantifying uncertainty in dynamic measurements
- Environmental testing

### Past Projects:

- Development of vibration control codes for random, sine, and shock testing using vibration machines.
- Early work on multi-shaker control.
- Analysis of Vibration data from numerous linear and non-linear test items.

### Contact Information

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## **Dr. Mehmet Imregun**

**Imperial College, UK**

MSc (1980) and PhD (1984) from Imperial College. Joined academic staff in 1987 after 3 years of postdoctoral research. Reader in Structural Dynamics & Aeroelasticity since 1996. Research interests include turbomachinery unsteady flows and aeroelasticity, finite element model updating and non-linear modal analysis.

Author of about 100 technical publications

## **Prof. Daniel J. Inman**

**Virginia Polytechnic Institute and State University, USA**

Daniel J. Inman Ph.D., Michigan State University (1980, Ph.D. Mechanical Engineering, 1975, MAT Physics, 1970 BS Physics) is the Goodson Endowed Professor of Mechanical Engineering at Virginia Tech. He is director of the Center for Intelligent Material Systems and Structures. A former Department Head at the State University of New York, Buffalo he has held adjunct positions in the Division of Applied Math at Brown University and in math at the University of Southern California. Since 1980, has published five books (on vibration, control, statics and dynamics) seven book chapters, over 90 journal papers, 185 proceedings papers, graduated 27 Ph.D. students and supervised over 50 MS degrees. He is a Fellow of ASME, a Fellow of the American Academy of Mechanics, an Associate Fellow AIAA, and an NSF President Young Investigator (1994-1995). He serves as Technical Editor of ASME Journal of Vibration and Acoustics (1990-2000), Associate Editor (1986-89), Technical Editor of the Shock and Vibration Digest (1998- 2001), Associate Editor of ASME Journal of Applied Mechanics (1988-94), Mechanics of Machines and Structures (1986-94), International Journal of Analytical and Experimental Modal Analysis (1986-1990), Smart Materials and Structures (1991-1999), Journal of Intelligent Material Systems and Structures (1992-2000)) and has given numerous short courses and invited lectures in the smart structures and control area for both ASME and SPIE including an ASME Satellite Broadcast short course on fundamentals of smart structures. Since 1980, has directed or been co principal investigator on a variety of external research projects through Virginia Tech, the University at Buffalo, Brown University and the University of Southern California. His projects have dealt with all aspects of vibration and control; theory, measurement and testing, as well as data acquisition and smart structures. These projects have been funded by a variety of industries (UTRC, Kistler, General Motors, Firestone, Moog, Atlantic Research, Harris, Cincinnati Milacron) and government agencies (NSF, AFOSR, ARO, ONR and NASA). He is a member of SIAM, AIAA, and on the Board of Directors of the Society of Engineering Science. He holds a patent in smart structures on self-sensing actuation. His research interest are in vibration of machines and structures, vibration testing including modal testing and parameter estimation, model updating of finite elements, damping models for FEM, computational vibration problems, vibration suppression of structures (both active and passive), continuum models of damping, and smart structures. He is a founding member of the ASME Adaptive Structures Technical Committee.

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## **Prof. Anne S. Kiremidjian**

**Stanford University, USA**

Anne S. Kiremidjian is Professor of Civil and Environmental Engineering and Director of the John A. Blume Earthquake Engineering Center at Stanford University. She received her B.S. degree from Columbia University in Civil Engineering and her M.S. and Ph. D. degrees from Stanford University in Structural Engineering. Professor Kiremidjian has been on the faculty at Stanford since 1978 where she teaches courses in structural analysis, earthquake engineering, probabilistic methods and structural reliability analysis. Her research has focused in the area of stochastic modeling of earthquake events, site hazard characterization, ground motion modeling, earthquake damage and loss estimation, structural damage modeling, risk analysis of transportation systems and reliability analysis of industrial systems. Currently she is working on the development of distributed remote sensing systems for structural damage monitoring using imbedded sensors and wireless communication. Another major research topic is the development of methods for socio-economic consequence analysis from natural disasters using geographic information systems. She has more than 100 papers and reports published on these topics.

During her career, Professor Kiremidjian has been active as a member of the Probabilistic Methods Committee of the Engineering Mechanics Division of ASCE; the Seismic Risk Committee and the Research Committee of EERI, the Committee on Building Instrumentation, CSMIP of the California Seismic Safety Commission; the Committee on Stochastic Methods in Structural Engineering, IASSAR; the Advisory Committee to the Biological and Critical Systems Division of NSF; the Executive Committee of Technical Council on Lifeline Earthquake Engineering of ASCE for which she served as a chair in 1995-1996; Board of Directors, CUREe, for which she served as a Treasurer and Secretary; the New York State Committee on Low Level Nuclear Waste Management, NAS, the Scientific Advisory Committee of NCEER; and the Research Board of the Pacific Earthquake Engineering Center. She has received the School of Engineering Distinguished Advisor Award, Stanford University, June 1989, the National Science Foundation Faculty Award for Women, 1991-1995, the Society of Women Engineers Distinguished Educator Award, 1992, the American Society of Civil Engineer, and the Technical Council on Lifeline Earthquake Engineering Distinguished Service Award, August 11, 1995. She was one of the founders of K2 Technologies, Inc. and served as its Chairman of the Board. She is also an advisor and consultant to other private companies and corporations.

## **Dr. Nick Lieven**

**University of Bristol, UK**

MEng Institute of Sound and Vibration Research (1986), Southampton, PhD Imperial College (1990).

Research Assistant at Imperial College (1986-91), Lecturer in Dynamics (1991-99) and Reader in Dynamics at Bristol University (1999-).

Royal Academy of Engineering Fellowship to British Aerospace Airbus (1992), Principal Editor of the Royal Society's Philosophical Transactions Special Issue on Modal Analysis (1998), Royal Academy of Engineering Stirling Lecturer (1998) British Association for the Advancement of Science Dorothy Hodgkin Lecturer (1998).

Research interests: FE model updating and Damage Detection, Non-contacting Excitation and Measurement of Structures, Dynamic Behaviour of Stressed Structures, Neural Networks and Optimisation methods in Dynamics. Author of ~50 research publications.

## **Prof. Michael Link**

**University of Kassel, Germany**

### **Education:**

Diploma in civil engineering, Technical University of Darmstadt, Germany (1967), Research assistant, Lightweight Structures Institute, Faculty of Mechanical Engineering, Technical University of Darmstadt (1968-1972), Dr.-Ing. Dissertation in the field of hybrid finite elements in structural stability, Technical University of Darmstadt (1972), Structural analyst at DORNIER aerospace company, Friedrichshafen (1972), Head of department Structural Analysis & Test (1976), responsible among others : European Launcher ARIANE 2nd stage tank structure, SPACELAB life support system, GEOS satellite, development of modal identification techniques.

Professor of Lightweight Structures and Structural Mechanics, University of Kassel (1980-present), Lectures on Finite element methods, structural dynamics, (post graduate), mechanics (under graduate), design of lightweight structures.

### **Research Specialization:**

Analytical and experimental structural dynamics ( computational model updating, experimental modal analysis: development of ISSPA code ), Finite element methods ( discretisation errors ).

Current research interests: non- linear system parameter and force identification

Consultant for aerospace (aero-engines, helicopters), automotive and railway industry and structural dynamics issues in civil engineering with emphasis on model validation and experimental modal analysis.

### **Honors, Awards, Fellowships, Membership of Professional Societies:**

Ehrenmedaille ( honor medal) VDI (Verein Deutscher Ingenieure); Member: VDI, SEM ( Soc. of Experimental Mechanics), GAMM (Gesellschaft f. angewandte Mathematik und Mechanik), DGLR ( Deutsche Gesellschaft für Luft- und Raumfahrt).

Editorial board member: "Mechanical Systems and Signal Processing" and "Encyclopedia of Vibrations"

### **Publications:**

About 60 papers in referenced journals and proceedings of scientific meetings, one book on Finite Element Analysis and contribution to a series of monographs on fundamentals of structural engineering.

## **Prof. C. D. Mote, Jr.**

**University of Maryland, USA**

In September 1998, C. D. (Dan) Mote, Jr. began his tenure as President of the University of Maryland, College Park and Glenn L. Martin Professor of Engineering after serving on the University of California, Berkeley faculty for 31 years. From 1991 to 1998, he was Vice Chancellor - University Relations, President of the UC Berkeley Foundation, and FANUC Chair in Mechanical Systems. As Vice Chancellor he conceived and led a comprehensive capital campaign with a goal of 1.1 billion dollars. He continues an active research program from which fifteen Ph.D. students have completed their dissertations in the last five years.

He has also held positions at Carnegie Mellon University, The University of Birmingham (England), the Norwegian Institute for Wood Science and Technology (Oslo), the Technical University of Darmstadt, and the Tokyo Institute of Technology.

President Mote's technical interests lie in dynamic systems, vibration, and biomechanics. He is internationally recognized for his research on gyroscopic systems, including high-speed translating and rotating systems like saws, computer memory disks and tapes. His work on the biomechanics of skiing injury spans three decades. His research efforts have resulted in more than 300 publications, plus patents in the U.S., Norway, Finland, and Sweden.

The National Science Foundation, the Royal Norwegian Council for Scientific and Industrial Research and the Japanese Society for the Promotion of Science have awarded him their research fellowships. His research has been supported continuously by the NSF since 1962. And he has received numerous prestigious awards, including, the University of California, Berkeley's Distinguished Teaching Award; the Federal Republic of Germany's Humboldt Prize; and the American Society of Engineering Education's Ralph Coats Roe Award.

He is a member of the U.S. National Academy of Engineering, and a Fellow of the International Academy of Wood Science, the Acoustical Society of America, and the American Association for the Advancement of Science.

President Mote received the B.S., M.S. and Ph.D. in mechanical engineering from the University of California, Berkeley.

## **Prof. Roger Ohayon**

**Conservatoire National des Arts et Metiers (CNAM), France**

Doctorate (1971) and Habilitation à Diriger des Recherches (1990) from University Pierre et Marie Curie, Paris, is currently, since 1992, Professor, Chair of Mechanics, at the Conservatoire National des Arts et Metiers (CNAM) in Paris. He is the Director of the Structural Mechanics and Coupled Systems Laboratory. He is also a Scientific Advisor at ONERA. Before joining academia, he carried on his career at ONERA since 1970, Head of the Computational Structural Mechanics and Coupled Systems Division (1982-1991) and Scientific Deputy (1991-1992). He serves as Managing Editor of Computational Mechanics Journal, as Member of the Editorial Board of *Revue Européenne des Elements Finis*, *International Journal of Numerical Methods in Engineering*, *Computer Methods in Applied Mechanics and Engineering*, *Computers and Structures*, *Engineering Computations*, *Archives for Computational Mechanics*, *International Journal of Computational Engineering Science*, *Journal of Intelligent Material Structures and Systems*. He co-organized numerous international conferences, for instance for ASME and IACM (International Association for Computational Mechanics). He serves as President of the French of Computational Structural Mechanics Association (1987-1996). He serves as a Member of the Intelligent Materials and Structures Group of the French MOD (1996-1997). He serves in the Committee as representative of France in the World Congress on Computational Mechanics four year Series (the last WCCM was in Buenos-Aires in 1998). Presently, he is a co-organizer of a symposium on Computational Structural-Acoustics and Fluid-Structure Interaction as part of the Fifth U.S. National Congress on Computational Mechanics in 1999. He is the Chairman of the International Conference on Adaptive Systems and Technology (ICAST'99) in Paris, October 11-13, 1999. He has strong links with Industries and European Aerospace Research Centers, for instance he was chairman of a GARTEUR (Group for Research and Technology in EUROpe) Action Group on Updating Methods between Computations and Tests for Modal Analysis Vibrations Problems. As a result, the so-called Garteur Truss serves as a benchmark in most of the updating community. He serves now as a Member of the Group of Responsables of GARTEUR/Materials and Structures. His scientific interest concerns Mechanical and Computational Modeling for Vibrations of Fluid-Structure Systems, Structural Acoustics and Vibration (Vibroacoustics), Parametric Updating and Inverse Problems, Adaptive Systems through Hybrid Passive/Active Control of Sandwich Structures ( "Intelligent/Smart Structures") for structural and internal vibroacoustics problems. He got an award from the French Academy of Sciences for his overall works on Mechanical and Numerical Modeling of Coupled Fluid Structure Problems (1989). He is currently a member of the National Academy of Engineering of Brazil (elected in 1994). He is an ASME Member and IACM Fellow (1998). He is the co-author of 80 papers and co-editor of 6 books (5 in French plus J. Crolet and R. Ohayon, eds., *Computational Methods for Fluid-Structure Interaction*, Longman, 1994).

Books written : 1) H. Morand and R. Ohayon, *Interactions Fluides-Structures*, Coll. P.G. Ciarlet and J.L. Lions, Masson, 1992, 2) H. Morand and R. Ohayon, *Fluid-Structure Interaction*, Wiley, 1995, an english updated version, and 3) R. Ohayon and C. Soize, *Structural-Acoustics and Vibration*, Academic Press, 1998.

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## **Dr. Thomas L. Paez**

**Sandia National Laboratories, USA**

Thomas L. Paez, Ph.D, P.E., Ph.D. (Eng) (Purdue University, 1973), M.S. (Civil Engineering), B.S. (Civil Engineering), (The University of New Mexico, 1971), Currently, Distinguished Member of the Technical Staff, Experimental Structural Dynamics Department, Sandia National Laboratories, Albuquerque, New Mexico (1984-present). Previously, Associate Professor of Civil Engineering, the University of New Mexico, Albuquerque, New Mexico (1977-1984); Member of the Technical Staff, Applied Mechanics Division, Sandia National Laboratories (1975-1977); Staff Scientist, Kaman Sciences Corporation (1973-1975). Member ASCE, ASME. Co-author of "Random Vibrations: Theory and Practice," 1995; author or co-author of numerous papers on probabilistic structural dynamics, probabilistic dynamic testing, and probabilistic system analysis. Author and co-presenter of numerous short courses on Random Vibrations, Nonlinear Random Vibrations, System Simulation with Artificial Neural Networks, and Chaos.

### **Research Interests:**

- Probabilistic Structural Dynamics
- Probabilistic System Analysis
- Artificial Neural Networks
- Experimental System Modeling
- Genetic Algorithm/Genetic Programming
- Random Environment Test Specification

### **Contact Information:**

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## **Mr. Charles R. Pickrel**

**The Boeing Company, USA**

Education: B. S., Aeronautical and Astronautical Engineering, U. of Illinois, 1966.

Current: Mr. Pickrel is a Technical Fellow of the Boeing Company in Seattle Washington. He has technical responsibility for methods, systems and software used in structural dynamic testing in the Boeing Commercial Airplanes Group. He has been an engineer in the Structural Dynamics Laboratory since 1980, where he has been test conductor for modal testing on a wide variety of airplanes. He was architect of software for test data acquisition and management, and is principal investigator for the "X-Modal" modal analysis software (which is being developed at the University of Cincinnati). Recent interests include: Modal testing, modal parameter estimation, flutter testing and software.

1977-1979: Mr. Pickrel worked as an engineer performing design and testing of home-entertainment loudspeakers for Speakerlab, Inc. of Seattle, Washington.

1966-1977: As an aerodynamics staff engineer, Mr. Pickrel developed methods for estimating aerodynamic drag and performed aerodynamic design and wind-tunnel testing of wings and airfoil shapes.

## **Dr. Robert B. Randall**

**University of New South Wales, Australia**

Bob Randall is currently an Associate Professor in the School of Mechanical and Manufacturing Engineering at the University of New South Wales (UNSW), Sydney, Australia, where he is also Director of the DSTO Centre of Expertise in Vibration Analysis, conducting research into diagnostics of helicopter gearboxes. Prior to joining the University in 1988 he worked for the Danish company Bruel & Kjaer for 17 years where he was responsible for developing systems for machine condition monitoring and diagnostics. He previously had ten years' experience in the chemical and rubber industries in Australia, Canada and Sweden. He is the author and/or co-author of the Bruel & Kjaer books Frequency Analysis and Mechanical Vibration and Shock Measurements, and the invited author of chapters on vibration measurement and analysis in the McGraw-Hill handbooks Shock and Vibration Handbook and Handbook of Acoustical Measurements and Noise Control. He is the author of more than 40 papers in the fields of vibration analysis and machine diagnostics. He received a B.Tech in Mechanical Engineering (1961) and a BA in Pure Mathematics, Applied Mathematics and Swedish (1971) from Adelaide and Melbourne Universities, respectively.

While at UNSW he has supervised four completed PhD projects and is currently supervising a further two, as well as co-supervising a student from the University of Grenoble, France.

He has ongoing research collaboration with a number of universities, and has spent sabbatical periods at Chalmers University of Technology, Gothenburg, Sweden, CETIM (Centre Technique des Industries Mécaniques), Senlis, France, (under a French Government Scholarship), the Department of Mechanics and Aeronautics, University of Rome, the University of Manchester, UK, Katholieke Universiteit Leuven, Belgium, (under a University Fellowship), the University of Technology, Compiègne, France, and the University of St Etienne (at Roanne), France

## **Dr. Mark H. Richardson**

**Vibrant Technology, USA**

### **Education**

Mark Richardson received his B.S. (1964), M.S. (1966), and Ph.D. (1970), all in Mechanical Engineering from the University of Notre Dame, Notre Dame, Indiana. He was holder of a National Science Foundation Traineeship during the graduate school years 1964 -1966.

### **Work Experience**

Upon completion of his Ph.D., Mark joined Systems Control, Inc., Palo Alto, California, as a consulting engineer. At SCI, he worked on client problems in the areas of dynamics and control systems, operations research, and systems analysis.

In 1973, he joined Hewlett Packard Co. in Santa Clara, California. As an engineering project leader, he directed the development of the first commercially available FFT-based modal testing system, introduced by Hewlett Packard in 1974. Later, he directed the development of the first dedicated modal testing instrument, the Hewlett Packard 5423A Structural Dynamics Analyzer. The 5423A became one of HP's top revenue producing instruments during the 1979 to 1984 period.

In August 1979, he co-founded Structural Measurement Systems, Inc. and was President and CEO. SMS marketed mechanical testing software used for analyzing structural noise and vibration problems. By 1989, SMS had grown to \$3.0 million in annual sales, with over 30 employees and a worldwide sales force. A majority of the Fortune 1000 companies in the U.S. use SMS software. In May 1989, SMS was sold to GenRad, Inc., Concord, Massachusetts.

In October 1991, he co-founded Vibrant Technology, Inc., Jamestown, California, and is President and CEO. Vibrant Technology has developed the ME'scope<sup>®</sup> family of post-test noise and vibration analysis tools, which are sold worldwide through sales representatives and OEM suppliers.

### **Professional**

Mark has authored numerous technical papers and magazine articles on the subjects of modal analysis, structural testing, and digital signal processing. He has traveled worldwide conducting tests, giving seminars, and teaching short courses. He is a member of the honorary scholastic fraternities, Tau Beta Pi, Pi Tau Sigma, and Sigma Xi. He is also chairman of the Advisory Committee for the International Modal Analysis Conference, an annual conference of vibration testing practitioners.

## **Prof. Gerhard Schweitzer**

**ETH Zurich, Switzerland**

Gerhard Schweitzer has been working for the ETH Zurich since 1978. First, he was Professor of Mechanics, and since 1989 he has been Professor of Robotics. The emphasis of his current research work is on the field of mechatronics, especially interactive and contact-free magnetic bearings.

G. Schweitzer studied mechanical engineering, received his doctorate from the TH in Stuttgart, and his habilitation from the TU Munich in 1974. Before coming to the ETH, he was involved in research and teaching for 16 years at several institutions, among which were the German Aero-space Establishment DLR in Oberpfaffenhofen, the NASA Marshall Space-flight Center, Huntsville, Alabama, (inertial navigation and satellite dynamics), the TH Stuttgart, and the TU Munich (dynamics and control).

At the ETH he was one of the founders of the Mechatronic Group in 1984, the Neuro-Informatics Group in 1988, the International Center for Magnetic Bearings in 1991 and Nanorobotics in 1993. He was president of

the Association of ETH Professors and head of the Department of Mechanical and Industrial Engineering. He has published about 120 papers, a book on magnetic bearings, and edited several conference proceedings on dynamics, magnetic bearings, motion and vibration control. He is a member of the Swiss Academy of Technical Sciences.

Gerhard Schweitzer was visiting professor at the Department of Applied Mechanics of Stanford University, at the Universities of Campinas and Florianopolis, Brazil, and at the Center for Interdisciplinary Research (ZiF) at the University of Bielefeld.

## **Prof. Aldo Sestieri**

**University of Rome, Italy**

Aldo Sestieri, Laurea in Mechanical Engineering, University of Rome La Sapienza, 1970. Professor of Mechanical Vibrations and Noise and Vibration Control at the Department of Mechanics and Aeronautics, University of Rome La Sapienza since 1986. Director of undergraduate courses in Mechanical Engineering. Previously Director of the Ph.D. course in Theoretical and Applied Mechanics, University of Rome (1994-1998). He has been visiting professor at the University of New South Wales (Sydney, Australia), University of Montevideo (Uruguay) and University of Xi'an (China). Associate Editor of Meccanica, Editorial Board member of Mechanical Systems and Signal Processing. Member of SEM, AIMETA (Italy), Elected Director of IIAV (International Institute of Acoustics and Vibrations). He has given numerous short courses and invited lectures in the field of modal analysis and vibroacoustics and has directed several research projects at the University of Rome. His research interests include: modal analysis (vibration testing, structural modification, model updating of finite elements, structural coupling), vibroacoustics (integral formulations), high frequency vibrations (envelope models), inverse problems and ill-conditioning. In these fields he published over 100 papers as author and co-author, and have contributions in two books on modal analysis. Supervisor of 8 Ph.D. students and over 60 MS (laurea) degrees.

Aldo Sestieri

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## **Prof. Stephen Shaw**

**Michigan State University, USA**

Ph.D., Cornell University, Ithaca, NY, 1983, Theoretical and Applied Mechanics. M.S.E., University of Michigan, Ann Arbor, MI, 1979, Applied Mechanics. A.B., University of Michigan, Flint, MI, 1978, Physics.

### **Professional Experience**

- 1984-present (except 1991-93), Professor, Associate Professor, Assistant Professor, Department of Mechanical Engineering, Michigan State University, East Lansing, MI.
  - Acting Associate Chairperson, Jan. - Sept., 1990.
  - Graduate Advisor, 1996-98.
- 1991-93 Associate Professor, Department of Mechanical Engineering and Applied Mechanics, The University of Michigan, Ann Arbor, MI.
- 1983-1984 Assistant Professor, School of Engineering, Oakland University, Rochester, MI.
- Visiting Appointments: June 1983, Department of Theoretical and Applied Mechanics, Cornell University; July 1984, Department of Aerospace Engineering and Mechanics, University of Minnesota; Jan.-June, 1989, Applied Mechanics, Caltech.
- Consulting: 1984-present, providing analytical support for NVH (Noise, Vibration and Harshness) studies for the automotive and aircraft industries.

### **Research Areas**

Dynamics and vibrations with emphasis on nonlinear behavior. Applications to vibration absorbers, structural vibrations, impact problems, ship dynamics, and rotor dynamics.

### **Honors and Awards**

- Arch T. Colwell Merit Award, Society of Automotive Engineers, 1997.
- Keynote Lecturer, IUTAM Symposium on Nonlinear Dynamics and Chaos in Mechanics, Ithaca, NY, 1997.
- Fellow, ASME, elected 1995.
- Sethna Lecturer, Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, MN, 1994.
- Westinghouse Distinguished Lecturer, Department of Mechanical Engineering and Applied Mechanics, University of Michigan, Ann Arbor, MI, September, 1990.
- Henry Hess Award, ASME, 1986, for the best paper by an author under 31 years of age. Professional affiliations and awards

### **Research Accomplishments**

- Publication of over fifty journal papers in twelve different journals.
- Research program funded by NSF and other agencies continually since 1984.

## **Dr. Håvard I. Vold**

**Vold Solutionsd, USA**

- Dr. Håvard Vold is the president of Vold Solutions, Inc., a company providing services and software products in mechanical engineering, specializing in noise, vibration, rotating machinery analysis and robotics motion control.
- Dr. Vold received his doctorate in applied mathematics from the University of Oslo in Norway in 1974, where he was also a research associate at the Institute of Mathematics. He was also assistant professor in the Institute for Statics and Dynamics at the University of Stuttgart, Germany, where he was a primary developer of the ASKA finite element system. He then spent three years consulting in the North Sea offshore industry before he moved to the United States of America and joined SDRC in 1980 as a senior technical fellow and later was promoted to vice president in the Technical Development Division where he was the designer of the SDRC integrated finite element solver and the principal developer of algorithms for analytical and experimental methods in statics and dynamics. He left SDRC in 1992 to found Vold Solutions, Inc. Dr. Vold has been adjunct professor in the Structural Dynamics Research Laboratory at the University of Cincinnati since 1986.
- In the dynamics community, Dr. Vold is known for his contributions to the field of experimental dynamics, where among his inventions are the Polyreference method for modal parameter estimation, the multivariate mode indicator function, for multiphase stepped sine methods, the frequency response function estimator and the adaptation of Kalman filtering to rotating equipment analysis. He holds two US patents on motion controls in kinematically redundant robotic manipulators and has received NASA awards for his work in the robotics field. He was the recipient of the 1993 SAE Arch T. Colwell Merit Award for his SAE publication on the application of Kalman filters to order tracking in rapidly slewing systems. He also received the 1994 Institute of Environmental Sciences Maurice Simpson Technical Editors Award for his work on multiaxis vibration control. This year, Dr. Vold is the recipient of the 1997 SAE Arch T. Colwell Merit Award for his SAE publication of "The Time Variant Discrete Fourier Transform as an Order Tracking Method."
- Dr. Vold has been a keynote speaker at the International Modal Analysis Conference, and participates in the professional societies as a referee, associate editor of a journal and a member of several advisory boards. He is the author of more than a hundred technical publications.



## **Dr. Bingen Yang**

**University of Southern California, USA**

Department of Mechanical Engineering, OHE 430, University of Southern California, Los Angeles, CA 90089-1453

### **Education**

Ph. D. Mechanical Engineering University of California at Berkeley 1989

M.S. Applied Mechanics Michigan State University 1985

### **Honors and Awards**

Northrop Grumman Corporation Excellent Research Award, 1995

US Army Research Award, 1993

NSF Research Initiation Award, 1990

USC Faculty Research Innovation Award, 1990

Charles Lee Powell Research Award, 1989

### **Professional Experience**

1989 - 1995 Assistant Professor, University of Southern California

1995 - Present Associate Professor, University of Southern California

1997 - 1998 Vice Chairman, USC Mechanical Engineering Department

### **Professional Activities**

Professor Yang's research interest lies in the fields of dynamics, vibrations, structures, controls, and applied mechanics. He is the founding director of the Dynamic Systems Laboratory, which is one of the principal facilities for research in modeling, analysis, control and design of complex mechanical systems at the USC Mechanical Engineering Department. At the Lab, Professor Yang's group has undertaken various research projects sponsored by the National Science Foundation, the US Army Research Office, the Powell Foundation, Jet Propulsion Laboratory, Caltrans, Structural Research & Analysis Corporation, Tektronix, Ford Motor Company, General Motor Company, Hughes Aircraft Company, and the USC Research and Innovation Fund. Professor Yang's research has resulted in over 70 referred journal and conference papers.

Professor Yang is recognized for his work on the distributed transfer function method, a technique for modeling, analysis and control of complex distributed parameter systems; the eigenvalue inclusion principles for gyroscopic dynamic systems; a time-delay approach for non-colocated control of flexible structures; and modal controllability and observability of general mechanical systems. He has been listed in Who's Who in Science and Engineering and Who's Who in America. For his research accomplishments, Professor Yang received the 1995 Northrop Grumman Corporation Excellent Research Award.

Professor Yang is an associate editor of the ASME Journal of Vibration and Acoustics. He is a reviewer for many technical journals, including Journal of Applied Mechanics, AIAA Journal, IEEE Transactions on Automatic Control, Journal of Engineering Mechanics, and Journal of Sound and Vibration. He is a member of the ASME Design Engineering Technical Committee on Vibration and Sound. He organizes symposiums and chairs sessions for various technical conferences. Professor Yang served as the program chairman for the 16th ASME Biennial Conference on Vibration and Noise in 1997.

## **Prof. David C. Zimmerman**

**University of Houston, USA**

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Dr. David C. Zimmerman is currently an Associate Professor in the Department of Mechanical Engineering and Director of the Dynamic Systems and Controls Laboratory at the University of Houston. He received his Ph.D. in 1987 from the State University of New York at Buffalo, during which time he was a NASA Graduate Research Fellow. Prior to his appointment at the University of Houston, Dr. Zimmerman held positions at the University of Franche-Comte (Invited Professor), University of Florida (Associate/Assistant Professor), the State University of New York at Buffalo (research/teaching assistant) and Hooker Chemical (engineering aide). Dr. Zimmerman's research interests include the dynamics and control of structures, inverse problems, structural identification and health monitoring, design optimization, and experimental methods. He has supervised the programs of 24 graduate students, and has authored or co-authored over 100 journal articles, monograph chapters and/or conference publications. He is also the co-developer of a NASTRAN integrated software tool for performing large scale model validation and verification of structures. Dr. Zimmerman is currently on the Board of Directors of the SES, a member of the AIAA-Houston Guidance, Navigation & Control Technical Committee, a former member of the AIAA-National Structural Dynamics Technical Committee, and a member of ASME, SEM, Tau Beta Pi, and Pi Tau Sigma. In addition, he has been a reviewer for over twenty journals and government laboratories, and has been a Session Chairman/Conference Organizer for several meetings.